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PROPOSE A NON-DESTRUCTIVE STERN-GERLACH APPARATUS FOR MEASURING SPIN POLARIZATION OF ELECTRON BEAM

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Abstract

Mott polarimeter is used for measuring the spin polarization of <10 MeV electron beam destructively. We propose a nondestructive spin polarization measurement device for electron beam based on Stern-Gerlach effect, which include a magnetic quadrupole, Lorenz force compensated electric quadrupole and Beam position monitor. The magnetic quadrupole provides a spin-magnetic interaction force (or Stern-Gerlach force) for the spin polarized electrons. The electric quadrupole provides an electric field force for electrons to offset the Lorentz force induced by the magnetic quadrupole. So that the polarized electron beam only experience the gradient force in the device, which has ability to split the spin polarized electron beam. By measuring the split spin polarized electrons using high resolution beam position monitor, the polarization of electron beam can be calculated. We will present the theoretical analysis and calculation of electron motion in this device.

INTRODUCTION

Measuring the spin polarization of electron beam is important for the use of polarized electron beam in nuclear and high energy physics experiments. All electron beam polarimeters developed to date rely on the spin dependence in one of three electron scattering processes: Mott, Compton, or Møller scattering [1]. In which, Mott polarimeter is used for measuring the spin polarization of electrons beam with energy lower than 10 MeV. Mott polarimeter need extract electrons from the beam line and let them scatter with targets, and thus the electron beam was destructive. Here, we propose a new spin polarization measurement apparatus based on Stern-Gerlach (SG) effect, which is nondestructive to the electron beam. The apparatus includes a magnetic quadrupole, Lorentz force compensated electric quadrupole and beam position monitor.

The magnetic quadrupole is used to produce a nonuniform magnetic field with a constant field gradient in any direction. When a transverse spin polarized electron beam propagating in the nonuniform magnetic field, the spin-magnetic interaction would introduce a nonuniform spin precession of electrons, which arises a space geometrical variant in the transverse plane [2, 3]. This space geometrical variant, measured by the high resolution beam position monitor, can be used to estimated the polarization of electron beam.

STERN-GERLACH EFFECT IN MAGNETIC QUADRUPOLE

The magnetic quadrupole provide a constant magnetic field gradient in any direction and can be given by $K_B = \frac{\partial B}{\partial r} = \frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y}$, which can be up to 600 T/m [4]. When a transverse spin polarized electron beam propagating in the longitudinal direction of the magnetic quadrupole, the Lorentz force and Stern-Gerlach (SG) force vectors of electrons in the quadrupole are shown in Figure 1. The Lorentz force vectors are indicated by blue solid arrows. The SG force vectors, for beam polarized along the *y axis*, are indicated by red hollow arrows. The Stern-Gerlach (SG) deflection is horizontal (along the *x axis*) in this case.

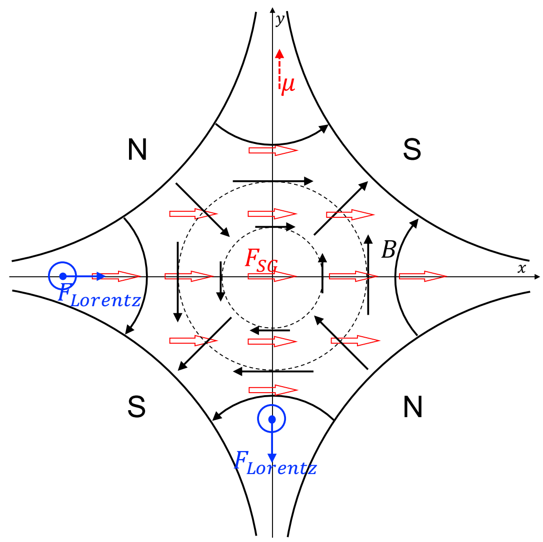

Figure 1: Field and force vectors of electrons in a magnetic quadrupole. The magnetic field vectors are indicated by black solid arrows. The Lorentz force vectors are indcated by blue solid arrows. The Stern-Gerlach force vectors, for beam polarized along the *y axis*, are indicated by red hollow arrows.

