ISBN: 978-3-95450-208-0 doi:10.18429/JACoW-IPAC2019-TUPRB089

UNDULATOR RADIATION GENERATED BY A SINGLE ELECTRON

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Abstract

The facilities providing single electron beams are currently being commissioned at Fermilab and will be at SLAC. Recently, Fermilab's IOTA ring routinely demonstrated circulation of a single electron at 100 MeV beam energy. Alternatively, SLAC is working on constructing LCLS-II an X-ray FEL driven by a 4 GeV SRF linac. A parasitic beamline, S30XL, is planned that will extract 4 GeV dark current from between the primary LCLS-II electron bunches. The dark current will be delivered to End Station A and can work independently of LCLS-II experiments. The dark current will be bunched at a frequency of 46 MHz while extracted current varied from single electrons to 10's of nA. In the present paper, we estimate the feasibility of propagating single electron beams through a conventional undulator, placed in the IOTA and S30XL beamlines. We explore the possible observable effects and experimental parameters range. In addition, we focus on potential applications of such beams in systems for high fidelity quantum measurements.

MOTIVATION

In an accelerator, electrons are subject to interaction with external electromagnetic fields via Compton scattering, and collective interaction via electromagnetic forces. Both effects can be theoretically described, in the leading order, by using a classical electromagnetic Hamiltonian, as shown by Glauber [1]. Under such conditions, electrons may be assumed to be spin-less charged scalar-like particles or Volkov wavefunctions dressed by a classical background field [2]. However, due to the improvement of electron beam manipulation techniques, magnets, and SRF technology, particle accelerators can enter new regimes where collective quantum effects start to play a key role. For instance, plasma acceleration, laser fields inside dielectric structures, and solid state acceleration, may involve electron current densities and acceleration gradients that alter quantum mechanical states. When sent to a very short-period undulator, e.g. an optical undulator, with large K parameter, the process of undulator radiation cannot be fully explained by means of classical perturbation theory and requires complete non-perturbative quantum mechanical treatment Dirac electron states [3, 4]. A path to getting a better understanding of the nonperturbative effects in accelerators is to experimentally study the behavior of a single electron. Examples of such studies include operation of a storage ring with single electron (IOTA, Fermilab), single electron source for dark matter search (proposed S30XL experiment at SLAC), low emittance dielectric laser accelerators, searches for anomalous electron dipole moment, single electron quantum dots, and ultra-precise nano-assembly [5–7]. With the development of fourth and fifth generation X-ray light sources around the world, it becomes increasingly important to learn about the quantum nature of electrons and their interaction with electromagnetic fields. Ultimately, a single electron in an accelerator is free of collective effects, therefore is the "cleanest" probe of non-perturbative QED.

THEORETICAL OVERVIEW

We first introduce two parameters to quantify the strength of electromagnetic processes [8]. First, the classical intensity parameter defined as

$$\xi = \frac{e\sqrt{A_\mu A^\mu}}{mc^2} \approx \frac{eE}{mc\omega} \tag{1}$$

is equivalent to the undulator K parameter and is a Lorentz and gauge invariant. Second, the quantum parameter is also an invariant and is associated with the field strength:

$$\chi = \gamma B / B_{cr}, \tag{2}$$

where $B_{cr} = 4.4 \times 10^{13}$ G is the critical Schwinger field intensity. Moreover, ξ is only well-defined in an oscillating laser/undulator field and $\to \infty$ for a constant field ($\omega \to 0$). The two parameters are related with the quantum electron recoil ρ as: $\chi = \xi\rho$. In the current generation of electron accelerators, ξ-parameter can be $\xi \geq 1$ in undulators, wigglers and strong dipole magnets, while χ is almost always taken to be negligible because of $\rho \to 0$. It may, however, become non-zero for the case of optical or quantum undulator.

