

# BEAM LONGITUDINAL DISTRIBUTION RECONSTRUCTED BY GESPAR METHOD AT CAEP THz FEL\*

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## Abstract

Coherent radiation can be used to measure the longitudinal distribution of the electron beam bunch of any length, as long as the coherent radiation spectrum can be measured. In many cases, the Kramers-Krönig relationship is used to reconstruct the temporal distribution of the beam from the coherent radiation spectrum. However, the extrapolation of the low frequency will introduce the uncertainty of the reconstruction. In this paper, GrEedy Sparse PhAse Retrieval (GESPAR) method was used to reconstruct the beam longitudinal distribution measured by coherent transition radiation on the THz FEL facility of China Academy of Engineering Physics. The results indicate that the GESPAR method works well for the complex and ultrashort distribution. It will be an effective tool to accurately measure the femtosecond bunch temporal structure.

## INTRODUCTION

During the past decades, many methods were developed to measure ultrashort electron beam bunch length, such as streak camera [1], RF zero-phasing [2], deflecting cavity [3], electro-optic sampling [4] and coherent radiation [5–7]. Coherent radiation, such as coherent transition radiation, coherent diffraction radiation, coherent synchrotron radiation, etc, can be used to measure the longitudinal distribution of the electron beam bunch of any length, as long as the coherent radiation spectrum can be measured. When the electron bunch length become as short as a few femtosecond nowadays, the coherent radiation method becomes the best length-measurement tool.

However, the coherent radiation measurement is to record the spectrum of the radiation, which loses the phase information. To reconstruct the longitudinal distribution, the phase information must be reconstructed first. For a long time, Kramers-Krönig (KK) relation was invited to retrieve the phase information. Unfortunately, there are at least three disadvantages of the KK relation. Firstly, extrapolation in the low frequency band will bring uncertainty to reconstruction. Secondly, KK relation is less accurate when time domain distribution is complicated. Thirdly, KK relation costs longer calculation time.

GrEedy Sparse PhAse Retrieval (GESPAR) method was presented by Y. Shechtman and Y. C. Eldar in 2014 [8, 9]. And then it has rapidly gained great applications in coherent

imaging, signal processing, macromolecular imaging, and 5G communication [10, 11]. GESPAR treats phase reconstruction as a nonlinear least squares problem, and assumes that the time domain signal is composed of a finite number of specific distributions.

In this paper, we invite the Differential evolution (DE) genetic algorithm to solve the nonlinear least squares problem. And the reconstruction with or without the GESPAR assumption is presented, respectively. At last, a temporal reconstruction with DE algorithm is shown from the signal measured on the Chengdu THz FEL (CTFEL) [12] facility.

## RECONSTRUCTION ALGORITHM

### Phase Retrieval with Nonlinear Least Squares

Assuming  $\rho(t)$  is the original time signal, whose frequency signal is  $\widehat{\rho}(\nu) = F(\rho(t)) = |\widehat{g}(\nu)|e^{-\phi(\nu)}$ , where  $F$  represents the Fourier transform,  $|\widehat{g}(\nu)|$  is the amplitude and  $\phi(\nu)$  is the phase.

Four the nonlinear least squares consideration, the objective is to minimize the function  $f = \|\widehat{\rho}_G^2 - |\widehat{g}|^2\|$ , where  $\widehat{\rho}_G$  is the  $G$ -th alternative signal. The flow-chart is shown in Fig. 1.

According to the Fourier transform relationship, we can normalize any segment of the time domain signal to the signal in the (0,1) time period. If the setting unit is 1 s, the corresponding frequency domain unit interval is 1 Hz, for example. And similarly, 1 ps in time domain corresponds 1 THz in frequency domain. This is the reasons why the coherent radiation can measure any short bunch length as long as the frequency signal can be recorded correctly.

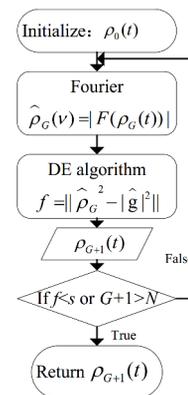


Figure 1: Phase retrieval flow chart with nonlinear least squares.

