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PARAMETRIC PUMPED OSCILLATION BY LORENTZ FORCE IN SUPERCONDUCTING RF CAVITY

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Abstract

Mechanical instabilities have been observed in superconducting RF cavities, when multiple cavities are driven by a single klystron and these cavities are regulated by vector-summing the outputs from these cavities. A nonlinear theory has been developed to study the source of this mechanical instability, which is due to the coupling between Lorentz force detuning and mechanical oscillation by parametric pumping. Analytical and numerical analysis of this model show regions of stability, limit cycles and instabilities. These results are in agreement with the observed oscillations by TRIUMF's eLinac Acceleration Module.

INTRODUCTION

In the TRIUMF ARIEL facility, electron acceleration is achieved currently by 3 9-cell Tesla type cavities [1]. The injection Cryomodule is powered by its own klystron, however, the 2 acceleration Cryomodules are driven by one single klystron. Vector sum feedback control is used for field stabilization. Although vector sum feedback control has been proven to work well for pulse operated machines [2],

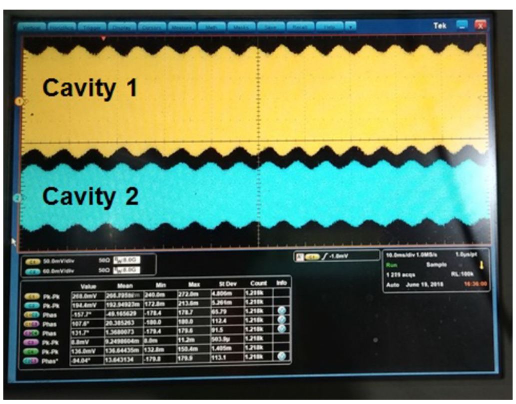

Figure 1: Experimental results showing the voltages from 2 cryomodules that have counter phase oscillations.

CW operation presented some new challenges. One of these challenges is observed at TRIUMF in July 2018, when the push for higher gradient was attempted. At low gradient the vector sum system was stable. When the field gradient was increased, amplitude oscillations started to grow in both cavities and eventually settled into counter-phase limit cycle oscillations as shown in Fig. 1. Hence the vector sum was perfectly stable while both cavities are ringing. As these oscillations only build up over several seconds this phenomenon was not observable in pulsed machine. Previous nonlinear theory [3] did not draw any conclusion on the regime in this paper, the effect of Lorentz force on a single cavity with no field regulation will be analyzed. It will be shown that under certain operating conditions, parametric pumped oscillation and limit cycle can occur.

TIME DEPENDENT LORENTZ FORCE

A RF cavity can be simulated by a parallel RLC lumped circuit at its operating frequency as

$$\frac{1}{R}\frac{dV}{dt}+\frac{d^2C}{dt^2}V+2\frac{dC}{dt}\frac{dV}{dt}+\frac{d^2V}{dt^2}C+\frac{1}{L}V=\frac{2}{Z_0}\frac{dV_f}{dt}. \quad (1)$$

Using $V = ve^{i\omega t}$, $\omega^2 = \frac{1}{LC}$ and $\tau = \frac{1}{RC} \equiv \frac{1}{\tilde{\omega}}$, together with a dimensionless detuning length a and variation in a as

$$a \equiv \frac{\omega_0-\omega}{\tilde{\omega}}, \quad x \equiv \delta a \equiv \frac{\delta\omega_0}{\tilde{\omega}} \quad (2)$$

Eq. (1) can be simplified to

$$\tau\dot{v}+(1-ja)v = v_f \quad (3)$$

Let the variation in v be δv, then the variation part for Eq. (3) is

$$\tau\delta\dot{v}+(1-ja)\delta v - jx\delta v - jv_0x = 0 \quad (4)$$

The phase shift between δv and v, v_f is represented by the imaginary part j, which is due to the detuning. Since x oscillates around a, we can let

$$x = r(t)\cos t \quad (5)$$

where $r(t)$ is a slow varying function of t, which is normalized so that a period of the oscillation is $2\pi t$. After this renormalization, the values of the dimensionless numbers a and x remain unchanged, but ϖ becomes the ratio between the rf bandwidth to the mechanical frequency. For later use, we define

