

A HIGH-PERFORMANCE CODE FOR BEAM DYNAMICS SIMULATION OF SYNCHROTRONS

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Abstract

This paper introduces a high-performance code Li-track for beam dynamics simulation of synchrotrons. It is a parallel multi-particle tracking program written entirely in C++ and therefore has a high computational speed. The overall design of Li-track is based on object-oriented mode, and the implemented element model can be easily reused to build different synchrotron lattice. The symplectic integral algorithm is used to ensure there are no physical errors in a long-term simulation. This code has been used for the slow extraction simulation of XiPAF synchrotron and the results will be given in this paper.

INTRODUCTION

In the field of radiation application, the beam is generally expected to be quasi-continuous in time structure, which requires the slow extraction technology to slowly extract particles from a proton synchrotron. The spill length of slow extraction is generally 0.1~10 s, which corresponds to tens of thousands or even tens of millions of turns in the ring. In order to achieve sufficient statistical accuracy, it is generally necessary to simulate a beam bunch with tens of thousands of particles. Therefore, the simulation is large in scale and time-consuming. Traditional synchrotron design program such as MAD [1] can facilitate the lattice design, and carry out small-scale tracking calculations, but it is difficult to satisfy the simulation demand of slow extraction. The dedicated multi-particle simulation program of circular accelerator, such as ORBIT [2], has powerful computing capacity, but does not provide the function of conveniently customizing elements, so it is difficult to realize the transverse RF excitation element required for slow extraction.

In order to facilitate the research on the slow extraction process of synchrotrons and other dynamics problems, we develop a high-performance simulation code, Li-track. This code is based on the concept of object-oriented design and can easily extend various types of elements. In the following, we will introduce the simulation algorithm, program structure, and give some examples to verify the effectiveness and computational capacity of the program.

SIMULATION ALGORITHM

There are a variety of simulation algorithms that can simulate particle motion in synchrotrons, such as transfer matrix method, higher-order transfer matrix method [3], Lie-algebraic method [4] used in MAD8 and various integral methods [5]. The advantage of the transfer matrix method is that the calculation is simple, but it only considers the linear effect in motion. The high order transport matrix method makes up for the deficiency of the transfer matrix method, but its computational cost increases significantly. Meanwhile, this method is usually not symplectic. In a long-term simulation, non-symplectic algorithm may lead to the spontaneous increase or decrease of the emittance. The Lie-algebraic method can preserve the non-linear effect and symplecticity, but the complex mathematical tools have to be applied on the implementation.

The method we use in Li-track code is symplectic integral algorithm [5, 6]. It has symplectic property and is relatively simple to implement. And the higher order integral method [6] can also achieve higher calculation accuracy and speed.

The motion of particles can be described with six-dimensional coordinates (x, p_x, y, p_y, t, p_t) , and we use the same Hamiltonian as MAD8 to describe the motion of particles.

$$H = \frac{p_t}{\beta} - (1 + hx) \sqrt{\left(1 + \frac{p_t}{\beta}\right)^2 - p_x^2 - p_y^2 - \frac{p_t^2}{\beta^2 \gamma^2}} + V(x, y; h). \quad (1)$$

Where, $V(x, y; h)$ denotes the magnetic potential; h is the curvature of the element; $h=0$ for straight elements, such as quadrupoles, sextupoles, etc., and $h=1/r$ for dipoles, r is the bending radius. The motion of particles can be obtained by solving the following equations.

$$\frac{d\vec{x}}{ds} = -\frac{\partial H}{\partial \vec{p}}, \quad \frac{d\vec{p}}{ds} = -\frac{\partial H}{\partial \vec{x}}. \quad (2)$$

Where, $\vec{x} = (x, y, t)$ is the position coordinate and $\vec{p} = (p_x, p_y, p_t)$ is the momentum coordinate. For the straight elements, $h=0$, so the Hamiltonian can be split into $H_1(p_x, p_y, p_t)$, the component consisting only of momentum coordinate, and $H_2(x, y, z)$, the component consisting only of position coordinate. It can be proved that the following integral algorithm is symplectic [6].

$$\vec{x}_{n+1} = \vec{x}_n - s \frac{\partial H_1(\vec{p}_n)}{\partial \vec{p}}, \quad \vec{p}_{n+1} = \vec{p}_n - s \frac{\partial H_2(\vec{x}_{n+1})}{\partial \vec{x}}. \quad (3)$$

The subscript n and $n+1$ are the identification before and after the coordinates update, and s is the integral step. The

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first step of the algorithm only changes the position coordinates, and then updates the momentum coordinates according to the new position coordinates, so it is called the "Drift-Kick" method, and its duality "Kick-Drift" method is also symplectic. The higher order symplectic integral algorithm is also the idea of using this "Drift-Kick", but the integral step of each step is different, refer to [6] for details. For dipoles, the position coordinates and the momentum coordinates are coupled, but the Hamiltonian can still be split into two terms, and then they can be solved in turn, the specific formula may refer to [5].