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## DE Algorithm

Differential evolution (DE) algorithm is a heuristic global optimization based on population, works on Darwin's concept of survival of the fittest [13, 14]. DE and other evolutionary algorithms are often used to solve the beam dynamic optimization [15–18].

DE starts with a population of  $NP$  candidate solutions, which may be represented as  $X_{i,G}$ ,  $i = 1, 2, \dots, NP$ , where  $i$  index denotes the population and  $G$  denotes the generation to which the population belongs. DE uses mutation, crossover and selection to solve problems, which are shown in Fig. 2.

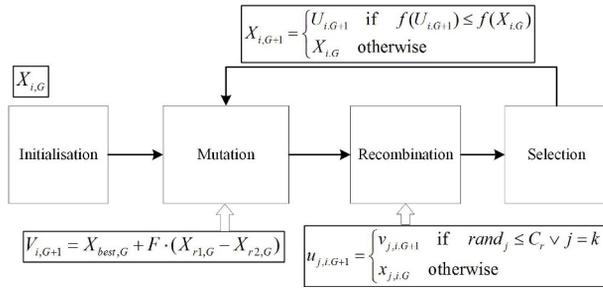


Figure 2: DE flow chart.

The mutation operator is the prime operator of DE. In this paper, a so-called ‘best-strategy-type-1’ is used [19], where  $F \in [0, 1]$  is the control parameter.  $r_i \in \{1, \dots, NP\}$  is a random selection and  $r_1 \neq r_2$ . The operator recombination and selection are also shown in Fig. 2 The crossover rate  $C_r \in [0, 1]$  is the other control parameter of DE.

## RECONSTRUCTION SIMULATION

The Sparsity-based method is also considered to reduce the dimension obviously. The dictionary can be made of any distribution function, such as Gaussian, flattop, etc, as shown in Eqs. (1) and (2), where  $\varepsilon$  is the step function.

$$\rho_{\text{Gaussian},i}(t) = a_i \exp\left[-\frac{(t - b_i)^2}{c_i}\right] \quad (1)$$

$$\rho_{\text{Flattop},i}(t) = a_i [\varepsilon(t - b_i) - \varepsilon(t - c_i)] \quad (2)$$

Some examples of the DE sparsity-based retrieval are shown in Fig. 3. The original distribution (red curve) are selected as (a) Gaussian, (b) rectangle, (c) triangle and (d) two-peak Gaussian, respectively. The blue curve is the reconstruction one. For the dimension has been reduced less than 30, the calculation goes much fast than the all-random method. All the results have been achieved within generation 20 and cost less than 1 minute. The performance of the sparsity-based retrieval is much more powerful than the all-random one when the original signals are not too complicated.

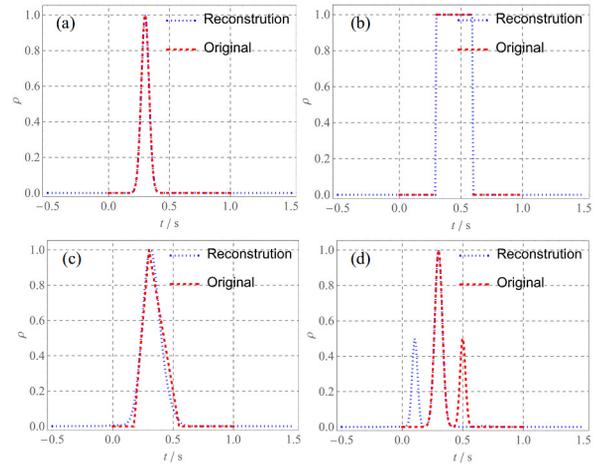


Figure 3: Sparsity-based retrieval with DE algorithm.

