

# SAWTOOTH GENERATION AND REGULATION WITH A SINGLE FPGA FOR TRIUMF'S ARIEL PREBUNCHER

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## Abstract

TRIUMF's ARIEL prebuncher is powered by a sawtooth waveform which is the combination of an 11.79 MHz, a 23.57 MHz and a 35.36 MHz components. The generation, control and regulation of these three components are all incorporated digitally inside a single FPGA. This FPGA can be standalone or inserted inside a VXI module. Commands and controls of these components can be directly through Ethernet, or indirectly through register-base or message-base VXI addresses.

## INTRODUCTION

The required sawtooth signal for TRIUMF's ARIEL project is the combination of an 11.79 MHz, a 23.57 MHz and a 35.36 MHz components which are the harmonic of the 5.893 331 MHz reference signal. Therefore, they are all coherently in phase. Instead of using three channels of ADC and DAC to sample and generate the three harmonic signals [1], the digital LLRF system adopts one channel ADC and DAC to achieve the amplitude and phase close loop control of the three signals. In this way, the system is more compact and the hardware cost is reduced significantly. To ensure the phase of the output sawtooth waveform is phase coherent with the reference signal, a digital phase-locked loop is implemented to lock the output phase to the reference signal. The system block diagram is shown in Fig. 1.

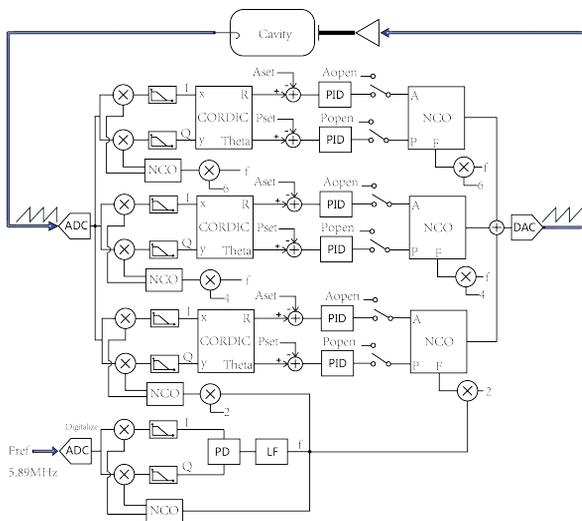


Figure 1: Diagram of Digital LLRF system for ARIEL.

## DIGITAL PHASE-LOCKED LOOP

As the digital LLRF system works in driven mode, by default, the phase of the output signal is not coherent with the reference signal. To resolve this issue, a digital phase-locked loop is introduced to the system. The digital phase-locked loop is based on a Costas loop which is widely used in communication systems. The basic Costas loop is used to lock the frequency of the NCO to the 5.89 MHz reference signal. Then, the three harmonic components can be locked to the reference signal by adjusting the frequency tuning words of the NCOs. Assume that the reference signal is:

$$x(t) = A \cos(\omega_c t) \quad (1)$$

And the output of the local NCO is:

$$I_0 = \cos(\omega_0 t + \phi(t)) \quad (2)$$

$$Q_0 = \sin(\omega_0 t + \phi(t)) \quad (3)$$

The mixing results of reference signal and NCO signal are:

$$I_o = A \cos(\omega_c t) \cdot \cos(\omega_0 t + \phi(t)) \quad (4)$$

$$Q_o = A \cos(\omega_c t) \cdot \sin(\omega_0 t + \phi(t)) \quad (5)$$

Eq. (4) ~ (4) can be written as:

$$\begin{cases} I_o = A/2 \{ \cos[(\omega_c + \omega_0)t + \phi(t)] \\ \quad + \cos[(\omega_c - \omega_0)t - \phi(t)] \} \\ Q_o = A/2 \{ \sin[(\omega_c + \omega_0)t + \phi(t)] \\ \quad + \sin[(\omega_c - \omega_0)t - \phi(t)] \} \end{cases} \quad (6)$$

