CAN A PAUL ION TRAP BE USED TO INVESTIGATE NONLINEAR QUASI-INTEGRABLE OPTICS?

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Abstract

Here we describe the design of an experimental setup using the IBEX Paul trap to test nonlinear quasi-integrable optics, an accelerator lattice design to create stable high intensity beams. In 2010 Danilov and Nagaitsev found a realisable nonlinear potential which can create integrable optics in an accelerator when embedded in a linear lattice that provides round beams. This concept will be tested in the IOTA ring at Fermilab. It is important to further test this concept over a wide parameter range, preferably in a simplified experimental setup such as IBEX. The IBEX Paul trap is capable of replicating the transverse dynamics of a high intensity accelerator without dispersion or chromaticity.

INTRODUCTION

Accelerators are primarily designed using linear components, such as dipole and quadrupole magnets. As there is no coupling between the horizontal and vertical directions in an ideal linear system, the motion of particles can be described by two uncoupled Hill's equations. The invariant of such a system is the Courant-Snyder invariant. Through the introduction of the Twiss parameters α, β and γ the Hamiltonian associated with the motion of the single particle becomes time independent. It can therefore be separated into two invariants, the horizontal and vertical Hamiltonians. These equations are solvable, so that the motion of the trapped particle is known to be bounded. The system is then described as integrable. A system consisting of N degrees of freedom is integrable if there are also N invariants of the motion [1].

Realistically, accelerators cannot be constructed only from linear magnets as linear systems are susceptible to perturbations. As non-linear components are inevitable, a nonlinear lattice that is also integrable is required, known as Nonlinear Integrable Optics (NIO). It is possible to find a combination of fields along the particle orbit that lead to a set of equations with two invariants (making the machine integrable, as the system has two degrees of freedom, horizontal and vertical). The challenge in doing this is that the fields must also obey Maxwell's equations. In [1] Danilov and Nagaitsev present a completely integrable solution which is also physically realisable. If a nonlinear lattice is integrable then even when effects such as momentum deviation, magnet misalignment and space charge are included the system will always be close to an integrable solution.

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THE QUASI-INTEGRABLE CASE

The fully integrable solution presented in [1] will be tested at the Integrable Optics Test Accelerator (IOTA) ring at Fermilab, first using electrons and then protons, with a series of complex magnets used to create the required nonlinear potential [2]. However, as creating such a highly non-linear potential is an ambitious project they will also test the quasi-integrable case, which requires only octupole magnets [3] [4]. The required quasi-integrable potential is given by

$$V(x, y, s) = \frac{k}{\beta(s)^3} \left(\frac{x^4}{4} + \frac{y^4}{4} - \frac{3x^2 y^2}{2} \right), \quad (1)$$

where k is the strength of the focusing and s the distance around the ring. Such a large nonlinear potential leads to amplitude dependent particle tunes, resulting in nonlinear decoherence [5].

PAUL TRAPS FOR ACCELERATOR PHYSICS

The Intense Beam Experiment (IBEX) at the Rutherford Appleton Laboratory, Oxfordshire, is a Paul trap used for accelerator physics. A Linear Paul Trap (LPT) confines ions in the transverse plane using an electrical quadrupole. Because of this, the Hamiltonian describing the transverse dynamics of particles in a Paul trap is equivalent to that for an alternating focusing channel in an accelerator [6]. This equivalence means that LPTs can be used to study resonances and beam loss in high intensity accelerators effectively. Ion loss in a LPT does not damage the system or activate components as the ions stored are of low energy. Furthermore, both cell tune and intensity can be varied over a large range in a LPT, accessing a parameter space that would be challenging for a single accelerator.