In the most general form, the Lagrangian density in the presence of strong external field is [9]:

$$\mathcal{L} = \bar{\psi}[\gamma^\mu(i\partial_\mu - eA_{ext,\mu}) - m]\psi - e\bar{\psi}\gamma^\mu\psi A_\mu - \frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu}, \tag{3}$$

where ψ is electron spinor, A_{ext} is the vector-potential of the external field, γ_μ are the Gamma matrices, and $F_{\mu\nu}$ is the electromagnetic field tensor for non-background field photons. The spinor solutions of the Dirac equation with the Lagrangian density defined as Eq. (3) are "dressed" by the background field and called (Dirac-)Volkov states. In this formalism, synchrotron and undulator radiation considered a Compton scattering of the external field photon by the electron, will be represented as a decay of a Volkov state. Figure 1 illustrates Compton scattering effect in first and second order of perturbation theory. Note, that there are two contributions to the two-photon effect, both included in the Volkov

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state calculation. First, named "incoherent", corresponds to the photons generated in different formation regions. Second, or "coherent" contribution represents two photons originated in the same formation region. The diagram representation of spontaneous radiation process, demonstrated in Fig. 1, is identical for the cases of synchrotron (dipole) and undulator radiation, with the differences in Volkov propagators and formation regions.

SOKOLOV SOLUTION FOR DIPOLE RADIATION

The probability density of a single photon emission for an electron in constant magnetic field is given by the following equation (in CGS units) by Sokolov [2]:

$$w_1 = \frac{5}{2\sqrt{3}}\frac{e_0^2}{c\hbar}\frac{e_0 H\sqrt{1-\beta_\parallel}}{m_0 c}, \quad (4)$$

where e_0 is the electron charge, m_0 is the electron mass, H is the field strength and $\beta_\parallel$ represents the electron's transverse velocity. For sake of simplicity we assume hereafter $\beta_\parallel = 0$. The cyclotron frequency of the electron is then $\omega = e_0 H/m_0 c$ yielding the time of one revolution period to be $\tau = 2\pi m_0 c/e_0 H$. The total integral probability of the photon emission per one period is $W_1 = w_1\tau = 2\pi\alpha\frac{5}{2\sqrt{3}}$, where $\alpha = e_0^2/c\hbar$ is the fine-structure constant. Assuming H to be weak meaning $H \ll H_{cr}$ and absense of spin and higher order coherent emission effects, one can postulate that the integral probability of the two-photon process is $W_2 = \frac{1}{2}W_1^2$. Here the factor of $1/2$ corresponds to indistinguishability of the two photons. Then the probability density can be expressed in the following form:

$$\frac{1}{2}\frac{W_1^2}{\tau} \equiv w_2 = \frac{25}{24}\alpha^2\left(\frac{e_0 H}{m_0 c}\right)\frac{1}{2\pi}. \quad (5)$$

The rigorous derivation of the probability w_2 of two-photon emission in the constant magnetic field is provided in Eq. (21) of [2] and is equivalent to Eq. (5). In the Eq. (21) of [2], one should chose $\chi = 0$ for the weak-field case and $\gamma\Delta\phi \equiv 1/2\pi$. In addition, the spin of the electron ζ under our assumptions remains unchanged ($\zeta_1 = \zeta_2$).

RITUS SOLUTION FOR SINGLE ELECTRON

The differential cross-section for emission of a photon from a laser or an undulator are almost the same for high-energy electrons [10], and can be written in the general form as [8]:

$$\frac{d^2W}{d\omega' d\Omega} = \frac{1}{2}\sum_{r,r'}\sum_{\lambda'}\frac{\alpha\omega'}{16\pi^2(k\cdot p)(k\cdot p')}|M^1(s;,r,r',\lambda')|^2,$$

where M^1 is the single photon matrix element given by:

$$M^1(s) = \Gamma_0 A_0(s) + \Gamma_+ A_+(s) + \Gamma_- A_-(s) + \Gamma_2 A_2(s), \quad (6)$$