After a the low pass filters, the higher frequency  $\omega_c + \omega_0$  is attenuated, which can be ignored:

$$\begin{cases} I_o = A/2 \cdot \cos[(\omega_c - \omega_0)t - \phi(t)] \\ Q_o = A/2 \cdot \sin[(\omega_c - \omega_0)t - \phi(t)] \end{cases} \quad (7)$$

After the phase detector, which is also a multiplier, the result is:

$$P_e(t) = A/8 \sin(2(\omega_c - \omega_0)t - 2\phi(t)) \quad (8)$$

Based on Eq. (7),  $P_e(t)$  is a function of  $2(\omega_c - \omega_0)t - 2\phi(t)$ . Therefore, the frequency of the local NCO is controlled by the frequency and phase error between the reference signal and the local NCO. While in phase lock mode,  $\omega_c = \omega_0$ , the phase error and frequency error between the two signals are zero.

The loop filter is a key factor in the digital phase-lock loop and is more than a low-pass filter. The active PI filter is used as the loop filter instead of FIR filter due to its delay.

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## AMPLITUDE AND PHASE CONTROL

For a single frequency system, assume the the cavity pickup signal is [2]:

$$u_0(t) = A_0[1 + f(t)] \cos[\omega t + \phi'(t)] \quad (9)$$

where  $f(t)$  is the amplitude modulation signal of the cavity. And the reference signal is:

$$u(t) = A \cos[\omega t + \phi(t)] \quad (10)$$

The output signal of NCO is:

$$I = \cos(\omega_1 t) \quad (11)$$

$$Q = \sin(\omega_1 t) \quad (12)$$

The mixing results of reference signal and NCO signal are:

$$I_1 = A \cos(\omega t + \phi(t)) \cdot \cos(\omega_1 t) \quad (13)$$

$$Q_1 = A \cos(\omega t + \phi(t)) \cdot \sin(\omega_1 t) \quad (14)$$

The mixing results of cavity signal and NCO signal are:

$$I_2 = A_0[1 + f(t)] \cos(\omega t + \phi'(t)) \cdot \cos(\omega_1 t) \quad (15)$$

$$Q_2 = A_0[1 + f(t)] \cos(\omega t + \phi'(t)) \cdot \sin(\omega_1 t) \quad (16)$$

Eq. (12) ~ (15) can be written as:

$$\begin{cases} I_1 = A/2\{\cos[(\omega + \omega_1)t + \phi(t)] + \cos[(\omega - \omega_1)t + \phi(t)]\} \\ Q_1 = A/2\{\sin[(\omega + \omega_1)t + \phi(t)] + \sin[(\omega - \omega_1)t + \phi(t)]\} \end{cases} \quad (17)$$

$$\begin{cases} I_2 = A_0[1 + f(t)]/2\{\cos[(\omega + \omega_1)t + \phi'(t)] + \cos[(\omega - \omega_1)t + \phi'(t)]\} \\ Q_2 = A_0[1 + f(t)]/2\{\sin[(\omega + \omega_1)t + \phi'(t)] + \sin[(\omega - \omega_1)t + \phi'(t)]\} \end{cases} \quad (18)$$

After a the low pass filters, the higher frequency  $\omega + \omega_1$  is attenuated, which can be ignored:

$$\begin{cases} I_1 = A/2 \cdot \cos[(\omega - \omega_1)t + \phi(t)] \\ Q_1 = A/2 \cdot \sin[(\omega - \omega_1)t + \phi(t)] \end{cases} \quad (19)$$

$$\begin{cases} I_2 = A_0[1 + f(t)]/2 \cdot \cos[(\omega - \omega_1)t + \phi'(t)] \\ Q_2 = A_0[1 + f(t)]/2 \cdot \sin[(\omega - \omega_1)t + \phi'(t)] \end{cases} \quad (20)$$

The amplitude can be calculated by:

$$U_2(t) = \sqrt{I_2^2(t) + Q_2^2(t)} = \frac{A_0[1+f(t)]}{4} \quad (21)$$

Eq. (20) indicates that the amplitude is independent from phase, frequency, and their modulation.