We aim to use IBEX, with modifications to meet the requirements outlined in this paper, to test quasi-integrable optics. The experimental setup of the IBEX LPT has been described in detail elsewhere [7], so will be covered here only briefly.
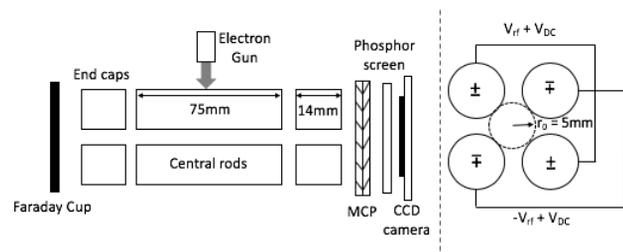
Figure 1: Diagram of the IBEX linear Paul trap.

Figure 1 shows the layout of the IBEX apparatus, which is contained within a vacuum vessel. The vessel is held at $\sim 10^{-11}$ mbar. When experiments are performed argon gas is introduced to a pressure of $\sim 10^{-7}$ mbar. The gas is then ionised in the trapping region using an electron gun, producing Ar^+.

Ions are confined transversely using four cylindrical rods, to which a time varying rf voltage is applied in the form of either a sinusoid or a step function (V_{rf}). A DC voltage of 10 V (V_{DC}) is also added to the rf to assist with extraction of ions from the trap. The voltage applied to the rods is created using an arbitrary wavefunction generator, which is then amplified. The same voltage is applied to opposing rods and the voltages applied to the two rod pairs are 180 degrees out of phase. The ions are confined longitudinally via the end caps, to which the rf voltage is also applied, with a DC offset of 25 V. As the end caps are significantly shorter than the central rods this creates a rectangular longitudinal potential and the ion distribution is assumed to be longitudinally homogeneous.

To extract the ions from the trap the DC offset is removed from one set of end caps, extracting the ions either towards the Faraday cup or the Micro Channel Plate (MCP), both of which are destructive detectors. The Faraday cup measures the number of ions extracted and the MCP provides an integrated image of the ion distribution.

EXPERIMENTAL DESIGN

T-Insert

The potential given in Eq. (1) leads to a quasi-integrable lattice, but magnets which create this potential alone are not sufficient for a real accelerator, either quadrupole magnets or a solenoid channel are still required to focus the beam. The IOTA accelerator uses strong focusing, with the nonlinear magnets which create the integrable potentials placed in the drift region of a linear lattice. To create a quasi-integrable lattice equal beta functions and round beams are required in the octupole region, furthermore, a phase advance of $n\pi$, where n is an integer, is also required between the drift regions.

An accelerator lattice with equal focusing in the horizontal and vertical directions can be created through strong focusing, using a lattice with the transfer matrix of a thin, axially symmetric lens. Such a strong focusing lattice, called a T-insert, should then leave a drift region of equal beta functions where the nonlinear magnets can be installed [2], or in the case of a Paul trap, a time period where the nonlinear fields can be applied.

Fortunately, in the case of a LPT dispersion is always zero, meaning that only 6 parameters must be controlled by the lattice ($\alpha_{x,y}$, $\beta_{x,y}$ and $\mu_{x,y}$), for each parameter a magnet is required, meaning that the simplest solution is a symmetric system of 12 magnets between non-linear insertion sections [8]. The Paul trap equivalent of each of these magnets is a square voltage pulse applied to the rods. The maximum beta function in both the horizontal and ver-

tical directions must also be constrained, if the transverse ion distribution is too large the ions will scrape on the confining rods and will be lost. Finally, the voltage applied must be in line with the limitations of the IBEX wave function generators and amplifiers. The maximum voltage of IBEX electronics is ~200 V, however, the rise and fall time is amplitude dependent. Due to this, a T-insert lattice was sought using pulses of 25 V, where the rise time is ~90 ns, the length of voltage pulses was varied, with the minimum operational limit assumed to be a pulse of width ~300 ns. A basin hopping algorithm was used to create a lattice which meets these specifications, this lattice is shown in Fig. 2.
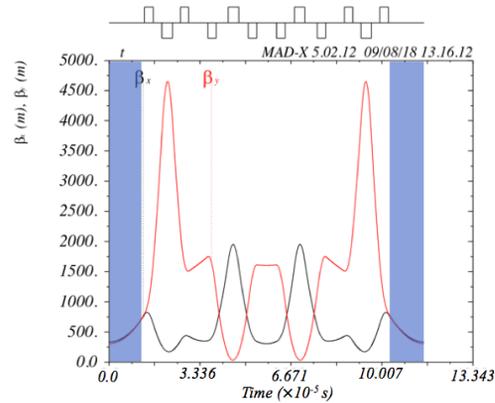
Figure 2: The T-insert lattice and beta functions in the horizontal and vertical directions, the region highlighted in blue is the drift region where the octupole would be inserted. Beta function is calculated assuming that $s = c\tau$, where τ is time, and c the speed of light.