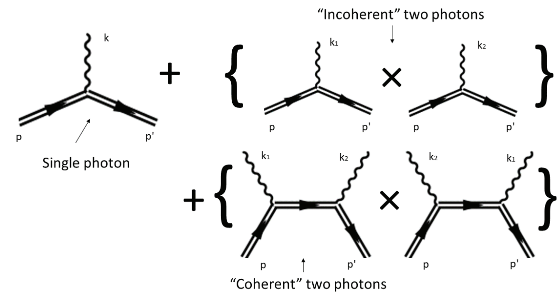

Figure 1: Feynman diagram schematics of undulator (dipole) radiation represented in Volkov states (thick lines).

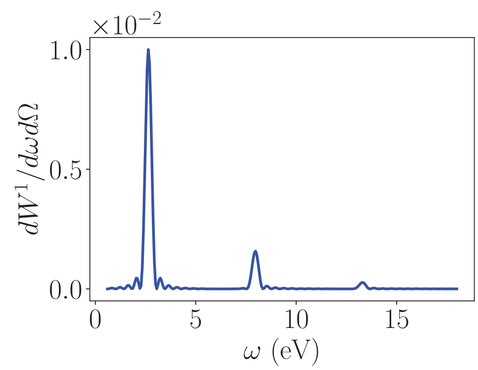

Figure 2: Single electron undulator spectrum for $K = 1$ on axis derived from Eq. (6) with e^- beam parameters of IOTA.

where terms $\Gamma_0, \Gamma_+, \Gamma_-$ are Dirac spinor currents and terms $A_0(s), A_{+/-}(s), A_2(s)$ are phase integrals that are defined by the external EM pulse envelope $g(\phi)$ and polarization ϵ; $p = (\gamma m, 0, 0, \beta\gamma m)$, $k = \omega_0(1, 0, 0, -1)$, are the electron and photon 4-vectors respectively, $\epsilon = (0, 1, 0, 0)$ is the polarization vector for the case of planar undulator or linearly polarized laser.

We note that numerical solution of Eq. (6) can provide complete undulator spectral-angular distribution with the undulator parameter $K < 2$. After that direct numerical integration of $A_i(s)$ becomes cumbersome due to highly oscillatory behavior of the integrals and therefore requires additional analytical simplifications or special integration methods. An example calculation of an undulator spectrum on axis for the case of IOTA single electron is presented in Fig. 2 and corroborates a well known harmonics peak structure. Two-photon effect differential cross-section can be computed in a similar fashion using Volkov solutions of the Dirac equation with Lagrangian density defined by Eq. (3) and expanding the calculation of both "coherent" and "incoherent" contributions. This calculation was provided in [11] for the case of strong laser fields and can be adopted to the case of an undulator field, similarly to the approach of Eq. (6). The resulting single electron two-photon undulator spectrum is presented in Fig. 3. We refer our readers to

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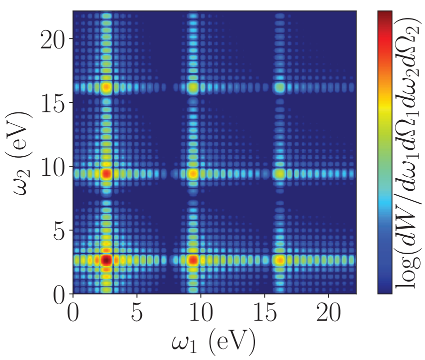

Figure 3: Two-photon undulator events spectrum for $K = 1$ on axis. Two "soft" photons are the most probable, while two "hard" photons are the least probable events. e^- beam parameters of IOTA.

Chapters 3 and 4 in [12] for the detailed explanation of the matrix element calculation.