The phase error between reference signal and cavity pickup signal is:

$$\begin{aligned} \Delta\theta &= \phi'(t) - \phi(t) = \arctan \frac{Q_2}{I_2} - \arctan \frac{Q_1}{I_1} \\ &= \arctan \frac{I_2 Q_1 - I_1 Q_2}{I_1 I_2 + Q_1 Q_2} \end{aligned} \quad (22)$$

Eq. (21) indicates that the phase error is independent from amplitude, frequency, and their modulation. Therefore, the phase control won't be effect by the phase-locked loop. After the amplitude error and phase error is obtained from the demodulator, a close loop amplitude and phase control is achieved by the PID controller.

TRIUMF's ARIEL prebuncher is driven by three signals. In this case, assume that the pickup signal from the cavity is:

$$\begin{aligned} u_1(t) &= A_0 \cos[\omega_0 t + \phi_0(t)] \\ &+ A_1 \cos[\omega_1 t + \phi_1(t)] \\ &+ A_1 \cos[\omega_2 t + \phi_2(t)] \end{aligned} \quad (23)$$

For  $\omega_0$ , the output of the tuning NCO is:

$$I = \cos(\omega_0 t) \quad (24)$$

$$Q = \sin(\omega_0 t) \quad (25)$$

The output of the mixer is:

$$\begin{cases} I_o = A_0/2\{\cos[(\omega_0 + \omega_0)t + \phi_0(t)] + \cos[(\omega_0 - \omega_0)t + \phi_0(t)]\} + A_1/2\{\cos[(\omega_1 + \omega_0)t + \phi_1(t)] + \cos[(\omega_1 - \omega_0)t + \phi_1(t)]\} + A_2/2\{\cos[(\omega_2 + \omega_0)t + \phi_2(t)] + \cos[(\omega_2 - \omega_0)t + \phi_2(t)]\} \\ Q_o = A_0/2\{\sin[(\omega_0 + \omega_0)t + \phi_0(t)] + \sin[(\omega_0 - \omega_0)t + \phi_0(t)]\} + A_1/2\{\sin[(\omega_1 + \omega_0)t + \phi_1(t)] + \sin[(\omega_1 - \omega_0)t + \phi_1(t)]\} + A_2/2\{\sin[(\omega_2 + \omega_0)t + \phi_2(t)] + \sin[(\omega_2 - \omega_0)t + \phi_2(t)]\} \end{cases} \quad (26)$$

As the frequencies of the three components are far away from each other, after the low-pass filter,  $\omega_0 + \omega_0$ ,  $\omega_1 - \omega_0$ ,  $\omega_1 + \omega_0$ ,  $\omega_2 + \omega_0$ , and  $\omega_2 - \omega_0$  are all attenuated. Eq. (25) can be written as:

$$\begin{cases} I_o = A_0/2 \cos[\phi_0(t)] \\ Q_o = A_0/2 \sin[\phi_0(t)] \end{cases} \quad (27)$$

Eq. (26) indicates that it's possible to use one ADC channel to sample the pickup signal and then digitally down convert it to three components by tuning each NCO of the DDCs. Relative to each component, the other two components are filtered by the low pass filter and therefore the system can split the pickup signal to three components inside FPGA. After each channel is calculated by the PID controller, the output of the LLRF system is the summation of the three opponents.

## HARDWARE DESIGN

The new daughter board accepts a maximum of four RF inputs and provides a maximum of four analog outputs and 4

digital clock in/out ports. The sampling rate of the ADC and DAC is 250MHz and the resolution is 14 bits. The Xilinx ZYNQ FPGA is adopted as the controller. The system is also equipped with two channel of optics fiber interface, USB2.0, Ethernet and serial port. Two 512 MB SDRAMs are used as the RAM for the CPU, and the firmware is stored in a TF card. All these components are mounted on a 14-layer 1 cm\*11 cm PCB board, as shown in Fig. 2.



Figure 2: Digital LLRF system daughter board.

