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EXPERIMENTAL OBSERVATION OF LOW-ORDER COLLECTIVE OSCILLATION MODES IN A STRONG-FOCUSING LATTICE*

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Abstract

In a conventional linear Paul trap (LPT), four electrode rods are placed symmetrically around the trap axis to generate a radio-frequency (rf) quadrupole field for transverse ion confinement. The periodic nature of the external focusing potential can give rise to serious ion losses under a specific condition. The loss mechanism is essentially the same as the coherent betatron resonance well-known in intense beam dynamics [1,2]. In fact, the collective motion of an ion plasma in the LPT is shown equivalent to that of a charged-particle beam traveling through an alternating-gradient (AG) focusing lattice.

In the present study, we perform the direct measurement of low-order coherent oscillation modes in the LPT by detecting image currents induced on the electrodes' surfaces. The four-rod structure of the LPT allows us to pick up weak signals from the dipole and quadrupole oscillations of a plasma bunch. These signals are Fourier analyzed to evaluate the coherent oscillation tune at different initial ion densities. The measured tune of the quadrupole mode is used to deduce the tune depression as a function of ion number stored in the LPT.

INTRODUCTION

Because of the long-range Coulomb interaction, the main body of a charged-particle beam, in other words the *beam core* in phase space, exhibits a complex collective feature. The motions of individual particles cannot be independent but have correlation with others especially when the beam is dense. The core motion can be expressed as the superposition of many collective modes whose oscillation frequencies depend on the density of the core. The stability of the whole beam will seriously be affected when the tune of a low-order collective oscillation mode comes close to that of a Fourier harmonic in the periodic driving potential. According to the work of Sacherer [3], the coherent resonance instability of the mth-order mode occurs under the condition

$$m(\nu_0 - C_m\Delta\nu) = n, \quad (1)$$

where ν_0 is the *bare tune*, i.e., the betatron tune at zero beam intensity determined solely by the AG lattice, C_m a m-dependent constant, $\Delta\nu$ the incoherent tune shift induced by the Coulomb repulsion among particles, and n an integer. The size of $\Delta\nu$ can uniquely be given in Sacherer's theory adopting the uniform-density model in which no incoherent tune spread exists (while, in a general non-uniform beam, $\Delta\nu$ depends on which particle we observe). The coherent phase-shift factor C_m is thought to be a complicated function of several parameters including the beam ellipticity, the degree of tune split, etc [4,5].

Okamoto and Yokoya later demonstrated the possibility of the coherent parametric resonance expected under the condition [6]

$$\Omega_m \equiv m(\nu_0 - C_m\Delta\bar{\nu}) = \frac{n'}{2}, \quad (2)$$

where $\Delta\bar{\nu}$ is the root-mean-squared (rms) tune shift uniquely determined for any distribution functions in phase space, and n' is an integer. Equation (2) includes Eq. (1) as a special case. The factor 1/2 on the right-hand side leads to a two-fold increase of the density of resonances in the betatron tune diagram, which is crucial in the design consideration of a high-intensity hadron machine. Recent numerical studies have suggested that the C_m-factor can simply be regarded as a constant over the whole tune space [2,7].

The coherent tune Ω_m determines the locations of resonance bands in the tune diagram and their parameter dependence. The detailed information of Ω_m is thus most valuable. The direct observation of a collective oscillation mode is, however, very difficult in a large-scale accelerator. We here employ a compact LPT that provides, in a local tabletop environment, a many-body Coulomb system physically equivalent to a relativistic charged-particle beam [8]. The coherent tunes of the dipole ($m = 1$) and quadrupole ($m = 2$) modes are measured by picking up weak image-current signals from the LPT electrodes.