The SG force is given by [5]

$$\vec{F}_{SG} = -\nabla U = \mu \frac{\partial B_y}{\partial x} \hat{n}_x = \mu K_B \hat{n}_x \tag{1}$$

where $U = -\vec{\mu} \cdot \vec{B}$ is the potential energy of electron in the magnetic field, and $\vec{\mu} = \frac{1}{2} g \mu_B \hat{n}_y$ is the electron magnetic moment, with $\mu_B = e\hbar/2m$ is the Bohr magneton, $g \approx 2$ is the g factor for electron.

However, electrons will also experience the Lorentz force, which gives a contribution to the motion in x direction and compete with the motion induced by SG force. The Lorentz force of electrons at location x is given by

$$\vec{F}_{Lorentz} = e \frac{p_z}{m} B_y \hat{n}_x = e \frac{p_z K_B x}{m} \hat{n}_x \tag{2}$$

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where p_z is the longitudinal momentum of electrons, B_y is the magnetic field at location x. Electrons in the beam will, in general, pass through the magnetic quadrupole along the longitudinal trajectories with different value of x and hence each electron will experience different Lorentz force, and consequently will have different deflection distance. Electrons at the edge of beam will experience the maximum Lorentz force. Combining the Eqs. (1) and (2), we obtain

$$\frac{F_{SG}}{F_{Lorentz}} = \frac{\hbar}{2p_z x} \quad (3)$$

For an electron beam with energy of 100 keV and width of $x_{max} = 1$ cm, we have $\frac{F_{SG}}{F_{Lorentz}} = 3 \times 10^{-11} \ll 1$, which means the SG force is far smaller than the Lorentz force. Thus, the electron deflection caused by the SG force cannot be observed. However, a electric quadrupole can be introduced to produce a electric field force to offset the Lorentz force, so that electrons only experience the SG force, and the SG deflection can be observed.

DESIGN OF STERN-GERLACH FILTER

The 3D structure of Stern-Gerlach filter is shown in Figure 2(a). The magnetic field gradient (K_B) is set to 50 T/m, the corresponding electric field gradient (K_E) is 8.2176×10^9 V/m^2 for electron beam with energy of 100 keV. The raidus (R) of electric quadrupole is set to 1 mm, so that the maximum electric field strength is $E_{max} = K_E \cdot R =$ 8.2176 MV/m. The 2D distribution of electric field strength (E) simulated by Opera Simulation Software [6] is shown in Figure 2(b), which is similar to the 2D distribution of magnetic flux density (B) in the center of SG filter (r <1 mm). The SG filter provide the constant field gradient in the center of SG filter (r < 1mm), E and B along r is shown in Figure 2(c).

The SG deflection distance in the SG filter can be calculated by

$$\delta x = \frac{1}{2}\frac{F_{SG}}{\gamma m_0}(\frac{L}{\beta c})^2 \quad (4)$$

in which, L is the length of magnetic quadrupole. The electron deflecting direction is determined by the spin direction. Spin-up and spin-down electrons will be deflected opposite direction. As a results of, the spin-up and spin-down beam can be separated by the SG force with separation of $S = 2\delta x$. The separation of spin polarized electron beam with energy of 100 keV, in the SG filter with length of $L = 0.5$ m and magnetic field gradient of $K_B = 50$ T/m, is $S = 3.94$ nm.

The fields at the quadrupoles edge are often modeled as a step function, called Hard-Edge field, as the dash black line shown in Figure 3(a,b). By modelling the fringe field as Hard-Edge field, the electric field force can totally offset the Lorentz force. However, it is an unrealistic and unphysical model, as the electromagnetic fields don't satisfy the Maxwell's equations. The realistic fringe field of quadrupoles can be achieved by solving the Maxwell's equations [7], and the fringe field shape can be modeled by the

