

## MEASURING BEAM PARAMETERS WITH SOLENOID\*

I. Pinayev<sup>†</sup>, Y. Jing, D. Kayran, V.N. Litvinenko, I. Petrushina, K. Shih, G. Wang, BNL, Upton, NY 11973, U.S.A.

### Abstract

We have developed methods of measuring electron beam energy and trajectory including angle and position based on the analysis of beam steering by a solenoid. Beam energy measurement is performed in the straight beamline and is suitable for the beams with substantial energy spread. In this paper, we describe the experimental set-up and the obtained results.

### INTRODUCTION

Solenoids are widely used for the optics control of the low energy beams because they provide focusing in the two planes and preserve cylindrical symmetry of the system. So far solenoids were mostly used for measuring beam emittance as well as  $\alpha$ - and  $\beta$ -functions.

In some occasions they were used for alignment of the beam trajectory in gun solenoid to preserve beam emittance [1, 2].

In this paper we describe our methods and experimental results. First, we measure the energy of the beam using the fact that solenoids rotate the plane of transverse motion and this angle is unambiguously defined by the beam rigidity. Second, we measure and correct beam trajectory in the solenoids. Since solenoids do not have overlapping fields, we utilize matrix approach to describe the transverse beam displacement at the observation point (either beam position monitor (BPM) or profile monitor) as a function of solenoid's current. We generalized the method to an arbitrary transfer function between solenoid and beam position monitor. The 4×4 transport matrix is calculated using the already known beam energy (rigidity) and the product of matrices of solenoids and drifts. This matrix is evaluated for variable current of the solenoid at the location of the measured trajectory, and fixed currents of other solenoids (if any) between the location and the observation point. The matrix of each solenoid is calculated using the beam energy and the magnetic measurement data. The beam trajectory at the location under study is found as a solution of a set of linear equations. Finally, we use beam profile monitors and solenoid scans for measuring transverse beam emittances.

Measurement of second moments of the beam image on the profile monitor with dependence of the solenoid current in combination of the knowledge of transport matrix allows to find second moments of the beam distribution and, hence, its parameters.

All the procedures described above make solenoids into a nearly universal tool for measuring beam parameters.

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<sup>†</sup> pinayev@bnl.gov

### TRANSPORT MATRIX CALCULATION

#### Calculation of Solenoid Transfer Function

The transfer function of a hard edge solenoid (uniform field of certain length) is well known [3]:

$$\begin{pmatrix} \tilde{x} \\ \tilde{x}' \\ \tilde{y} \\ \tilde{y}' \end{pmatrix} = M_{rot} M_f \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} \quad (1)$$

where rotation matrix is:

$$M_{rot} = \begin{pmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & \cos\theta & 0 & \sin\theta \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & -\sin\theta & 0 & \cos\theta \end{pmatrix} \quad (2)$$

and  $\theta = \int eB_{\parallel}(s)/2p ds$ , where  $e$  is electron charge,  $p$  is its momentum, and  $B_{\parallel}(s)$  is axial field. The focusing matrix can be calculated from the following equation:

$$M_f = \begin{pmatrix} \cos\theta & \sin\theta/k & 0 & 0 \\ -k\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta/k \\ 0 & 0 & -k\sin\theta & \cos\theta \end{pmatrix} \quad (3)$$

where  $k$  is  $eB_{\parallel}(s)/2p$ .

The real solenoid can be represented as series of hard edge solenoids with different magnetic fields.

The rotation of beam trajectory was utilized for beam energy measurement.

#### Transport Matrix Calculation

We treated focusing elements as infinitesimally short, separated by known drift space. In this case transfer matrix from the solenoid under test to BPM can be found as:

$$M_{tr} = M_{driftBPM} \prod (M_{foci} M_{drifti}) M_{sol} \quad (4)$$

where  $M_{driftBPM}$  is matrix of drift space between the last focusing element and beam position monitor (BPM) used for position observation,  $M_{foci}$  is matrix of the focusing element (either solenoid or quadrupole), and  $M_{sol}$  is matrix of the solenoid being scanned.