EXPERIMENTAL SETUP

The ion species used for the LPT-based experiment at Hiroshima University is mostly $^{40}Ar^{+}$ that can readily be produced from neutral Ar gas atoms by applying a low-energy electron beam from a compact e-gun. The mass and charge state of stored particles in the LPT are not essential to the present study. The initial plasma density

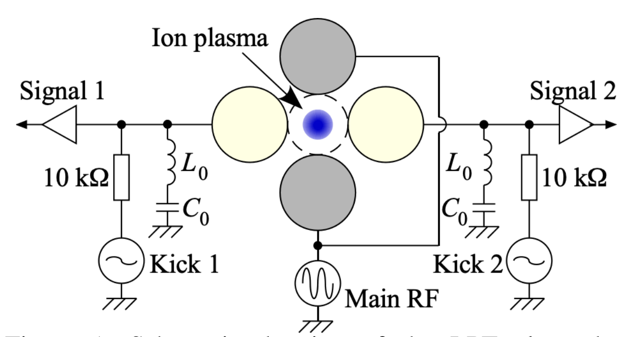

Figure 1: Schematic drawing of the LPT electrodes (cross-sectional view) together with the perturbation driver and image current detector.

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can be controlled by changing the neutral gas pressure and/or the electron-beam current for ionization. The four quadrupole electrodes of 5.75 mm in radius have been divided into several electrically isolated pieces so that we can apply different bias voltages to form an axial potential well [9]. The quadrupole section where a large number of ions are confined for various experiments is 50 mm long. The aperture size is 10 mm in diameter. The sinusoidal rf waveform was employed for simplicity to give a strong transverse AG focusing force to Ar^+ ions [2]. The operating frequency is set at 1 MHz. The horizontal and vertical bare tunes per AG cell (sinusoidal period) were both adjusted to 0.162 at which the plasma is stable. The experimental conditions are summarized in Table 1.

Table 1: Experimental Conditions

Trapped ion	*Species*	*Initial number*
	$^{40}Ar^+$	$10^5 - 10^7$
Plasma confinement region	*Axial length*	*Aperture*
	50 mm	10 mmϕ
Focusing parameters	*RF frequency*	*Bare tune*
	1.0 MHz	0.162

In order to excite a low-order collective oscillation in an ion plasma, we apply transverse kicks of either the dipolar or quadrupolar symmetry periodically at the beginning. The horizontal pair of electrodes are employed for this purpose as illustrated in Fig. 1. The strength and number of the kicks are carefully chosen not to cause ion losses. After shutting down the periodic perturbation, we detect the image currents induced on the surfaces of the quadrupole electrodes and Fourier analyze the signal to obtain the tune of the excited collective mode.

Note that the plasma envelope executes a large quadrupole oscillation driven by the AG confinement force. The resultant image signal, oscillating at 1 MHz, is much stronger than that from a collective mode. In order to separate the former from the latter, series resonators consisting of the inductance L_0 and capacitance C_0 are installed as notch filters (see Fig. 1). Filtered signals are amplified and then recorded by a 12-bit digital oscilloscope. Data acquisition is started right after the shut down of the driving kicks and continues for the duration of 1 ms (10^3 AG periods) at the sampling rate of 250 MHz. The frequency resolution in discrete Fourier transform is, therefore, 1 kHz and the Nyquist frequency is 125 MHz.

RESULTS

Dipole Mode

Low rf perturbation voltages of opposite signs are added to the horizontal pair of the electrode rods initially to shift the plasma centroid. The perturbation frequency is chosen close to the resonant value predicted by Eq. (1), so that we can readily excite a horizontal oscillation of the whole plasma bunch, in other words, the dipole mode ($m = 1$). In this experiment, the perturbation was switched on only for five cycles at the amplitude of 18 mV. The image

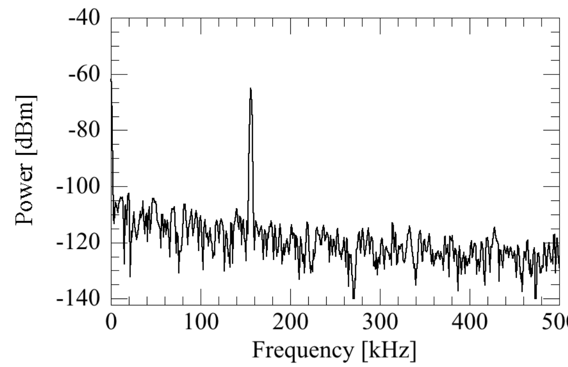

Figure 2: Example of the frequency spectrum of an ion plasma initially perturbed by dipole kicks.

currents on the two electrodes oscillate out of phase if the dipole mode is activated successfully. The difference of these image signals is Fourier analyzed to figure out the tune of this lowest-order ($m = 1$) collective oscillation.

