

GENERALISED SCATTERING MODULE IN SIXTRACK 5

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Abstract

A generalised scattering module has recently been added to SixTrack. This module enables the use of arbitrary generators and target profiles. Presently, a simple model of elastic scattering and a coupling to Pythia8 have been implemented. This makes it possible to use SixTrack for studies of aperture losses and beam lifetime as a result of beam-beam scattering.

INTRODUCTION

The SixTrack code [1, 2] is an important tool for predicting the dynamic aperture, distribution of particle losses, and tracking the evolution of the particle distribution during dynamic behaviour of the machine [3, 4]. It is used for these purposes for the Large Hadron Collider (LHC) [5], the High Luminosity LHC (HL-LHC) [6], Future Circular Collider for hadrons (FCC-hh) [7], and other machines. Typically, the main physics of interest is the symplectic tracking of particles through the applied magnetic fields, including non-linear terms as well as RF- and beam-beam fields. However, these electromagnetic fields are not the only effects seen by the particles. They can also be scattered by collimators and apertures [8], beam-gas [9–11], and by the counter-rotating beam at the interaction points (IPs). The modelling of beam-beam scattering at the IPs is a new feature described in this paper.

SCATTER MODULE

While some of the collisions at the IPs are hard processes that lead to an immediate loss of the colliding pair, some of the collisions are softer elastic (no energy loss, or production of new particles) or diffractive collisions (with energy loss, and the possibility of loss of primary particle or production of secondary particles). Scattered particles may stay in the machine, and within the aperture, after a scattering event, but with a larger amplitude.

The effect on the beam distribution is in part determined by the optics at the IPs, where this effect is more important. This is especially the case for high- β^* optics [12], as the initial angular distribution of the beam particles at the IPs are then narrower compared to standard collision optics.

In order to simulate these processes, as well as be able to expand to cover more cases in the future, the Scatter Module in SixTrack is designed to be a flexible framework for implementing additional processes and event generators. Currently, two models have been implemented and are described here: a fit to experimental data, and a coupling to the Pythia event generator.

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The SixTrack Input File

The general configuration of SixTrack simulations is described in the user manual [13], but briefly, each module of SixTrack is configured through a separate section of a text input file. Each such section, referred to as a “block”, is started by a module-specific keyword. The block is closed by the keyword NEXT.

The Scatter Module in SixTrack is configured through the SCATTER input block in the main SixTrack input file, and by adding special elements in the lattice description file. An example of the Scatter Module blocks in the main input file is shown in Figure 1.

```
-----  
PYTHIA-----  
LOSSES  
SPECIES PROTON PROTON  
ENERGY 7000000 7000000  
SEED 41  
PROCESS SINGLEDIFFRACTIVE  
NEXT  
  
SCATTER-----  
SEED 42  
GEN sc_elas PPBEAMELASTIC 0.046 18.52 4.601 2.647 0.2 30.0  
GEN sc_pyth PYTHIASIMPLE 12.0  
PRO ipFLAT FLAT 3e25 938.0 7000000  
ELEM ip1_sc ipFLAT auto 1.0 sc_elas  
ELEM ip5_sc ipFLAT auto 1.0 sc_pyth  
NEXT
```

Figure 1: Example of the PYTHIA and SCATTER blocks in a SixTrack input file [13].

The special elements injected into the lattice description file must be of type number 40, and all other parameters are set to 0. When the tracking code encounters an element of this type, the Scatter Module is called, and the events of the scattering processes requested in the configuration block are generated and applied to the tracked particles. The scatter elements are zero-length, and can therefore be placed at the same location as any other thin element in the lattice. Typically, one would want to put such an element in the interaction points.

Scatter Configuration

The configuration of the Scatter Module consists of three main keywords.

The event generator, keyword GEN, takes a name and a generator algorithm plus its parameters, and a cross-section. Currently, SixTrack provides an internal Monte Carlo event generator that will sample a fit to data taken by TOTEM [14, 15]. Alternatively, SixTrack can integrate with Pythia8, and use its SoftQCD event generator. The details of these implementations are covered in the next sections.

The target profile of the beam is described with the PRO keyword. It takes a name, followed by a profile keyword and its parameters. Currently, a “flat” target is supported, meaning in this context a uniformly dense target. Alternatively, a bi-Gaussian density profile can be provided, with and optional offset relative to the tracked beam.