Transfer matrix of the solenoid was calculated using the magnetic measurement data:

$$M_{sol} = M_{drift2} \prod (M_{rot} M_f) M_{drift1} \quad (5)$$

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where  $M_{drift1}$  and  $M_{drift2}$  are transfer matrices of drifts with negative length transporting beam from the solenoid center to the start of the data and from the end of the data to the solenoid center, respectively.

### ENERGY MEASUREMENT

For energy measurement we steer the electron beam with a trim located prior a solenoid and measure the tilt of the beam trajectory in the X-Y plane (see Fig. 1). We use two solenoid settings to suppress roll angles influence. Usage of vertical and horizontal trims suppresses systematic errors with scaling errors and stray magnetic fields [4].

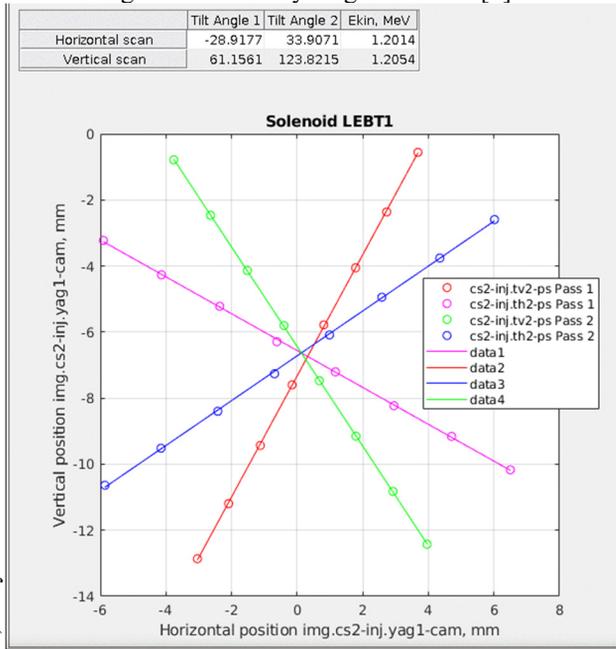


Figure 1: Scan of the beam position on a profile monitor with two trims employed and solenoid current settings. Angles between red and green lines and blue and magenta lines are used for measure of change in the orbit tilt.

The usage of the profile monitor allows tightly focus the beam and perform measurement with low charge, where BPMs have significant noise.

Another advantage of the proposed method is insensitivity to the energy spread. We have used it to measure and phase bunching cavities (see Fig. 2).

The maximal induced energy spread of the measurement shown in Fig. 4 was about 4%. Nevertheless, the obtained data have a good fit allowing straightforward calculation of the cavity voltage and phase.

For the guns with photocathodes one can employ the scanning of the laser spot on the cathode to provide beam motion instead of the using the trims. This measurement can be combined with quantum efficiency map scan of the cathode surface.

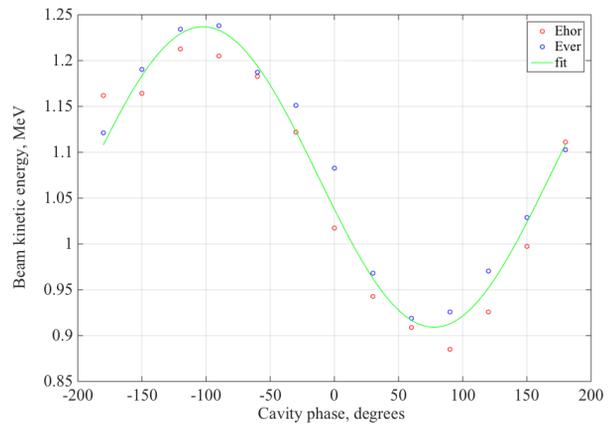


Figure 2: Measurement of the buncher cavities voltage and phase. Beam energy from the gun is 1.073 MeV and buncher voltage is 164 kV.