Ideally, the dipole-mode tune should be equal to the bare tune ν_0 because $C_1 = 0$ resulting in $\Omega_1 = \nu_0$. The whole bunch is considered as a sort of rigid macro-particle oscillating at ν_0 about the LPT axis regardless of the space-charge density. Figure 2 shows an example of the frequency spectrum that has a clear peak at around 156 kHz corresponding to the tune of 0.156. The same measurement procedure was repeated at different initial plasma densities to obtain Fig. 3. Contrary to the theoretical expectation, the tune of the coherent dipole mode has weak dependence on the ion density. This is probably due to the image-charge effect analogous to the Laslett tune shift [10]. As the initial plasma density decreases, Ω_1 approaches the bare tune ν_0 (= 0.162).

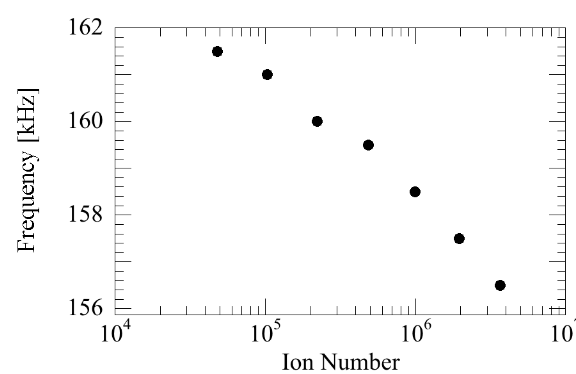

Figure 3: Intensity dependence of Ω_1.

Quadrupole Mode

Compared with the dipole mode, the detection of the quadrupole mode ($m = 2$) is more difficult because the primary AG focusing force causes a large envelope oscillation of the same symmetry. In addition to the linear focusing field (1 MHz), a weak quadrupole perturbation near the resonant frequency (~ 300 kHz) is applied initially for fifteen rf cycles to enhance the signal from the coherent quadrupole mode. Since the image currents induced on the horizontal (or vertical) electrode pair by

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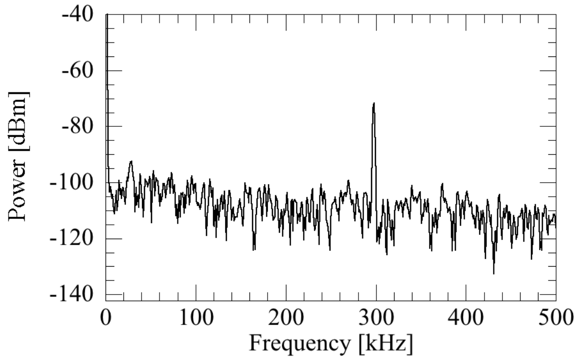

Figure 4: Example of the frequency spectrum of an ion plasma initially perturbed by quadrupole kicks.

this mode execute in-phase oscillations, we take the sum of these image signals to evaluate the coherent tune.

An example in Fig. 4 indicates the existence of a Fourier component oscillating at roughly twice the bare tune ν_0 (162 kHz). As shown in Fig. 5, the coherent tune of the quadrupole mode goes down rapidly as we increase the initial plasma density. The rms tune shift $\Delta\bar{\nu}$ in Eq. (2) due to the space-charge repulsion is responsible for this tune reduction. The reduction rate as a function of the initial ion number is much higher than the dipole case in Fig. 3.