## CTR EXPERIMENT RESULTS

An experiment of a picosecond electron beam bunch measurement has been carried out on the Chengdu THz free electron laser (CTFEL) facility. A self-made Martin-Puplett interferometer is applied to get the auto-correlation curve of the coherent transition radiation (CTR) when the electron beam passing through a golden foil (as shown in Fig. 4). The electron energy is about 8 MeV. The charge is 100 pC. The RMS bunch length is estimated as 2 ps by Astra code.

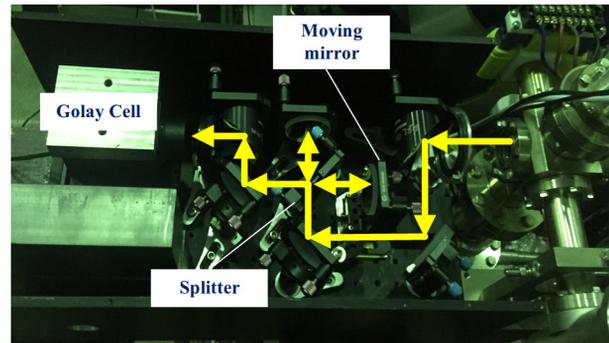


Figure 4: Experiment setup of the self-made Martin-Puplett interferometer.

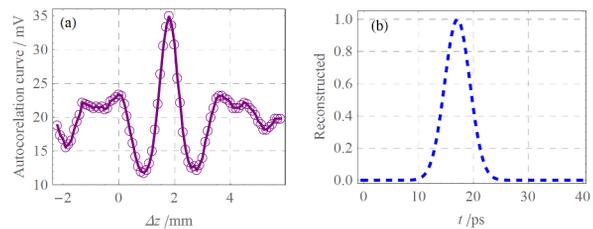


Figure 5: Auto-correlation curve and the DE reconstruction of the beam longitudinal distribution.

The auto-correlation curve is shown in Fig. 5 (a). The baseline is set as 20 mV. Then the Fourier transform and

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single electron transition radiation are considered and the coherent spectrum has been calculated. From the coherent spectrum, sparsity-based DE retrieval reconstructs the beam distribution, as shown in Fig. 5 (b). This retrieval uses the single Gaussian assumption, whose result agrees well with the KK relation. However, when using multi-peak assumption, the DE algorithm gives more information. as shown in Fig. 6.

In Fig. 6 (a), The retrieval gives two peaks at last. The distance between these peaks are about 11.4 ps, and the RMS of the main peak is about 2.2 ps. In Fig. 6 (b), the curve gives the result of the photo-cathode drive laser longitudinal distribution measured by a streak camera. Not surprisingly, the drive laser has two peaks, too. The distance between these peaks are about 30 ps, and the RMS of the main peak is about 5.7 ps. Consider the compression ratio of the accelerator system, the peak distance from the drive laser would become as:  $30 \times 2.2 / 5.7 = 11.6$  ps, which is very close to the retrieval result.

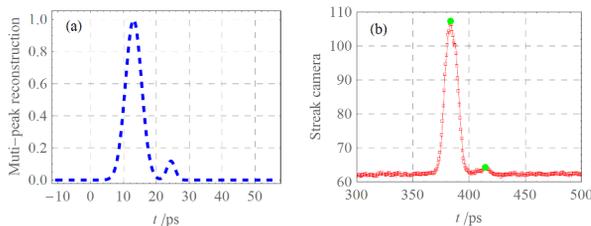


Figure 6: Result of multi-peak assumption (a) and the photo-cathode drive laser longitudinal distribution measured by a streak camera (b).

## SUMMARY

This paper has briefly introduced the application of differential evolution algorithm to reconstruct the phase information of the coherent radiation for ultrashort beam length measurement. The DE algorithm with all random assumption has the ability to reconstruct any distribution but has a large cost of time. The sparsity-base DE algorithm can solve the problem in most cases and goes much faster. One coherent transition radiation experiment has been carried on the Chengdu THz free electron laser facility. The DE algorithm agree well with the Kramers-Krönig relation and the laser distribution measurement result.

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