### ORBIT MEASUREMENT

For each solenoid current the transfer matrix was calculated according to the Eq. (4). Then two  $6 \times 2N$  matrix  $R_y$  is formed:

$$X = \begin{bmatrix} x_1 \\ \dots \\ y_N \end{bmatrix} = R \begin{bmatrix} x_0 \\ x'_0 \\ y_0 \\ y'_0 \\ x_{offs} \\ y_{offs} \end{bmatrix} \quad (6)$$

where  $X$   $1 \times 2N$  vectors of recorded positions ( $x_1-x_N, y_1-y_N$ ), and vector  $W$ , appearing on the right side, has offsets and angles of the beam with respect to solenoid axis (X and Y are also relative to this axis) as well as BPM offset.

$$R = \begin{pmatrix} M_{11\ 1} & M_{12\ 1} & M_{13\ 1} & M_{14\ 1} & 1 & 0 \\ M_{11\ 2} & M_{12\ 2} & M_{13\ 2} & M_{14\ 2} & 1 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ M_{31\ N} & M_{32\ N} & M_{33\ N} & M_{34\ N} & 0 & 1 \end{pmatrix} \quad (7)$$

The offsets correspond to the expected beam position if beam is injected directly on the solenoid axis.

For the real-time application, we developed a MATLAB script. The script is setting the solenoid current, waits for the predefined period to the end of the transient process and measures beam position. At the end of the scan fit is performed and data are displayed. The results of the script operation are shown in Fig. 3.

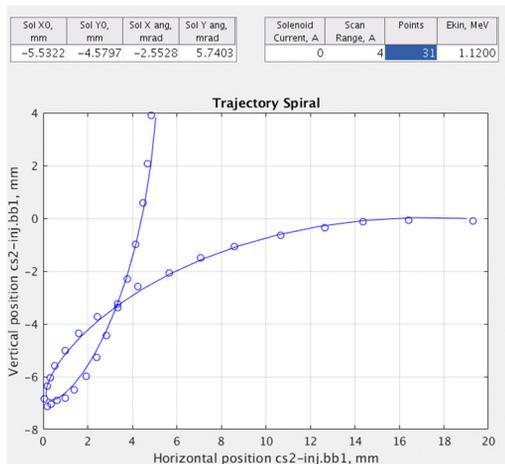


Figure 3: Partial view of a MATLAB GUI for performing solenoid beam-based alignment.

We tested the accuracy of the procedure by changing the incoming angle of the beam with a horizontal trim in front of the solenoid. The results of the test are shown in Table 1. As one can see the agreement is very good. The effect on the vertical plane is most likely due to the roll of the trim.

Table 1: Results of the Procedure Test with Beam

$I_{trim}, A$	Bend Angle, mrad	$X', mrad$	$Y', mrad$
-2.6	10.1	0.23	4.09
0.0	0.0	-10.59	3.29
1.0	-3.9	-14.69	3.05

After the verification, the procedure is routinely used for orbit measurement and correction [5].

## BEAM FUNCTIONS MEASUREMENT

The emittance measurement with scan of a focusing element is a well-known technique. We expanded this technique for measuring the second moments of the beam distribution. We collect data r.m.s. size of the beam image on a profile monitor as well as cross-correlation term  $\langle xy \rangle$ . And using elements of the transport matrix we solve set of linear equations connecting seconds moments with the observable values. The results are shown in Fig. 4.

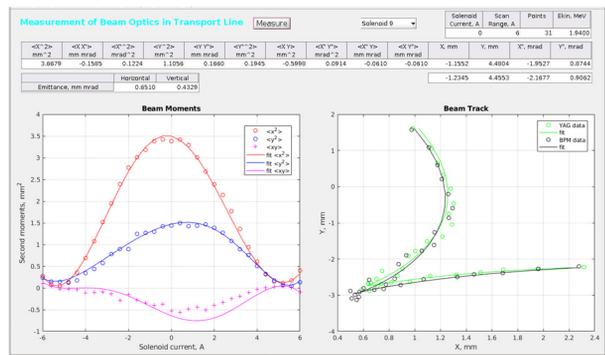


Figure 4: Measurement of the beam second moments.

It should be noted that we can find only 9 terms out of ten because value of  $\langle xy' - x'y \rangle$  is not changed neither by solenoid nor drift space. In our calculations we assumed this value zero.

## CONCLUSIONS

We have developed methods of measuring the multiple beam parameters including energy, trajectory, and beam functions using solenoid in a generalized beam transport line with multiple focusing elements. The methods require knowledge of the axial field of solenoid.

The methods were tested experimentally on two accelerators and three types of solenoids, and were successfully used for the routine measurement of beam parameters.

## REFERENCES

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