$$y \equiv -\dot{x} = r\sin t \quad (6)$$

The solution for δv can be expressed as a power series

$$\delta v = \sum_{k=0}\left(\mu_k\cos kt + \nu_k\sin kt\right) \quad (7)$$

We can write Eq.(8) in a more compact form by defining

$$\upsilon_k = \mu_k - i\nu_k \quad (8)$$

and Eq. (8) becomes

$$\delta v = \sum_{k=0}\Re\left(\upsilon_k e^{ikt}\right) \quad (9)$$

By substituting Eq. (9), Eq. (5) in Eq. (4) and applying Euler's identity we get

$$\begin{aligned} &i\tau\sum_{k=0}k\upsilon_k e^{ikt} + (1-ja)\sum_{k=0}\upsilon_k e^{ikt} = \\ &jr\frac{e^{it}+e^{-it}}{2}\sum_{k=0}k\upsilon_k e^{ikt} + jv_0 r\frac{e^{it}+e^{-it}}{2} \end{aligned} \quad (10)$$

Collecting terms of the same order in e^{ikt} for $0 \le k \le 2$

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$$\upsilon_0 = j\frac{r}{2}\frac{1}{1-ja}\upsilon_1 \tag{11}$$

$$\upsilon_1 = \frac{1}{2}\frac{1}{1+i\tau-ja}jr\left(v_0+\upsilon_0+\upsilon_2\right) \tag{12}$$

$$\upsilon_2 = j\frac{r}{2}\frac{1}{1+i2\tau-ja}\upsilon_1 \tag{13}$$

Eq. (12) is $O(r)$ and $O(r^3)$, while that of Eq. (11) and Eq. (13) are $O(r^2)$. By applying Method of perturbation we first solve Eq. (12) to first order

$$\upsilon_1 = \frac{1}{2}\frac{1}{1+i\tau-ja}jrv_0 \tag{14}$$

and gets

$$\upsilon_1 = \frac{1}{2}v_0 r\frac{a\left(-1+\tau^2+2i\tau\right)+j\left(1+\tau^2-i\tau^3-i\tau\right)}{\left(1+\tau^2\right)^2} \tag{15}$$

Then apply this result to Eq. (11) and Eq. (13). The results are then used to reiterate Eq. (12) to obtain the third order result. By combining the results of Eq. (15), Eq. (11) and Eq. (13) we get

$$\delta v^* v_0 + v_0^*\delta v = -\frac{1}{2}\left|v_0\right|^2\frac{\tilde{\omega}^2}{\left(\tilde{\omega}^2+1\right)^2}\cdot \left[\begin{array}{c} 2ar\left[\left(\tilde{\omega}^2-1\right)\cos t+2\tilde{\omega}\sin t\right] \\ +r^2\left\{\frac{\tilde{\omega}^2\left[\left(\tilde{\omega}^2+3\right)\cos 2t+3\tilde{\omega}\sin 2t\right]}{\varpi^2+8}+1\right\} \\ +4ar^3\frac{\tilde{\omega}^4}{\tilde{\omega}^2+1}\left[\frac{5\cos 3t+\left(\tilde{\omega}^2+6\sin 3t\right)}{\tilde{\omega}^2+8}\right] \end{array}\right] \tag{16}$$

MECHANICAL DYNAMICS

The mechanical dynamics describe the physical movement of the walls for a cavity driven by Lorentz force variations. Defining the Lorentz force variation as

$$G \cong \Lambda\left(\delta v^* v_0 + v_0^*\delta v\right) \tag{17}$$

By applying Eq. (16) to Eq. (17) we can write

$$G = g_1 + g_{20} + g_{21} + g_3 \tag{18}$$

where

$$g_1 = \frac{1}{2}\Lambda\left|v_0\right|^2\frac{\tilde{\omega}^2}{\left(\tilde{\omega}^2+1\right)^2}2a\left[\left(\tilde{\omega}^2-1\right)x+2\tilde{\omega}y\right] \tag{19}$$

$$g_{20} = \frac{1}{2}\Lambda\left|v_0\right|^2\frac{\tilde{\omega}^2}{\left(\tilde{\omega}^2+1\right)^2}\frac{\tilde{\omega}^2}{\tilde{\omega}^2+8}\left[\begin{array}{c}\left(\tilde{\omega}^2+3\right)\left(x^2-y^2\right) \\ +6\tilde{\omega}xy\end{array}\right] \tag{20}$$

