ISBN: 978-3-95450-208-0 doi:10.18429/JACoW-IPAC2019-TUPGW056

COMPARISON OF CONSTRAINED OPTIMIZATION METHODS FOR DESIGNING A MULTI-BEND ACHROMAT LATTICE

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Abstract

In the design of a multi-bend achromat (MBA) lattice for a diffraction-limited storage ring, there are usually many magnet parameters to be optimized and some stringent constraints to be satisfied. For example, to cancel out nonlinear dynamics effects, the phase advances between some sections are generally required to be set to certain values in the lattice design. For better designing a MBA lattice using an evolutionary algorithm, the handling of constraints will be important, because it is very hard to satisfy the constraints for most or even all of solutions in the early generations of the algorithm. This paper will first describe some methods for handling constraints, which are then applied to designing a hybrid 7BA lattice. The comparison of these methods shows that better lattice solutions can be obtained by including constraints into objective functions.

INTRODUCTION

Since the electron beam emittance of a storage ring light source scales inversely with the third power of the number of bends, multi-bend achromat (MBA) lattices have been adopted in designing diffraction-limited storage rings (DLSRs). Compared to the double-bend achromat (DBA) lattice used in most of the third-generation synchrotron sources, the MBA lattice is more complicated with many magnets employed. A lattice with an ultra-low emittance will generally face very serious nonlinear dynamics effects, and thus nonlinear cancellation schemes are usually required, where the phase advances between some sections are limited to certain values. So in the MBA lattice design of a DLSR, there are usually many parameters of magnets to be optimized and some stringent constraints to be satisfied.

Generally, the lattice design of a storage ring is a constrained multi-objective optimization problem (CMOP), which can be solved using evolutionary algorithms such as genetic algorithm [1] and particle swarm optimization (PSO) [2, 3]. In the design of a DBA lattice with an evolutionary algorithm, it is often in the first few generations that most of solutions satisfy the constraints. While for a MBA lattice, only some or even none of solutions can satisfy the constraints in the early generations, due to many decision variables and some stringent constraints. In the latter case, the handling of constraints in the algorithm will play an important role in searching for optimal solutions.

In this paper, four methods for handling constraints will be introduced and combined with multi-objective PSO (MOPSO). Then they are applied to designing a hybrid 7BA lattice [4] that has the same energy as the Hefei Advanced Light Source (HALS) [5], and a comparison between them is made.

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CONSTRAINT HANDLING METHODS

A CMOP can be defined as follows:

$$\begin{aligned} &\text{minimize: } F(x) = (f_1(x), f_2(x), \ldots, f_m(x))\,; \\ &\text{subject to: } g_j(x) \geq 0,\, j = 1, 2, \ldots, J; \\ &\qquad h_k(x) = 0,\, k = 1, 2, \ldots, K; \\ &\qquad x_i^L \leq x_i \leq x_i^U,\, i = 1, 2, \ldots, n; \end{aligned} \tag{1}$$

where m is the number of objective functions, n is the number of decision variables, J is the number of inequality constraints, K is the number of equality constraints, ${x_i}^L$ and ${x_i}^U$ are the lower and upper limits of the i-th decision variable.

If not considering the constraints, a solution x_i is said to dominate another solution x_j, if the following conditions hold:

- x_i is no worse than x_j in all objectives;
- x_i is strictly better than x_j in at least one objective.

To handle the constraints, the violation degree is introduced, defined as the sum of the squares of the constraint violation values:

$$H(x) = \sum_{j=1}^{J}[\min\{0, g_j(x)\}]^2 + \sum_{k=1}^{K}[h_k(x)]^2, \tag{2}$$

where all constraints are normalized before computing the constraint violations. Now we will present four constraint handling methods.

Method 1: Biasing Feasible over Infeasible Solutions (1)

The first method is to change Pareto dominance to constraint-Pareto dominance [6] by keeping the objective function unchanged. Assuming minimization, a solution x_i constrain-dominates another solution x_j if:

- x_i is feasible and x_j is infeasible;
- x_i and x_j are both infeasible, but x_i has a smaller constraint violation $H(x)$;
- x_i and x_j are both feasible and x_i Pareto-dominantes x_j.

Method 2: Biasing Feasible over Infeasible Solutions (2)

In the second method, a solution x_i is said to constraint-dominate another solution x_j if [7]:

- x_i is feasible and x_j is infeasible;

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- x_i and x_j are both infeasible, but x_i is no worse than x_j in all constraints and x_i is strictly better than x_j in at least one constraint;
- x_i and x_j are both feasible and x_i Pareto-dominantes x_j.

