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ENERGY AND RF CAVITY PHASE SYMMETRY ENFORCEMENT IN MULTI-TURN ERL MODELS

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Abstract

In a multi-pass Energy Recovery Linac (ERL), each cavity must regain all energy expended from beam acceleration during beam deceleration, and the beam should achieve specific energy targets during each loop that returns it to the linac. For full energy recovery, and for every returning beam to meet loop energy requirements, we must optimize the phase and voltage of cavity fields in addition to selecting adequate flight times. If we impose symmetry in time and energy during acceleration and deceleration, fewer parameters are needed, simplifying the optimization. As an example, we present symmetric models of the Cornell BNL ERL Test Accelerator (CBETA) with solutions that satisfy the optimization targets of loop energy and zero cavity loading.

INTRODUCTION

An Energy Recovery Linac (ERL) can create linac-quality beams of high power and current that are useful as synchrotron sources or for other experiments [1]. In an ERL, beams are accelerated and decelerated through the same set of cavities. These cavities reclaim energy from decelerating particle bunches to reduce the net power consumption of ERL operation [2]. In this study, we consider two ERL objectives: minimizing the power load on each cavity, and achieving the design target for the maximum beam energy.

One could determine ERL settings by approximating the beam as ultra-relativistic ($v = c$) throughout the ERL. In this case, energy will be fully recovered if cavities are all set at identical phases relative to beam entrance. The return loop carrying beams in the process of accelerating or decelerating would be an integer number of wavelengths long, with length $a\lambda$ for an arbitrary integer a. The loop with the highest energy would require a half-wavelength offset, with length $(a+0.5)\lambda_{RF}$, to switch from acceleration to deceleration [2]. However, ultra-relativistic settings will not provide sufficient energy recovery for all beam energies within an ERL. Low-energy beams will experience RF phases slipped away from maximal energy gain, and synchrotron radiation in the return loops can cause additional offsets in beam flight times.

Phase, voltage, and return loop length settings that minimize cavity load and achieve beam energy targets can be determined by optimization. We present a method of enforcing symmetric beam energy profiles during acceleration and deceleration. This symmetry leads to a compact optimization system, which we test in models of the Cornell BNL ERL Test Accelerator (CBETA) (Fig. 1).

If CBETA settings use the $v = c$ approximation, each cavity experiences a power load up to 46 kW per 40 mA beam (1.15 MeV per electron) when modeled in an ERL tracking script with thin lens cavities. If loop lengths alone are numerically optimized, each cavity has power load up to 2 kW per beam (50 keV). In practice, each CBETA cavity runs on a maximum of 5 kW power, of which only 2 kW are available for beam acceleration [3]. The $v = c$ solution is unfeasible, and the loop-only solution is risky. We turn to symmetry optimization in search of solutions with more reasonable power load.

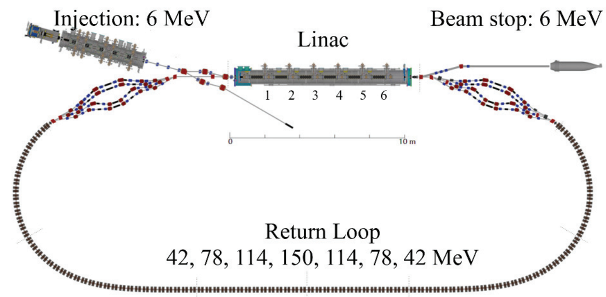

Figure 1: CBETA layout [3]. CBETA has 4 physically distinct return loops and a linac that holds 6 evenly spaced RF cavities. A 6 MeV injected electron beam accelerates to 150 MeV over 4 linac passes, and the beam returns to 6 MeV after 4 more decelerating passes. The energy of the 150 MeV beam is critical, as it intended for use in experiments.

OPTIMIZATION SYSTEM

Consider a $\frac{M}{2}$-turn ERL with N cavities and M linac passes. Individual passes and cavities use indices m and n, such that $1 \leq m \leq M$ and $1 \leq n \leq N$. The m^{th} return loop is between the m^{th} and $(m+1)^{th}$ linac passes.

