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DESIGNING OF A SOLENOID LENS FOR THE APPLICATION TO A COMPACT ELECTRON BEAM TESTING BENCH*

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Abstract

To calculate beams transport is vital for designing vacuum pipe and arranging focusing elements for each electron beam line system. Space charge effects of a low-energy, high-intensity DC electron beam focused by a solenoid lens with bucking coil are investigated theoretically in this paper. A second-order equation is numerical solved for the beam envelope focused by a short solenoid lens. In addition, a conventional transfer matrix of solenoid is not applicable to low-energy, high-intensity electron beams because the strong space charge effects are ignored. By cutting a solenoid into several segments, we have derived a micro-transfer matrix which takes space charge fields into account, and a complete beam envelope for a transport system. A simulation is used to verify our theoretical calculation results, and corresponding discussions are given in detail.

INTRODUCTION

The external cathode electron gun with two independent tunable cavities has many merits which are used in THz-FEL field [1]. As the injector of the RF gun, a high intensity and low energy electron gun is required [2]. To measure the key parameters of the high-intensity, low energy gun, a compact beam diagnostics instrument has been constructed in Huazhong University of Science and Technology.

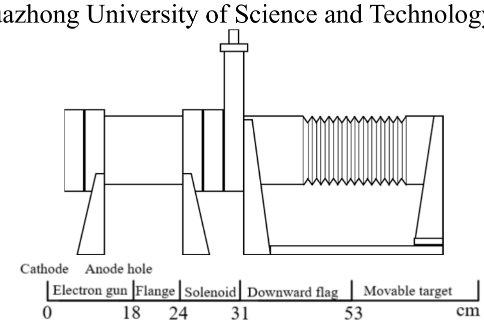

Figure 1: The layout of the compact beam diagnostics instrument built in HUST.

The primary layout is shown in Fig. 1. Take the cathode of the electron gun as the origin point of the coordinate, the distance between the cathode and anode is 18 cm. The electron gun and focusing solenoid are connected by a flange which thickness is 6 cm. The total length of the solenoid is 7 cm, and the downward flag and the flapper valve are 22 cm in total. The diagnostic instrument integrates beam current, emittance and energy dispersion measurements in one device [3].

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To calculate envelop of beams is critical, it will guide the design of vacuum pipe and matching of focusing elements. Due to lacking a suitable mathematic model for a bucking coil, calculating beam envelope were complicated. Plus, space charge effects of high intensity current and low energy beam are not negligible when calculating the envelop of strong current low energy beam through a solenoid.

DESIGN OF A SOLENOID LENS

It is necessary to perform an appropriate initial focus at the exit of the electron gun, which provides to counteract emittance growth stem from space charge effects and control beam envelope. The strong space charge effects of high intensity and low energy beams causes beams to diverge. A solenoid lens which is more suitable for installation in a compact space than other focusing magnets because the solenoid lens focuses the beam in both x and y directions (Fig. 2).

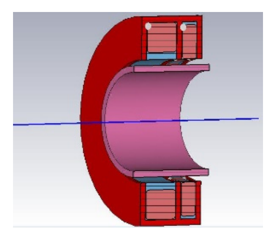

Figure 2: The 3D model of the solenoid lens.

A solenoid lens isn't placed far from the cathode [4], the magnetic field generated by the solenoid will stretch to the cathode. However, this leak magnetic field leads to not only the emittance increase, but also make electron run back stream and hit electron gun cathode, and even damage cathode eventually. In order to avoid situations stated above, a bucking coil was being used with electron gun.

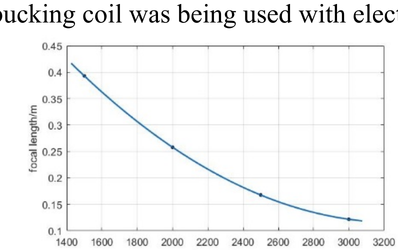

Figure 3: The focal length as a function of the NI number.

The key parameter of the solenoid is the NI number. If the NI number is large, the focal length of the electron beam is short. However, too short length will cause the beam diverge rapidly, and beams will hit the vacuum tube after passes through a drift tube. On the contrary, using a small NI number extends the focal length, but it won't constrain beams well, resulting beam divergence. The focal

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length of the diagnostic instrument is 13-40 cm, and the focal length as a function of the NI number is shown in Fig. 3. The peak field is 80 Gs and the position is located in 31 cm from the cathode. The magnetic field distributions are shown in Fig. 4.

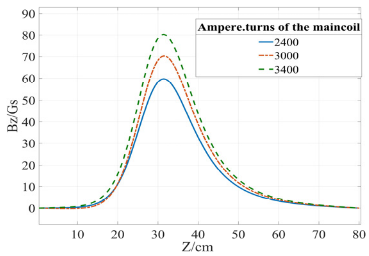

Figure 4: Magnetic field distributions as a function of the distance from the cathode. Lines of different colours represent different ampere turns of the solenoid.