PROGRAM STRUCTURE

There are two main kinds of objects involved in the beam dynamics of synchrotrons, one is the ring composed of dipoles, quadrupoles, and other elements, and the other is particle beam. We abstract these actual objects into objects in the C++ language, which facilitates the management of their internal data and enhances the scalability of the program. The model library based on object-oriented technology makes it easy for the program to model different synchrotrons.

As shown in Figure 1, Li-track implements the following classes: (1) Synchronous particle Class (Syn_par), responsible for managing the overall information of the beam; (2) Beam Class (Herd), responsible for recording the specific coordinates of each particle, loss and other information; (3) Element base Class (Element), the specific element is derived from the base class, each element realizes its specific particle tracking algorithm; (4) Ring Class (Ring), responsible for assembling various Element Class (Element) and Beam Class (Herd) for turn-by-turn simulation.

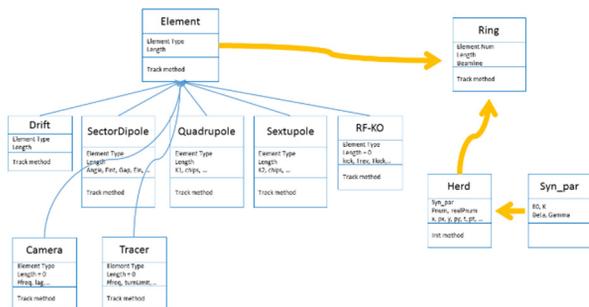


Figure 1: Li-track overall structure (blue arrows represent derivative relationship, and yellow arrows represent calling relationship).

SIMULATION EXAMPLES

In this section, we will provide a series of examples to demonstrate the performance of Li-track code, using the lattice of XiPAF [7] proton synchrotron. The examples shown mainly involve the trajectory of a single particle in the vicinity of third-order resonance and the simulation of the resonant slow extraction process. We compare the calculation results of Li-track with the ones of MAD8 to verify the correctness of the program. A long-term tracking of single particle was carried out to test symplectic conservation.

MC5: Beam Dynamics and EM Fields

D11 Code Developments and Simulation Techniques

Comparison with MAD8

For straight elements, if the symplectic integral algorithm is used, the appropriate step size should be adopted for integral operation, if the step is too small, the calculation amount is too large, and if the step is too large, the simulation accuracy will be reduced. In order to determine the appropriate integral step size, we divide each quadrupole, each sextupole, etc. on the ring into several segments to integrate. Figure 2 shows the phase trajectories calculated under different segment numbers k . Since the simulation case is very close to third-order resonance, particles may be lost when the integral step is too large ($k < 3$). As the fourth-order precision algorithm is adopted, the results converge quickly, and when $k > 4$, the phase trajectories are already very close, so the number of $k=5$ used in the subsequent simulations. The effective lengths of the quadrupole and the sextupole are 0.2 m and 0.15 m, respectively, which equivalents to an integral step of 0.04 m and 0.03 m.

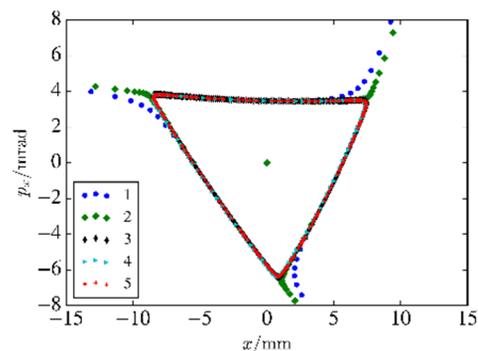


Figure 2: The phase trajectories calculated by Li-track under different integral step sizes ($k=1,2,3,4,5$).

According to the reasonable integral step size obtained by the simulation experiments, the phase trajectories calculated by Li-track and MAD8 are compared, and shown in Figure 3. As can be seen from these plots, the phase trajectories calculated by Li-track are in good agreement with ones by MAD8.

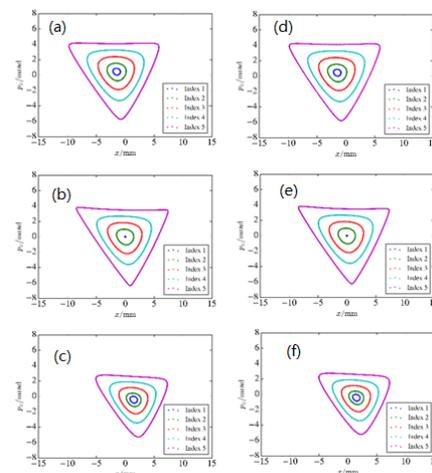


Figure 3: The phase trajectories calculated by Li-track (the momentum dispersion are (a) -1×10^{-3} , (b) 0, (c) 1×10^{-3}) and MAD8 (the momentum spread are (d) -1×10^{-3} , (e) 0, (f) 1×10^{-3}).