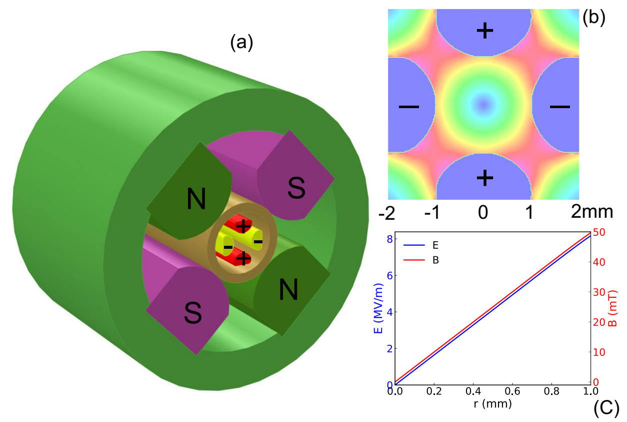

Figure 2: (a) The 3D structure of Stern-Gerlach filter, including a electric quadrupole, a ground shell and a magnetic quadrupole. The aperture of electric and magnetic quadrupoles are 2 mm and 8 mm, respectively. The length of the filter shown here is only 20 mm. (b) The 2D distribution of electric field strength (E), the magnetic flux density (B) has a similar distribution in the center of SG filter. (c) E and B along r. The magnetic field gradient $K_B = 50$ T/m. The magnetic field at r=0.5 mm is 25 mT, and the corresponding electric field is 4.1088 MV/m for electron beam with energy of 100 keV.

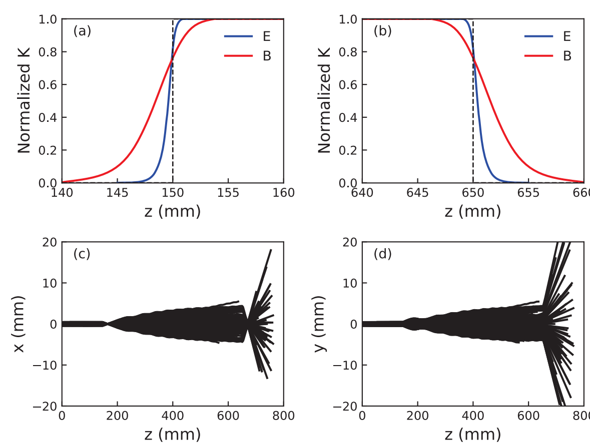

Figure 3: (a,b) Fringe field shape of quadrupoles. Electric and magnetic quadrupoles have the same length of 500 mm, The black dash lines show the edges (also Hard-Edge field). (c,d) The corresponding electron beam trajectories on the yz and xz plane.

fifth order Enge function [8–10]

$$F(s) = \frac{1}{1 + exp[a_0 + a_1(\frac{s}{D}) + ... + a_5(\frac{s}{D})^5]} \quad (5)$$

where $s = z - z_{edge}$ is the position with respect to the quadrupoles edge, D is the aperture of quadrupoles, and $a_i (i = 0, 1, ..., 5)$ are Enge coefficients that can be obtained by solving the Maxwell's equations or by fitting the measured or simulated fringe field data. The resulting fringe

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field gradient is given by

$$K_{fr}(s) = K_B F(s) \quad (6)$$

where K_B is the field gradient in the core region of quadrupoles.

For our studies, the fringe fields are simulated by Opera Simulation Software, and the Enge coefficients are obtained by fitting the simulated field data. The simulated fringe field shape of quadrupoles are shown in 3(a,b). Due to the fringe field shape of electric and magnetic quadrupoles are different, the electric field force can not totally offset the Lorentz force, which means the electron beam would be focused or defocused. To know how the electron beam passing through the SG filter, the General Particle Tracer (GPT) [11] is used to simulate the beam trajectories. The simulated electron beam trajectories passing through the fields described as before are shown in Figure 3(c,d). It indicates that the effective electric field force is smaller than the effective Lorentz force, and the electron beam could not pass through the SG filter.

To make the electric field force offset the Lorentz force and let the electron beam passing through the SG filter, the electric quadrupole is extended. The fringe field shapes of extended SG filter and the corresponding electron beam trajectories are shown in the Figure 4. The electron beam can pass through the SG filter after extending the electric quadrupole.