$$g_{21} = \frac{1}{2}\Lambda\left|v_0\right|^2\frac{\tilde{\omega}^2}{\left(\tilde{\omega}^2+1\right)^2}\left(x^2+y^2\right) \tag{21}$$

$$g_3 = \frac{1}{2}\Lambda\left|v_0\right|^2\frac{\tilde{\omega}^2}{\left(\tilde{\omega}^2+1\right)^2}\frac{\tilde{\omega}^4}{\tilde{\omega}^2+1}\bullet \frac{4ax}{\tilde{\omega}^2+8}\left[\begin{array}{c}5\left(x^2-3y^2\right) \\ +\left(\tilde{\omega}^2+6\right)\left(x^2+y^2\right)\end{array}\right] \tag{22}$$

The In-phase component of g_1 (the term x) represents oscillation. The quadrature-phase component of g_1 (the term y) determines the growth/decay of the orbit.

Newton's 2[nd] law written in phase space is

$$\begin{pmatrix}\dot{x} \\ \dot{y}\end{pmatrix} = \begin{pmatrix}0 & -1 \\ 1 & \varsigma\end{pmatrix}\begin{pmatrix}x \\ y\end{pmatrix} + \begin{pmatrix}F=0 \\ G\end{pmatrix} \tag{23}$$

where ς is the viscous damping coefficient and G is a force excluding the internal restoring force. When $G=G(x, y)$, the system is a parametric system. Linearizing the system around its equilibrium point $x, y = 0$ one gets

$$\begin{pmatrix}\dot{x} \\ \dot{y}\end{pmatrix} = \begin{pmatrix}0 & -1 \\ 1+\left.\frac{\partial G}{\partial x}\right|_{x,y=0} & \varsigma+\left.\frac{\partial G}{\partial y}\right|_{x,y=0}\end{pmatrix}\begin{pmatrix}x \\ y\end{pmatrix} \tag{24}$$

Letting $\varsigma = 0$, $|a|<<1$, then the eigenvalues of Eq. (24) are

$$\lambda_{1,2} \cong -a\Lambda\left|v_0\right|^2\frac{\tilde{\omega}^3}{\left(\tilde{\omega}^2+1\right)^2} \pm i \tag{25}$$

Hence the linearized system has an unstable spiral center at $x, y = 0$ for $a < 0$ and a stable spiral center for $a > 0$. In particular, the growth/decay rate has a maximum at $\varpi = \sqrt{3}$. As the cavity vibrates mechanically, the RF voltage lags behind due to ϖ. While the In-phase term of the Lorentz force changes the equivalent spring constant of the mechanical vibration, the quadrature term can either drive or damp this vibration. An important characteristics of parametric oscillations that distinguish them from forced oscillation is that if the initial amplitude is zero, it will remain so, a fact we can used to suppress these oscillations.

A. Limit Cycle, Bifurcation

From Eq. (19) and Eq. (21), where g_1 depends on the product $a.x$ and $a.y$, g_{20} depends on x^2, xy and y^2. These give rise to the possibility that at some x,y there exist a sign change for $\boldsymbol{G}$. This is expressed in a more rigorous definition as the Poincare-Andronov-Hopf bifurcation [4], where it shows that the above system satisfied all the 3 requirements for a supercritical bifurcation, and leads to the existence of stable limit cycles as illustrated in Fig. 2. When an unstable oscillation grows above a certain radius its trajectory crosses over to the stable region. Then non-linear effects causes the trajectory to linger longer in this stable region, therefore balancing between growth and decay and resulting in a limit cycle. We can get an estimate of the radius of the limit cycle by letting $g_1 \approx g_{21}$. Then for $\varpi = 2$, $r \approx 10a$. Note that the radius of the limit cycle is many times larger than the initial detuning, which implies the dynamic detuning will be quite large even for a small initial

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detuning before the system settles into a limit cycle oscillation.

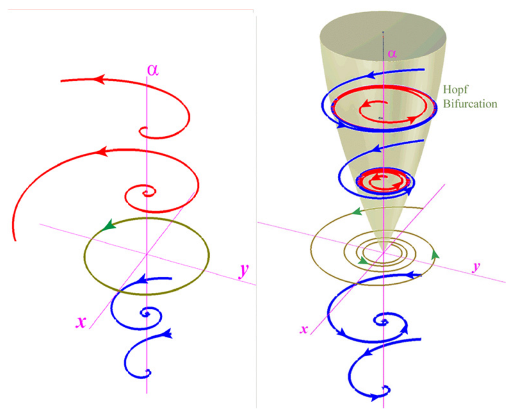

Figure 2: Trajectories for different detuning on linear and nonlinear systems due to Hopf Bifurcation.