In an ERL without shared return loops, the degrees of freedom include: $(M-1)$ independent loop lengths, N cavity phases, and N cavity voltages. The objectives include minimization of N cavity loads, where load is defined as the net beam energy gain within a single cavity over the full ERL run. In addition, $(M-1)$ beam energies during return loops must meet design targets to ensure proper beam control.

There are a total of $(2N + M - 1)$ degrees of freedom, which can be varied to satisfy $(N + M - 1)$ objectives. In CBETA, there are only $\frac{M}{2} = 4$ shared return loops; this gives 16 degrees of freedom and 13 objectives.

Symmetric ERL

ERL symmetry occurs when the decelerating beam encounters an exactly reversed sequence of energy steps and electric field profiles as it initially experienced during acceleration. To create symmetry, we make the phase and voltage

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settings of the $(N-n+1)^{th}$ cavity dependent on those of the n^{th} cavity. Degrees of freedom are then: $\frac{M}{2}$ independent loop lengths, $\frac{N}{2}$ phases, and $\frac{N}{2}$ voltages.

A symmetric ERL has only $[\frac{N}{2}]$ independent loads, where the Gauss bracket denotes the floor of a number. The m^{th} and $(M-m)^{th}$ returning beams have identical energy: only $\frac{M}{2}$ loop energy objectives are required. If return loops with index $m < \frac{M}{2}$ can be calibrated post-optimization to match the beam energy, then these loops need not be considered as target energy objectives. Then, the system needs only considers the design target of the highest energy beam, which must be met if the beam is to be used in experiments.

If the objectives only consider the maximum beam energy, the symmetric ERL has a total of $N+\frac{M}{2}$ degrees of freedom, which must satisfy $[\frac{N}{2}]+1$ objectives. If an optimization system with equal numbers of variables and constraints is desired, one can set all $\frac{N}{2}$ voltages and the first $(\frac{M}{2}-1)$ loop lengths to reasonable constant values. In CBETA, symmetry yields 10 possible degrees of freedom and 4 objectives.

SYMMETRY ENFORCEMENT

Our goal is to create a decelerating sequence of beam energy and electric field encounters that is identical, but reversed in order, to the accelerating sequence. In an ERL, symmetry in the cavity fields can exist if the geometry of the n^{th} cavity is the mirror image of that of the $(N-n+1)^{th}$ one, with respect to the center of the linac.

Linear Sequence: 2 Cavities

Before examining a full ERL, consider two cavities (A and B) arranged end-to-end, in a mirror symmetric way about a central point. In the later cavity, B, the beam should decelerate to its original energy over an identical transit time as the acceleration in A took: $T_B = T_A$. The RF phase of cavity B when the particle enters, $\phi_{\text{in},B}$, must have a specific relation to the cavity A input phase. To find this relation, consider the electric field within A,

$$\mathcal{E}_A(s,t) = \mathcal{E}_{A0}(s)\sin\left(\omega(t-t_{\text{in,A}})+\phi_{\text{in},A}\right), \quad (1)$$

where ω is the RF frequency, the particle enters cavity A at time $t_{\text{in,A}}$, and $\phi_{\text{in},A}$ is the input phase that we use as a degree of freedom. The spatial RF field dependence is given by $\mathcal{E}_{A0}(s)$, and by convention it is chosen to start with a positive value in the first cell of a multi-cell cavity. For cavities with an odd number of cells, the spatial dependence is a symmetric function about the center; for an even number of cells, it is an anti-symmetric function. This means that $\mathcal{E}_{B0}(L-s) = \pm\mathcal{E}_{A0}(s)$. The sign (+) is for odd and (–) is for even numbers of cells per cavity.

For symmetry in A and B, the electric field at distance s from the start of A must be opposite the field at distance s from the end of B. Suppose each cavity has length L_s, and the linac has total length $L \geq 2L_s$,

$$\mathcal{E}(L_s-s) = -\mathcal{E}(s) \quad (2a)$$
$$\mathcal{E}_B(L-s, t(L_s-s)) = -\mathcal{E}_A(s, t(s)) \quad (2b)$$

Solving for the unknown cavity B input phase, $\phi_{\text{in},B}$, we find conditions for cavities with an odd or even number of cells,

$$\begin{aligned} \phi_{\text{in},B} &= -\phi_{\text{in},A} - \omega T_A = -\phi_{\text{out},A} && \text{[odd]} \\ \phi_{\text{in},B} &= \pi - \phi_{\text{in},A} - \omega T_A = \pi - \phi_{\text{out},A}. && \text{[even]} \end{aligned} \quad (3)$$

If $\phi_{\text{in},B}$ fulfills this phase condition, for any arbitrary choice of $\phi_{\text{in},A}$, then the deceleration in B will exactly reverse the acceleration from A. We now extend the 2-cavity argument to find phase conditions for a full multi-turn ERL.