### TEST

The digital LLRF board is tested on the test bench of the LLRF system. The board is first tested open loop to see the frequency spectrum. The test results are shown in Figs. 3-6. The open-loop test show that the three channel of signal works well on one channel ADC. After the open-loop test, the close-loop test is done on the test bench. The blue line in Fig. 7 is the 5.89 MHz reference signal and the purple line is the sawtooth waveform which is the combination of the three components. The result shows that the phase of the output signal is locked to the reference signal.

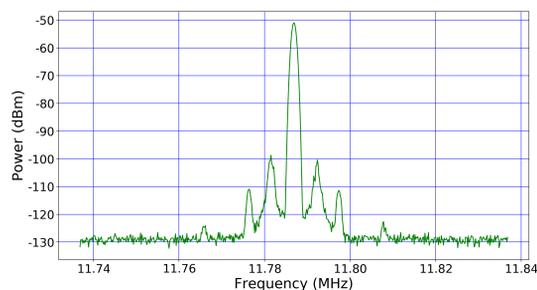


Figure 3: Test result of 11.79 MHz.

### CONCLUSION

An FPGA based digital LLRF system is developed for TRIUMF ARIEL project. This design implements the sawtooth wave form control with one channel ADC and DAC. Preliminary tests show that the requirement of the low-level

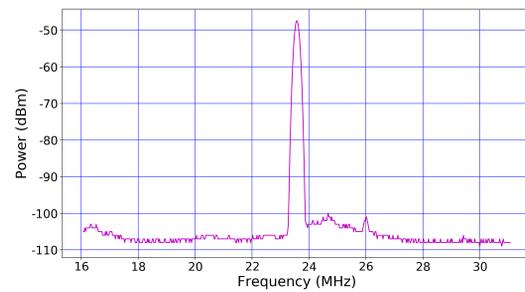


Figure 4: Test result of 23.57 MHz.

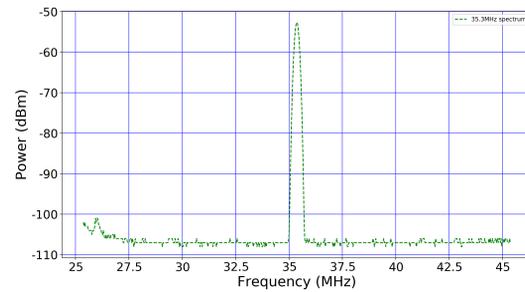


Figure 5: Test result of 35.36 MHz.

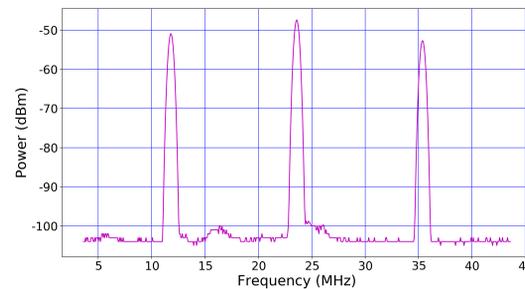


Figure 6: Test result of all three frequency.

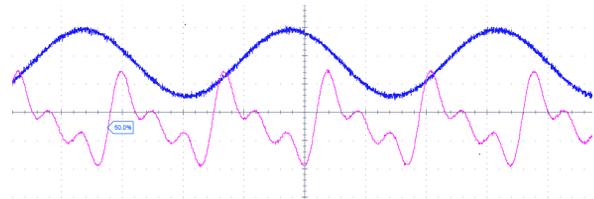


Figure 7: Close-loop test.

RF control system has been satisfied. The digital LLRF system will be applied to the ARIEL system.

### REFERENCES

- [1] K. Fong, M. Laverty, S. Fang, and W. Uzat, "Sawtooth Wave Generation for Pre-Buncher Cavity in ISAC", in *Proc. 17th Particle Accelerator Conf. (PAC'97)*, Vancouver, Canada, May 1997, paper 8P036, pp. 3057-3059.
- [2] CONG Yan, XU Zhe, Li Gang, *et al.*, "Research of Digital Low Level Radio Frequency Control System for HIRFL", *Nuclear Physics Review*, 2012, 29:248-252.