SYSTEM SIMULATION

We can apply directly Eq. (4), Eq. (17) and Eq. (23) to form a set of coupled non-linear differential equations

$$\begin{pmatrix} \tau\delta\dot{v}_i \\ \tau\delta\dot{v}_q \\ \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} -1 & -a & \frac{-a}{1+a^2}v_f & 0 \\ a & -1 & \frac{a}{1+a^2}v_f & 0 \\ 0 & 0 & 0 & -1 \\ -\Lambda\frac{2}{1+a^2}v_f & -\Lambda\frac{2a}{1+a^2}v_f & 1 & \varsigma \end{pmatrix} \begin{pmatrix} \delta v_i \\ \delta v_q \\ x \\ y \end{pmatrix} + \begin{pmatrix} -x\delta v_q \\ x\delta v_i \\ 0 \\ 0 \end{pmatrix} \tag{26}$$

This set of equations can then be numerical integrated using some arbitrary non-trivial initial conditions. The results obtained can be used to compare with that obtained analytically. Fig. 3 shows the growth rate vs. ϖ, which clearly shows the dependence of the growth/decay rate on ϖ matches well with theory. Fig. 4 shows the phase space plot of the cavity wall movement for an initial detuning angle of a = -0.026 or -1.5° at 2 different initial displacements. The theory predicts that although $a < 0$ is unstable, a stable limit cycle of $r \approx 10a$ exists. This is confirmed by the trajectories computed in Fig. 4. The simulation can be easily extended into feedback controlled vector sum multiple cavities by imposing the feedback conditions

$$v_{f_{i,q}} = v_{i_{ref},q_{ref}} - K_p \sum \delta v_{i,q} \tag{27}$$

DISCUSSION

The conditions for Lorentz force induced parametric oscillation are quite specific. We need the following condition to occur:

- A high electric fields ($\approx$ 10MV/m), which is only in the domain of superconducting cavities.
- Poor voltage regulation, such as regulating multiple cavities using vector sum feedback.
- CW-operation, as it takes several seconds for these oscillation to grow.
- Driven frequency higher than cavity's resonant frequency.

Since these oscillations are parametrically pumped, to overcome or suppress these oscillations one can either suppress the initial displacement and its speed in the form of microphonic suppression, or provide artificial damping to the system.

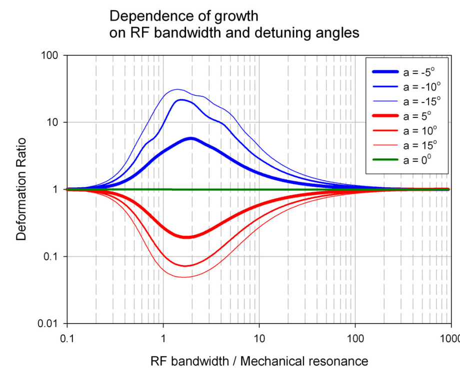

Figure 3: Grow/Decay Rate for different detuning.

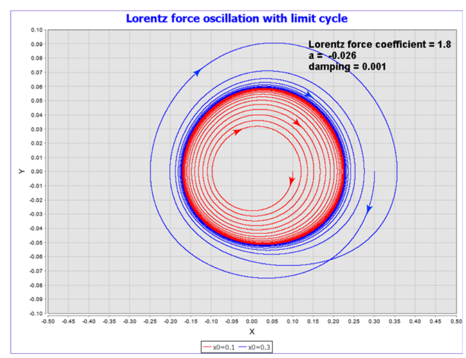

Figure 4: Limit cycle for x0=0.1 (red trajectory) and x0=0.3 (blue trajectory). The trajectory of the blue curve crosses over itself due to 3rd and higher order nonlinearity.

CONCLUSION

We have presented a theoretical analysis of Lorentz force parametric pumped oscillation on a single cavity. The analysis provides a good understanding how and why these oscillations grow or decay. This provides possible solutions to stabilize vector sum cavities.

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