The rms tune shift is related to the rms tune depression η as $\Delta\bar{\nu} = (1-\eta)\nu_0$. Substitution of this relation into Eq. (2) leads to

$$\eta = 1 - \frac{1}{C_2}\left(1 - \frac{\Omega_2}{2\nu_0}\right). \qquad (3)$$

Here, we have put $n' = 2$ because an external driving force can enhance the resonances of the corresponding order only for even n' [2,6]. The resonance condition then becomes identical to the Sacherer's formula in Eq. (1) except for the definition of the space-charge tune shift. Coherent resonances with odd n' in Eq. (2) are driven only by the internal self-field force.

The perturbative analysis of the rms envelope equations predicts that C_2 is equal to 3/4 for the quadrupole mode [5,11]. There is the possibility of another linear-mode excitation (breathing oscillation) with C_2 = 1/2, but this mode seems practically unimportant in quadrupole

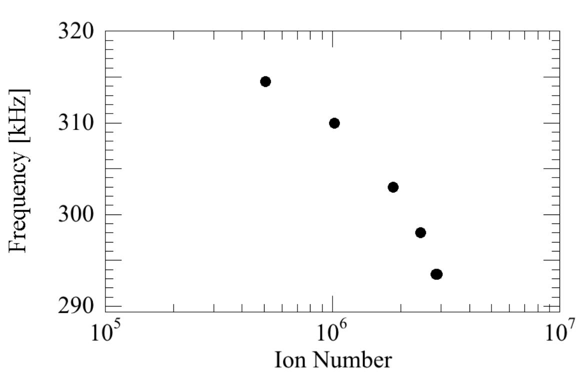

Figure 5: Intensity dependence of Ω_2.

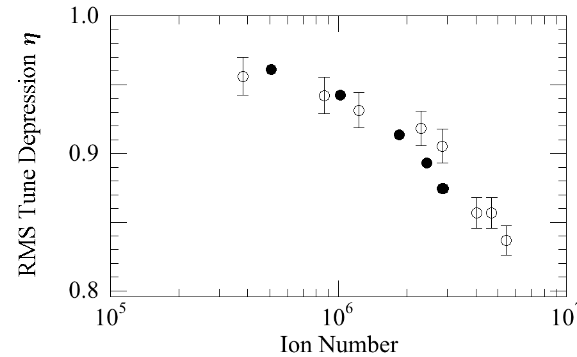

Figure 6: Rms tune depression estimated from the measurement data in Fig. 5 (black dots). For comparison, we have added the previous experimental data (open circles) given in Ref. [13].

focusing channels according to our past experience through self-consistent numerical simulations and ion-trap experiments [2]. Considering the insensitivity of the envelope equations to the detail of the particle distribution function [12], it is reasonable to assume C_2 = 3/4 in Eq. (3). Interestingly, the Sacherer's Vlasov theory for a one-dimensional beam has also concluded the same C_2 value [3]. Equation (3) now enables us to make a good estimate of η from the direct measurement of Ω_2. Recalling that ν_0 is fixed at 0.162 here, the use of the experimental data in Fig. 5 together with C_2 = 3/4 gives the plasma-intensity dependence of η as plotted in Fig. 6. This result is consistent with the previous estimate in Ref. [13]. As the present estimate is based on the direct observation of the coherent oscillation mode, it should be more accurate.

SUMMARY

We succeeded in picking up the signals from the coherent dipole and quadrupole modes excited in an intense ion bunch. The oscillation tunes of these modes were evaluated from the measurement data, both of which show the space-charge-induced shifts as in Figs. 3 and 5. Despite the theoretical prediction that C_1 = 0, we observed weak dependence of the dipole tune on the plasma intensity. This unexpected shift can be attributed to the image charges on the electrode surfaces [1,10]. Naturally, the quadrupole mode is found more sensitive to the change in ion density because of a relatively large coherent tune-shift factor. The rms tune depression η was determined as a function of the initial ion number in the LPT from the measured tune of the coherent quadrupole oscillation.

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