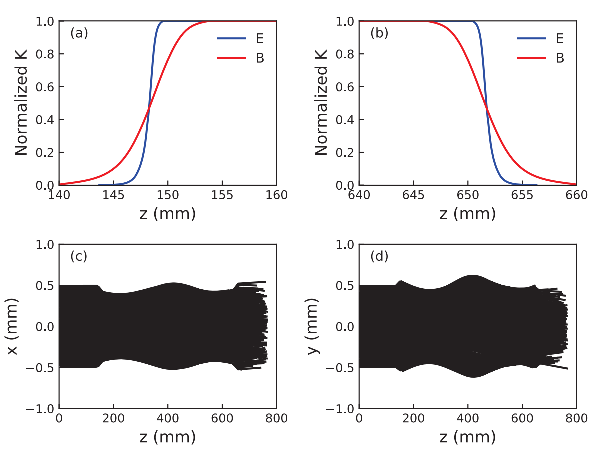

Figure 4: (a,b) Fringe field shape of quadrupoles. The magnetic quadrupole has the length of 500 mm, and the length of electric quadrupole is 502.6 mm. (c,d) The corresponding electron beam trajectories on the yz and xz plane.

SUMMARY

The SG filter including a magnetic quadrupole and a electric quadrupole is proposed to split the spin polarized electron beam. The magnetic quadrupole provides a SG force for the spin polarized electrons, which split the polarized electron beam. The electric quadrupole provides a electric field force for electrons to offset the Lorentz force induced by the magnetic quadrupole. However, the difference of fringe field shapes between electric quadrupole and magnetic quadrupole lead to electric field force couldn't totally offset the Lorentz force. Adjusting the length of electric quadrupole can decrease the effects of fringe field to the electron beam, and the electron beam can pass through the filter after extending the electric quadrupole.

REFERENCES

[1] J. M. Grames, C. K. Sinclair, J. Mitchell, E. Chudakov, H. Fenker, A. Freyberger, D. W. Higinbotham, M. Poelker, M. Steigerwald, M. Tiefenback, *et. al.*, "Unique electron polarimeter analyzing power comparison and precision spin-based energy measurement", *Phys. Rev. Spec. Top. - Accel. Beams*, vol. 7, p. 47, 2004.

[2] E. Karimi, L. Marrucci, V. Grillo, and E. Santamato, "Spin-to-orbital angular momentum conversion and spin-polarization filtering in electron beams", *Phys. Rev. Lett.*, vol. 108, p. 044801, 2012.

[3] E. Karimi, V. Grillo, R. W. Boyd, and E. Santamato, "Generation of a spin-polarized electron beam by multipole magnetic fields", *Ultramicroscopy*, vol. 138, p. 22, 2014.

[4] D. Cesar, J. Maxson, P. Musumeci, Y. Sun, J. Harrison, P. Frigola, F. H. O'Shea, H. To, D. Alesini, and R. K. Li, "Demonstration of Single-Shot Picosecond Time-Resolved MeV Electron Imaging Using a Compact Permanent Magnet Quadrupole Based Lens", *Phys. Rev. Lett.*, vol. 117, p. 024801, 2016.

[5] B. M. Garraway and S. Stenholm, "Observing the spin of a free electron", *Phys. Rev. A*, vol. 60, p. 63, 1999.

[6] Opera Simulation Software: `https://operafea.com`

[7] B.D. Muratori, J.K. Jones, and A. Wolski, "Analytical expressions for fringe fields in multipole magnets", *Physcial Rev. Spec. Top. - Accel. Beams*, vol. 18, p. 064001, 2015.

[8] H.A. Enge, "Effect of extended fringing fields on ion-focusing properties of deflecting magnets", *Rev. Sci. Instrum.*, vol. 35, p. 278, 1964.

[9] M. Berz, B. Erdélyi, and K. Makino, "Fringe field effects in small rings of large acceptance", *Phys. Rev. Spec. Top. - Accel. Beams*, vol. 3, p. 124001, 2000.

[10] F. Carlier and R. Tomás, "Accuracy and feasibility of the β^* measurement for LHC and High Luminosity LHC using k modulation", *Phys. Rev. Accel. Beams*, vol. 20, p. 1, 2017.

[11] S.B.van der Geer and M.J. de Loos, General Particle Tracer (GPT), http://www.pulsar.nl