EXPERIMENTAL PROGRAM

Single electron beams in storage rings have been first obtained in Novosibirsk and DESY, where they were used for quantum fluctuations and photon metrology studies [13–15]. Recently, this capability has been enabled at Fermilab IOTA storage ring. For the most recent developments of the IOTA experimental program, we refer the reader to [16–20]. Alternatively, multi-GeV single electron source S30XL is proposed as a complimentary beamline to LCLS-II SRF linac.

For the first round of experiments, at IOTA ring and SLAC S30XL, two interesting topics will be targeted: the radiation formation region and the electron wave-packet localization in an undulator. An electron, placed in a "bath" of field photons in an undulator, will constantly undergo quantum mechanical interactions [21] and associated electron wave function reduction. So far, no conclusive experimental measurements of single electron wave packet localization in an undulator has been performed. Multi-photon Compton effect and analysis the time of arrival correlations will inherit this property [13, 14, 22, 23]. In a very strong undulator or wiggler, e.g. such as available for XLEAP-II experiment at SLAC, the arrival time of multi-photons intrinsically carries information on the formation region when $K \gg 1$. Alternatively, when two and more photons are born in the same formation region, they possess quantum concurrence of polarization, angular and spectral properties, associated with the strong field effects [24]. Understanding the spectral-angular distribution of the undulator multi-photons is potentially relevant to the operation of a quantum FEL. Both IOTA and S30XL would be able to probe this with electron-undulator, electron-wiggler and electron-laser interactions in a single electron experimental configuration.

IOTA Status

Fermilab's IOTA ring has recently commissioned a short 55 mm undulator, previously used for ECHO experiments at SLAC, with IR radiation produced with 100 MeV beam. It is expected to raise the electron energy up to 200 MeV, bringing the undulator photons to the visible frequency range. The design of the photodetector is summarized in [25]. Currently, the IOTA e^-−beam injector, consisting of a normal conducting RF-gun and SRF crymodules, can routinely deliver electron dark current to the injection point. Single electron lifetime in IOTA ring is estimated to be about 10 minutes, mainly limited by the residual gas scattering.

S30XL Beamline

An overview of the proposed S30XL project at SLAC has been provided in [26, 27]. In brief, S30XL beamline, will provide low-current multi-GeV electrons produced from dark current parasitically to nominal LCLS-II SRF linac operations. The S30XL electron beam may be varied from single electron to 10 nA beam current, which then will be smashed into a thin aluminum target for the purposes of light dark matter experiment (LDMX). We note that raising electron beam energy increases the χ parameter and there are plans to use LCLS-II HE 8 GeV beam [28]. Currently we are evaluating this capability to observe possible Unruh effects [29]. In addition, with a TW laser, one can verify wavefunction interference effects predicted in [30]. Precise control of a small number of electrons, such as that provided by the S30XL beamline, may also shed light on collective quantum interference and collective recoil of electron wavepackets [30]. The higher beam energy and higher duty cycle at the S30XL beamline will provide an opportunity to extend the experimental programs planned at IOTA and other facilities aimed at exploring QED.

SUMMARY

In summary, we have presented an overview of strong field quantum electrodynamics (SFQED) theoretical calculation of a synchrotron or undulator radiation. This theory can be also extended to the cases of strong XLEAP-II wigglers, currently being commissioned at SLAC. We note that the same formalism is also applicable to the inverse Compton scattering (ICS) experiments, where double-photon effect is much more pronounced. Experimental single electron beamlines at Fermilab and SLAC will foster excellent possibilities for SFQED studies and beyond.

ACKNOWLEDGEMENTS

This work was supported by the U.S. Department of Energy Contract No. DE-AC02-76SF00515. A.H. would like to thank S. Meuren (Princeton University), G. Stupakov, A. Marinelli, E. Hemsing (SLAC), T. Shaftan (BNL), A. Benediktovitch (DESY) for valuable and in-depth discussions. Authors are grateful to Alex Murokh (RadiaBeam, USA) for supporting the first single electron workshop organized at FNAL in 2018.

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