This method is very similar to the first method, except in comparing infeasible solutions.

Method 3: Adding Constraints to Each Objective

The third method is a penalty function method, where the penalty function, i.e. the constraint violation $H(x)$, is added to each objective function, forming a new objective function $G(x, \mu)$:

$$G(x, \mu) = F(x) + \mu H(x), \tag{3}$$

with μ being the penalty coefficient. The performance of this method depends largely on the setting of the penalty coefficient, and in practice the value of the penalty coefficient is difficult to grasp. In this paper, we use a strictly incremental positive sequence $\{\mu_k\}$ with a small initial value.

Method 4: Treating Constraints as an Objective

The last method is to treat the constraint violation $H(x)$ as an additional objective. Therefore, a constrained problem with m objective functions, J inequality constraints and K equality constraints becomes an unconstrained problem with $m + 1$ objective functions.

COMPARISON OF METHODS

We will combine the four constraint handling methods with MOPSO, and use them to design a DLSR, composed of 24 identical hybrid-7BA lattice cells. The designed DLSR has an energy of 2.4 GeV, the same as HALS, and a circumference of 576 m. In the lattice design, there are more than 20 decision variables, including dipole and quadrupole field strengths of various magnet elements and their positions. The optimization objectives are the natural emittance and the sum of the integral strengths of three families of sextupoles, $|I_{sum}| = |I_{SD1}| + |I_{SF}| + |I_{SD2}|$. To calculate the integral strengths, the sextupoles are first treated as a combination of (SF, SD1) and (SF, SD2). The defocusing sextupole SD1 is close to the first bend and SD2 close to the second. Then the pair (SF, SD1) is assumed to contribute 2/3 of chromaticity correction, and (SF, SD2) contributes 1/3, because we found that a larger contribution from (SF, SD1) could give a better nonlinear dynamics performance. The constraints are listed as follows:

- the transverse tunes: (57.0, 20.2)± 0.2;
- the phase advances between two dispersion bumps $(\Delta\mu_x, \Delta\mu_y)/2\pi : (1.5, 0.5) \pm 0.01$;
- the maximum beta functions < 25 m;
- the dispersion function at the long straight section > -0.005 m.

The second constraint is to make a -*I* transformation between the dispersion bumps, so that most of nonlinear effects caused by the sextupoles can be cancelled out.

For each method, a MOPSO algorithm with a population size of 5000 was run for 100 generations. The optimized Pareto fronts with the four methods are shown in Fig. 1. We can see that the Pareto fronts of the third and fourth methods present both better values and better distributions of objective functions than the first and second methods. Fig. 2 shows the iterative processes of these optimizations. It can be seen that the third method iterates faster than the other three methods, and the third and fourth methods have shown better distributions of objective functions in early generations. So in this lattice design, the methods based on including constraints into objective functions are better than those based on biasing feasible over infeasible solutions.

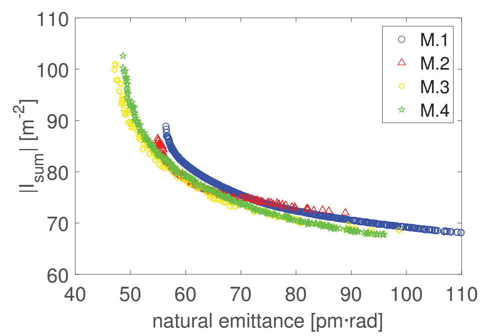

Figure 1: Comparison of the Pareto fronts optimized by the four methods.

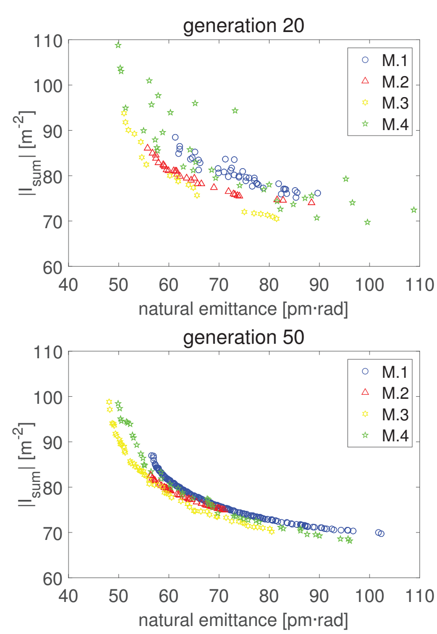

Figure 2: Comparison of the objective functions of solutions for the four methods at the 20th and 50th generations.