The generator and the profile are combined for each element described in the lattice that one wishes to run the scatter generator for. This is configured with the element keyword ELEM. It takes the name of the element, as set in the lattice file, followed by the name of the target density profile. The third argument provides a way to control the scattering probability. It can either be set to “auto”, in which case the probability is computed from the cross-sections of the generators and the profile density. Alternatively, a fixed probability can be set. The fourth argument is a scaling factor for the probability. This value can be controlled by the Dynamic Kicks Module (DYNK) [4]. DYNK provides the possibility to change the scaling on a turn-by-turn basis, which can be useful if one for instance wants to switch on scattering in one turn only, and track how many turns the scattered particles survive. The remaining arguments are a list of the generators to be used for the given element. Several generators can be applied to a single element, and their branching ratios are computed from the cross-section of each generator.

Scatter Event Output

The Scatter Module produces two dedicated output files during tracking. It also injects information into the Collimation Module log files if this module is enabled.

- **Scatter Log:** The main log file is detailing every single particle scatter event, which particle was affected, which process it underwent, and what scatter angle and energy loss it received. It also contains information about the statistical scaling of these events.
- **Scatter Summary:** The file sums up the turn-by-turn and element-by-element scatter events, the ratio of each type of process, and their final cross section.

PHYSICS IMPLEMENTATION

The Scatter Module can support many different generators and profiles. These are all treated equally, being reduced to providing a cross-section as a function of centre-of-mass energy, and for the selected process, a momentum change t . Here, the Mandelstam variables $t = (p_1 - p_3)^2$ and $s = (p_1 + p_2)^2$ are used, where p_1 is the four-momentum of the incoming tracked particle, p_2 of the incoming “target” particle, and p_3 the outgoing simulated particle.

Process Selection

The probability P of a scattering process occurring in an element is either set explicitly in the input block, or computed from the target density and the profile, given by

$P = \sigma \rho(x, y)$, where $\sigma(s)$ is the cross-section of the process and $\rho(x, y)$ is the target profile density at the point x, y . The scaling factor is also applied to this probability. If multiple event generators are added to the same element, the branching ratio between them is calculated. The Monte Carlo method is then used to decide which process to apply to each particle according to the computed probabilities.

Kinematics

Both of the generators generate the events for head-on collision at centre-of-mass, assuming both particles are at the reference energy. In the Pythia interface, the scattering angle θ , and in case of diffractive processes the momentum loss $\delta p/p$, is retrieved from the event generator. The internal elastic generator only provides the Mandelstam variable t .

For the internal elastic generator, the scattering angle θ is computed according to

$$\theta = \cos^{-1} \left(1 - \frac{t}{2|\vec{p}|^2} \right), \quad (1)$$

which assumes that $m_0 c^2 \ll |\vec{p}|c$, and that the particle three-momentum \vec{p} only changes direction and not magnitude.

When a momentum loss is given, it is applied directly to the tracked particle. Here we assume that the error introduced by applying the momentum loss of an event generated at reference energy directly to the tracked particles, with a relative energy spread typically of an order of 10^{-3} – 10^{-4} , is negligible compared to the physics models or experimental fit used by the event generators. The scattering angle θ is added directly to the existing particle angle, while the azimuthal angle ϕ is sampled uniformly between 0 and 2π :

$$x'_3 = \theta \cos(\phi) + x'_1 \quad (2)$$

$$y'_3 = \theta \sin(\phi) + y'_1, \quad (3)$$

where x'_1, y'_1 are the horizontal and vertical plane of the incoming particle, and x'_3, y'_3 are the horizontal and vertical plane of the outgoing particle.

EXPERIMENTAL FIT MODEL

In this model, the differential cross-section $d\sigma(t)/dt$ as a function of the Mandelstam t is assumed to be on the form

$$\begin{aligned} g(t) &= \frac{1}{a_1^2} \frac{d\sigma}{dt} \\ &= e^{-b_1 t} + 2a e^{-(b_1+b_2)t/2} \cos(\phi) + a^2 e^{-b_2 t} \\ &\equiv g_1(t) + g_2(t) + g_3(t). \end{aligned} \quad (4)$$

Here, the parameters $a = a_2/a_1$, b_1 , b_2 , an ϕ are given by a fit to parts of a data set from TOTEM [15] as shown in Figure 2. The overall normalisation of $g(t)$ is arbitrary, since the total cross-section is set explicitly in the input.