ERL: N Cavities, M Passes

Since the beam encounters each cavity multiple times in an ERL, the input phase, ϕ_{in}, is not as useful a setting. Instead, let $\phi_{0,n}$ be the RF phase of cavity n at beam injection time $t=0$,

$$\begin{aligned} \phi_{0,n} &= \phi_{\text{in},mn} - \omega t_{\text{in},mn} \\ &= \phi_{\text{out},mn} - \omega t_{\text{out},mn}, \end{aligned} \quad (4)$$

where the mn subscript indicates the m^{th} pass of cavity n. In the 2-cavity example, A and B represented a symmetric acceleration-deceleration pair. In the full ERL, the m^{th} encounter of cavity n is the mirror symmetric pair of the $m' = (M-m+1)^{th}$ encounter of cavity $n' = (N-n+1)$, where the primed encounter occurs first, *i.e.* $m > m'$. If N is odd, then the central cavity will act as its own pair, but the phase condition will follow the same form as the other pairs. Our goal here is to find the initial phase of cavity n in terms of the known $\phi_{0,n'}$. Inserting these pair designations into the phase conditions from Eq. (3),

$$\begin{aligned} \phi_{0,n} &= -\phi_{0,n'} - \omega t_{\text{total}} && \text{[odd]} \\ \phi_{0,n} &= \pi - \phi_{0,n'} - \omega t_{\text{total}}, && \text{[even]} \end{aligned} \quad (5)$$

where the beam travels through the full ERL, from injector to beam stop, over a total time interval, $t_{\text{total}} = \omega t_{\text{in},mn} + \omega t_{\text{out},m'n'}$. For t_{total} to accurately describe the total transit time over the ERL, the time of flight of the return loop between acceleration and deceleration must be set,

$$t_{\text{loop},\frac{M}{2}} = t_{\text{total}} - 2t_{\text{out},\frac{M}{2}N}, \quad (6)$$

where $t_{\text{out},\frac{M}{2}N}$ is the time from beam injection to the end of the last accelerating pass. Time t_{total} then becomes a degree of freedom.

The Eq. (5) and Eq. (6) conditions are sufficient to guarantee that every stage of beam acceleration is matched by an equivalent deceleration.

ERL CAVITY MODELS

To test ERL symmetry optimization, we construct models of CBETA beam flight using Mathematica and Bmad softwares [4]. Models consider a transversely on-axis particle that encounters only drift pipes or cavities. The beam traverses a cavity in time T_{mn}, where its energy changes by some ΔE_{mn}. If a modeled cavity is shorter than the physical length in CBETA, the model element is centered within the space, and drifts on either side are extended to compensate

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for the missing length. CBETA cavities have an elliptical geometry with 7 cells [3].

In the thin lens (TL) model, cavities have zero length and deliver a delta-function acceleration. If voltage is designated V and particle charge q, the cavity models time and energy as,

$$\Delta E_{\mathrm{TL}} = qV\cos(\phi_{\mathrm{in}}), \qquad T_{\mathrm{TL}} = 0. \tag{7}$$

The ultra-relativistic (UR) model treats the beam as having speed $v = c$ within a 7-cell cavity of length corresponding to 7 stacked pillboxes,

$$\Delta E_{\mathrm{UR}} = qV\cos(\phi_{\mathrm{in}}), \qquad T_{\mathrm{UR}} = \frac{L}{c} = \frac{7\pi}{\omega}. \tag{8}$$

The finite time-tracked (FT) model accounts for non-ultrarelativistic particle speeds by calculating energy, E_{FT}, from particle momentum p_{FT},

$$T_{\mathrm{FT}} = \frac{L}{2}\left(\frac{1}{v_{\mathrm{in}}} + \frac{1}{v_{\mathrm{out}}}\right) \tag{9a}$$

$$\Delta p_{\mathrm{FT}} = \frac{q}{\omega}E_{\mathrm{in}}[\cos(\omega T_{\mathrm{FT}} + \phi_{\mathrm{in}}) - \cos(\phi_{\mathrm{in}})] \tag{9b}$$