THEORETICAL CALCULATION OF BEAM ENVELOPE

A solenoid lens was developed for a 14 keV thermionic cathode DC electron gun. The solenoid lens integrated a main coil and a bucking coil in the same iron yoke in order to avoid misalignment error. When the main coil current provides the peak field and the bucking coil adjusts the magnetic field at the cathode to zero.

If we ignore the axial magnetic field caused by the beam rotating in the solenoid, the envelop equation of the beam in the solenoid is satisfied [5,6]:

$$\frac{d^2R}{dz^2} = \frac{eR}{\beta\gamma m_0 c^2}\left(\frac{\mu_0 cI}{2\pi R_s^2 \beta^2\gamma^2} - \frac{eB_z^2}{4m_0\beta\gamma}\right) + \frac{\varepsilon^2}{\beta^2\gamma^2 R^3} \quad (1)$$

Where R denotes beam size and B_z is the magnetic field on the axial of the solenoid.

The equation is a second-order differential equation with time-varying terms, which is solved by fourth-order Runge-Kutta method. The result is shown in Fig. 5. The beam energy in the calculation is 14 keV, and the beam current is 5.5 A. The beams have a converging effect under the solenoid lens, leaving the effective field of the solenoid, it is diverged by the space charge fields. By changing the strength of the magnetic field, the beam size at a particular location changes.

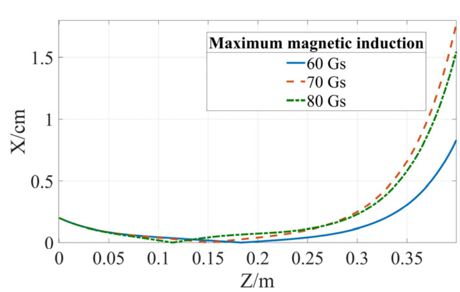

Figure 5: Envelope curves solved by K-V equation for the maximum magnetic induction in the Z direction on the solenoid axis for 80 Gs, 70 Gs and 60 Gs. Z starts from the entrance of the solenoid.

TRANSFER MATRIX CALCULATION

The effect of space charge effects on the beam envelope is not taken into consideration when calculating the beam size using conventional transfer matrix method of the solenoid [7]. The space charge fields change as the beam size changes. Therefore, it is unreliable to calculate the beam size of a low energy intense current by the complete transfer matrix of the elements [8]. To solve this problem, the beam envelope is calculated using a solenoid transfer matrix that takes into account space charge fields. Since space charge fields are constantly changing, each segment of the solenoid is calculated using a micro-matrix, and the transfer matrix of the entire element and the complete beam envelope are obtained.

When calculating a non-intense current beam, the elements' transfer matrix and beam are independent i.e. the elements in the transfer matrix do not change as the beam state changes. However, in high intense current beams, the trajectory of particles motion and space charge fields interact with each other, and the transfer matrix of the element and the state of the particle are closely interdependent. In order to simplify the model, the coupling case is not considered. The transfer matrix of the intense current in a short segment is [9]

$$M = \begin{bmatrix} \cos(k_m l)\cos(s_x l) & \frac{1}{s_x}\cos(k_m l)\sin(s_x l) \\ -s_x\cos(k_m l)\sin(s_x l) & \cos(k_m l)\cos(s_x l) \end{bmatrix} \quad (2)$$

Where $K_m = qB/2p_0$, B is the magnetic field on the axial of the solenoid and P_0 is the reference particle momentum.

$$k_x = \frac{qI}{X(X+Y)m_0 c^3 \beta_0^3 \gamma_0 \pi \varepsilon_0} \quad (3)$$

Where X and Y denote half axis of elliptical beam, I is beam current, and ε_0 is the vacuum dielectric constant. $s_x^2 = k_m^2 - k_x^2$ refers to the wave number of the vibration of the particle in the X direction when considering the space charge effect.

Calculation Method

Using the micro-matrix to calculate beams' envelope requires to compute the iteration of the current, the error reduction iteration and iteration of space.

Current Iteration Supposed that x and x\` is the initial state of the beam, the solenoid is equally divided into equidistant m segments, and the beam current is uniformly divided into n segments. The beam current of each segment of the solenoid is increased from 0 A to the rated current, and calculate the beam envelope simultaneously. The last calculation result and the final calculation result of the previous segment are used each time. The average value of them as an input to calculate current beam envelope. The envelope of beam of rated current is obtained after n-1 iteration. Suppose the jth segments of computational segment, and the ith current intensity iteration, denoted by $x_{i,j}$.