The underlying amplitude for this differential cross-section is

$$A(t) = a_1 e^{-b_1 t/2} + a_2 e^{b_1 t/2 + i\phi}, \quad (5)$$

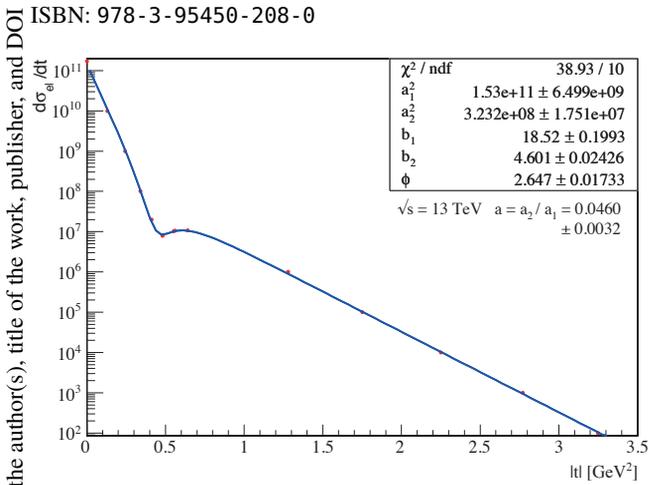


Figure 2: Differential cross-section data in arbitrary units and corresponding fit for the elastic “experimental fit model”, using Equation 4.

such that $d\sigma(t)/dt = A(t)A^*(t)$. This shows the interpretation of the fit parameters a_1 and a_2 as the sub-process amplitudes, b_1, b_2 describe how quickly the amplitudes fall off with increasing t , and ϕ is the relative phase between them.

Scattering angles are generated by sampling t from $g(t)$, and then converting this to θ using Equation 1. A minimum cutoff t_{\min} can be provided to sample larger scatter angle for special study. The sampling is done by first sampling from a function $h(t) = g_1(t) + g_3(t) > g(t)$ (since $\cos(\phi) < 0$), and then correcting the oversampling of the “dip” in $g(t)$ by rejecting it and drawing a new t if $g(t)/h(t) > R$, where R is a uniformly distributed pseudo-random number. The initial sampling of $h(t)$ is done by first selecting either $g_1(t)$ or $g_3(t)$ through comparing their relative probability

$$\int_{t_{\min}}^{\infty} \frac{g_3(t)}{g_1(t) + g_3(t)} dt \quad (6)$$

to a uniform random number, and then sample the chosen branch via inverse transform sampling. This procedure is efficient. The rejection sampling has a hit ratio of 87% with $t_{\min} = 0$ and the parameters shown in Figure 2.

PYTHIA INTEGRATION

The main tool for generating diffractive events is Pythia8 [16]. To make the integration easier, SixTrack can be compiled with the PYTHIA build flag, which will link the two applications into a single executable. This makes the Pythia event generator available through the SixTrack input file through a PYTHIA configuration block. The block allows for basic configuration of the event generator. Optionally, a full Pythia8 input file can be set in the block, which is then passed on to Pythia during initialisation.

When a valid PYTHIA block is provided, Pythia becomes available as a generator in the SCATTER block, alongside the internal elastic scattering module. See Figure 1 for an example of a PYTHIA input block used as an event generator for the SCATTER module.

The Pythia generator allows for elastic scattering, and single, double, and central diffractive events. It also allows for non-diffractive events if particle losses are permitted. See the Pythia manual for further details on the SoftQCD event generator [17].

Double diffractive and non-diffractive events can generate losses. Specifying the LOSSES flag in the Pythia block will allow this to occur, while if the flag is not present, in the double diffractive case, events resulting in a loss of the primary tracked particle are discarded, and a new event requested. Particle losses are logged to the scatter log file.

DISCUSSION AND OUTLOOK

The presented module is a new, experimental feature of SixTrack, which have been recently developed in order to study what happens to particles undergoing small-angle scattering at the interaction points of the HL–LHC. The code is written as a generalised scattering module, capable of treating most types of scattering processes for which generators are provided. Currently a “thin” scattering target is assumed, i.e. no gradual energy loss or multiple scattering can be treated, and only a single process can be applied per particle in a single element on a single turn. This approximation is sufficient for rare processes, allowing for the study of beam–beam scattering, and possibly beam–gas.

For the future, as well as continuing simulations of beam–beam scattering, it may be interesting to implement a “thick” scattering model, treating the target not as a plane but as an object with an extent in the s -direction. This would make it possible to treat also scattering with material (as happens in collimators), while also making it easier to compare the loss distributions in time and location between different scattering models.

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