Lastly, the Runge Kutta (RK) model tracks the beam time and energy by integrating through the RF fields directly. The Runge Kutta algorithm can be applied to any map or model of electric fields; in this study, we model a RK cavity as a stack of 7 first-harmonic pillboxes with on-axis fields,

$$\mathcal{E}(s) = \frac{2V}{L}\sin(cs/\omega)\sin(\omega(t - t_{\mathrm{in}}) + \phi_{\mathrm{in}}). \tag{10}$$

The RK model should have the most accurate time and energy results of these models; however, it is computationally intensive and slow in simulation. Since simulation speed is key when optimizing for hundreds of iterations, we begin with the simplest calculation, the TL model, and use the solutions as starting points for optimization of the more complex UR, FT, and RK models. Models are optimized using Newton's Method or built-in Bmad optimizers [4].

CBETA RESULTS

Optimization of the four CBETA models yields loads of under 1 eV per electron, corresponding to 40 W for a 40 mA beam (Table 1). The different objective values of the four models indicate respective numerical noise thresholds; for instance, the RK model has the highest optimization noise, as its objectives are several orders of magnitude larger than those of the other models. The physical system is expected to have imperfect setting precision. If a single input setting is varied from the Table 1 solution, then we can calculate the range by which that input can vary before the objectives exceed chosen tolerance bounds: ±2 kW load, and ±150 keV maximum beam energy offset. Ranges for single-input errors are in Table 2.

Table 1: Optimized Solutions. |Load| is the largest of the 6 cavity loads, in net energy change per electron; the peak energy objective is $\Delta E_{\mathrm{loop},4} = E_{loop,4} - 150$ MeV. Phase and total time are optimized. Loops 1-3 have flight times set at 0.26 µs, selected to reasonably achieve the 150 MeV target; voltage is 6.05 MV.

Objective	TL	UR	FT	RK
$\Delta E_{\mathrm{loop},4}$ (µeV)	37.104	-43.869	-72.360	851488
|Load| (µeV)	28.707	28.782	30.886	76798

Table 2: Tolerable Error Range of One Imperfect Input. Assumes that only the indicated input deviates from the optimized solution, while all others are perfect (Table 1). The narrowest range of each input category (phase, voltage, or loop length) is reported. Ranges do not describe scenarios with multiple imperfect inputs.

Input	TL	UR	FT	RK
ϕ_0 (°)	1.7004	1.6782	2.2341	0.4725
qV (keV)	37.433	37.541	37.716	37.391
Loop (mm)	0.2557	0.2389	0.3162	0.3277

CONCLUSION

The load on each RF cavity must be minimized in order for an ERL to properly recover energy from an accelerated beam during deceleration. Phase, return loop length, and voltage can be varied to reduce the load, but direct optimization of the ERL results in a large system of variables and constraints. Applying ERL symmetry with the Eq. (5) and Eq. (6) conditions can reduce the size of this optimization system, resulting in a less computationally intensive search. In the CBETA models, optimization of a symmetric ERL setup has resulted in solutions with cavity loads several orders of magnitude smaller than the pre-optimized version.

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REFERENCES

[1] S. M. Gruner, D. Bilderback, I. Bazarov, K. Finkelstein, G. Krafft, L. Merminga, *et al.*, "Energy recovery linacs as synchrotron radiation sources," *Rev. Sci. Instrum.* vol. 73, p. 1402 (2002).

[2] L. Merminga, "Energy Recovery Linacs," in *Proc. of PAC07*, Albuquerque, New Mexico, USA, 2007, paper MOYKI03, pp. 22–26.

[3] G. H. Hoffstaetter, D. Trbojevic, C. Mayes, N. Banerjee, J. Barley, I. Bazarov, *et al.*, "CBETA Design Report, Cornell-BNL ERL Test Accelerator," Jun. 2017, `arXiv:1706.04245`

[4] D. Sagan, *The BMAD Reference Manual*, Dec. 2017, `https://www.classe.cornell.edu/bmad/manual.html`