Updated parameter k_x and transfer matrix of the solenoid

$$k_x = f(x_{i-1,j}),\ M_{i,j} = M(k_x) \quad (4)$$

According to beam optics

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$$\begin{bmatrix} x_{i,j} \\ x'_{i,j} \end{bmatrix} = M_{i,j} \begin{bmatrix} x_{i-1,j} \\ x'_{i-1,j} \end{bmatrix} \tag{5}$$

Calculate the next iteration input value

$$x_{i,j} = \frac{1}{2}(x_{n,j-1} + x_{i,j}), \quad x'_{i,j} = \frac{1}{2}(x'_{n,j-1} + x'_{i,j}) \tag{6}$$

Error Reduction Iteration After calculating the current iteration of the jth segment, It is necessary to reduce the error of the transfer matrix of the solenoid and the beam envelope size. The calculation method is that when the absolute value of the difference between envelopes of beam calculated by the last two times is less than the given error, the envelope calculation is considered complete, otherwise the iterative calculation is performed until the absolute value of the two envelope difference is less than the given error. Assume that the jth segment of the computational element is represented by $x_{n,j}$, since the current iteration has been done.

Calculate the initial iteration input value

$$t_{1,j} = \frac{1}{2}(x_{n,j-1} + x_{n,j}), \quad t'_{i,j} = \frac{1}{2}(x'_{n,j-1} + x'_{n,j}) \tag{7}$$

Updated parameter k_x

$$k_x = f(t_{1,j}), \quad M_{n,j} = M(\mathrm{k}_x) \tag{8}$$

$$\begin{bmatrix} t_{2,j} \\ t'_{2,j} \end{bmatrix} = M_{n,j} \begin{bmatrix} t_{1,j} \\ \mathrm{t}'_{1,j} \end{bmatrix} \tag{9}$$

Calculating the absolute value of the difference between envelopes of beam by the last two times

$$\varepsilon = \left| t_{2,j} - t_{1,j} \right| \tag{10}$$

Until ε is less than a given error. Where $t_{2,j}$ is the beam envelope and $M_{n,j}$ is the micro transfer matrix of the solenoid.

Space Iteration After calculating the transfer matrix and the envelope of the jth segment, the value of the jth segment is assigned to the j+1th segment as an initial case. The current iteration and error reduction iteration are performed on the j+1th segment of the solenoid lens until j=m. The transfer matrix and the envelope of each segment of the entire solenoid are obtained. Then the transfer matrix of the solenoid is

$$M = M_{n,j} M_{n,j-1} \ldots M_{n,1} \tag{11}$$

Calculation Results The entire transmission system consists of a solenoid lens and drift. Beam envelope is calculated using the above process. The effective length of the solenoid is 7 cm, and the length of the drift section is 40 cm. The energy of the electron is 14 keV, and the beam is uniformly distributed. The rated current of the beam is 5.5 A. Under space charge effects, the envelope diagram of the transmission system is shown in Fig. 6. The three line are the envelope for maximum magnetic induction in the z direction on the solenoid axis of 60 Gs, 70 Gs and 80 Gs respectively.

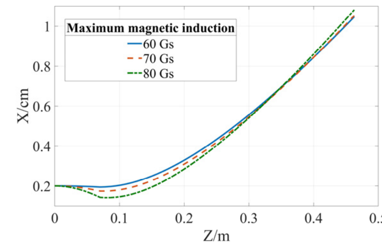

Figure 6: Envelope curves solved by transfer matrix with difference maximum magnetic fields in Z direction. Z starts from the entrance of the solenoid.

PARMELA SIMULATION RESULTS

The solenoid and drift sections are simulated using beam tracking code PARMELA [10]. Set the initial position to the anode hole of the electron gun, and the simulation components are the drift, the solenoid lens and the drift. The focus magnetic field of the solenoid is calculated by the POISSON program. Since the origin of magnetic field is the cathode of the electron gun when POISSON is calculated, we shift the magnetic field, so that the origin of the PARMELA calculation is aligned with the magnetic field of the anode hole obtained by POISSON. The total particle number is 20000, the current of beam is 5.5 A, and the kinetic energy of the ideal particle is 14 keV. Beams are uniform distribution. Beam envelope simulation results are shown in Fig. 7.

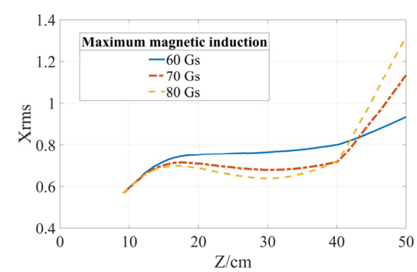

Figure 7: Envelope curves solved by PARMELA with difference maximum magnetic field in Z direction. Z starts from the entrance of the flange.

SUMMARY

In this paper, beam transport studies were performed including space charge effects for low energy & high intensity current beams from electron gun. To calculate transport of DC beam of solenoid, we design two programs to calculate beam optical system consists of solenoid lens and drift space. The results show that space charge effects have a huge impact on beam envelope in solenoid focusing, while the use of magnetic field can constrain the beam well in a certain distance. Furthermore, we verify the envelope of the DC low energy high current beam moving in the solenoid through code PARMELA. The calculation method presented in the paper can provide guide line for design of a beam transport system.

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