In order to better compare the four constraint handling methods in different cases of constraints, we also designed the lattice with only two relaxed constraints:

- the phase advances $(\Delta\mu_x, \Delta\mu_y)/2\pi : (1.5, 0.5) \pm 0.2$;
- the maximum beta functions < 40 m.

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The decision variables and objective functions are the same as those in the previous design. The optimized Pareto fronts are shown in Fig. 3. It is obvious that the four methods can obtain almost the same results when the constraints are very relaxed. By comparing the two designs, it can be known that when the constraints become stringent, the performance of the third and fourth method will be better than the other two methods.

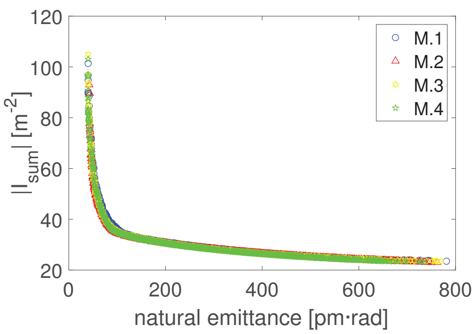

Figure 3: Pareto fronts optimized by the four methods in the case with relaxed constraints.

One lattice was selected from the Pareto front obtained with the third method as shown in Fig. 1, which has a natural emittance of 65 pm·rad. The linear optical functions and magnet layout of the lattice are shown in Fig. 4. Three families of sextupoles and one family of octupole were used in the nonlinear optimization. The optimized dynamic aperture (DA) and tune shifts with momentum are shown in Fig. 5 and Fig. 6. We can see that the DA and dynamic momentum aperture at the long straight section are large.

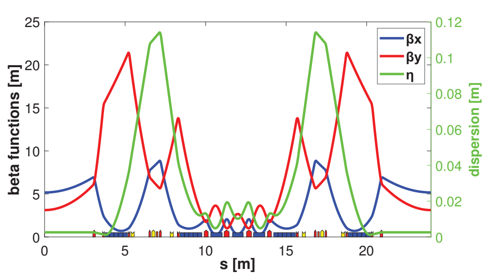

Figure 4: Linear optical functions and magnet layout of the designed hybrid 7BA lattice.

CONCLUSION

For better designing MBA lattices for DLSRs using evolutionary algorithms, four constraint handling methods have been studied. The four methods were combined with MOPSO and applied to designing a hybrid 7BA lattice. It was shown that for the lattice design with stringent constraints, the methods based on including constraints into objective functions could find better lattice solutions than those based on biasing feasible over infeasible solutions. In this study, a hybrid 7BA lattice solution with a natural emittance of 65 pm·rad at 2.4 GeV was selected, and its DA and dynamic momentum aperture were large.

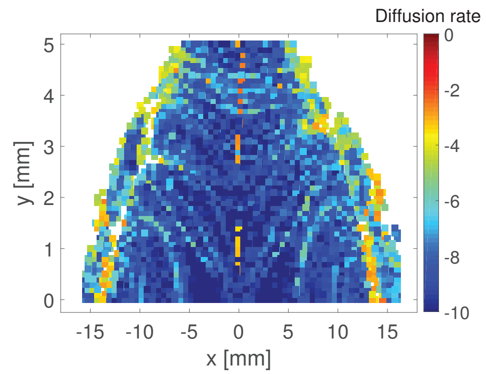

Figure 5: Optimized DA, tracked for 1024 turns.

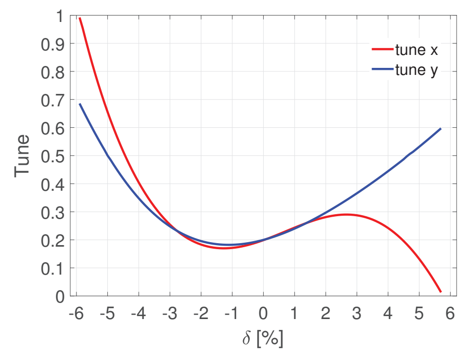

Figure 6: Momentum dependent tune footprints.

ACKNOWLEDGEMENTS

This work was supported by the National Natural Science Foundation of China (11605203, 11875259), the National Key Research and Development Program of China (2016YFA0402000), and the Chinese Universities Scientific Fund (WK2310000058).

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