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Symplectic Conservation Test

A very important application of Li-track is the slow extraction process simulation, which requires tens of thousands or even millions of turns. If the algorithm is not symplectic, even if there is no transverse excitation, the amplitude of the particles may change significantly, resulting in errors of simulation results. To test symplectic conservation of the algorithm, the particle coordinates were tracked for 20000 turns, and the results are shown in Figure 4. Figure 4 (a) shows the phase trajectories calculated according to the algorithm described at the above, during 20000-turn tracking process, the phase trajectories are basically closed, and the amplitude of the particle remains stable, see Figure 4 (b). If the equation (3) is changed to:

$$\vec{x}_{n+1} = \vec{x}_n - s \frac{\partial H_1(\vec{p}_n)}{\partial \vec{p}}, \quad \vec{p}_{n+1} = \vec{p}_n - s \frac{\partial H_2(\vec{x}_n)}{\partial \vec{x}}. \quad (4)$$

That is, the position coordinates and momentum coordinates are changed synchronously, the symplectic condition will be destroyed. Applying this change to sextupoles on the ring, the 20000-turn tracking shows that the phase trajectories of particles cannot be closed, see Figure 4 (c), and for particles with positive momentum dispersion, the amplitude of the particles shrinks significantly after a long-term simulation, see Figure 4 (d), which is inconsistent with the theoretical expectation.

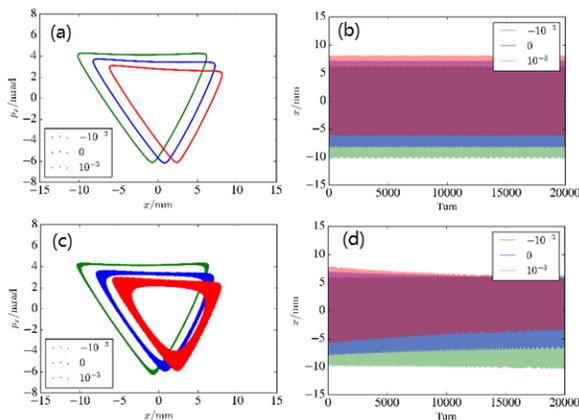


Figure 4: Symplectic conservation test ((a) phase trajectories and (b) amplitude are calculated by the symplectic integral algorithm; (c) and (d) are results using equation (4)).

Slow Extraction Process Simulation

Based on the flexible programming ability of Li-track, we can simulate the whole process of slow extraction. In this section, we have simulated the slow extraction process of XiPAF synchrotron based on third-order resonance and RF-KO (Radio Frequency Knock Out) method [8].

Before the formal extraction, the sextupoles strength gradually increases and the triangular stable region is established in the phase space. Then RF-KO element begins to work, kicking the particle out of the stable region and out of the ring. Figure 5 is the phase space evolution during the gradual enhancement of sextupoles, only a small amount of particles was lost in this process due to the slow change of the sextupoles strength. Figure 6 shows the simulated time structure of spill, in this calculation, coloured

noise method was used to cover the spectrum range of the transverse oscillation of particles. Through adjusting the waveform of excitation intensity with time, the spill can be more uniform in time structure.

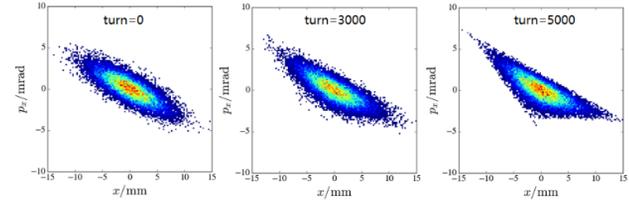


Figure 5: The phase space evolution during the gradual enhancement of sextupoles.

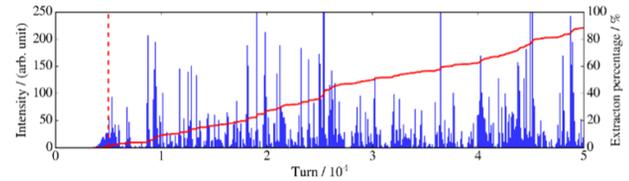


Figure 6: The time structure of spill.

Based on Li-track code, many beam dynamics problems during the slow extraction process can be studied, such as the optimization of slow extraction parameters [8, 9], the influence of magnetic field ripple on spill uniformity [9, 10], low energy slow extraction simulation under strong space charge effect [11], and so on.

At present Li-track code has been upgraded to the parallel version, which uses multi-core computation, the slow extraction process with the spill length of seconds can be simulated with high speed.

CONCLUSION

In order to study various beam dynamic problems during the slow extraction process of synchrotrons, a high-performance Code, Li-Track, has been developed, which is a parallel multi-particle tracking program based on MPI. The simulation program has the following characteristics: (1) By adopting the symplectic integral algorithm, the stability of long-term simulation can be guaranteed; (2) It implements object-oriented design, easy to extend and build different synchrotron lattice; (3) The program is written in C++ language and upgraded to parallel version with high computational efficiency. Li-track code has been applied to XiPAF synchrotron slow extraction design, and many helpful results have been obtained.

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