Octupole Region

In IOTA the octupole potential is realised using a channel of 20 octupole magnets [9], however, in a LPT there is the freedom to create an octupole potential that varies smoothly in time, an example of the octupole voltage profile across the drift region required for the T-insert in Fig. 2 is shown in Fig. 3. As the IBEX trap is currently linear, creating an octupole potential will require an upgrade to the trap.

Furthermore, the trap will still need to be capable of creating a near perfect quadrupole potential, so that the T-insert section of the lattice can be produced. We must be able to create the required octupole potential by applying an achievable voltage to the rods. The Beam Physics group at Hiroshima University have already performed a study of a number of different designs for a nonlinear Paul trap, with the intention of controlling sextupole and octupole fields [10]. This study attempted to find a solution that is also easy to manufacture and that has sufficient space to position the electron gun. It was found that the best solution involves four metal plates placed between the existing rods. Figure 4 shows the layout which minimises higher order multipoles when only the quadrupole rods are powered.

The voltages that must be applied to both the rods and plates to give a specific octupole were then found, these voltages are shown in Fig. 3.
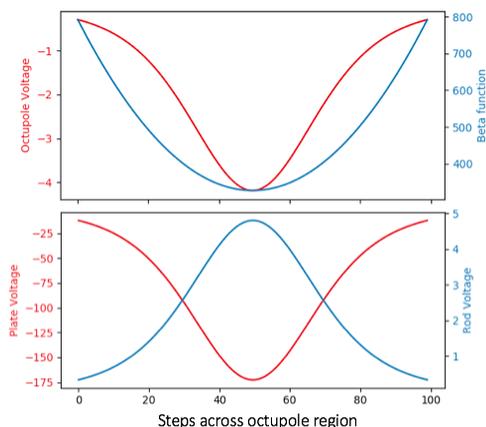
Figure 3: Beta function and corresponding voltages for the lattice shown in Fig. 2. Rod and plate voltages correspond to the voltages applied to the trap in Fig. 4.
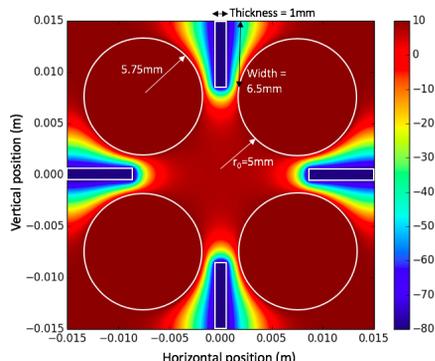
Figure 4: Diagram of the octupole trap designed at Hiroshima university [10], showing how voltages are applied to create the octupole and DC offset. The colour bar indicates voltage.

SIMULATION

To study varying octupole amplitudes we use the particle in cell code Warp [11]. Instead of simulating the full T-insert and octupole region the transfer matrix of the T-insert was used. Within the octupole regions an ideal, time varying, octupole field was applied to the ions. The transfer matrix of the T-insert region was then used to progress ions through the simulation to the next octupole region. Using this method we injected a number of ions into the system with zero momentum but a range of initial locations and tracked them for a number of turns. Using NAFF [8] [12] we were able to calculate the tune spread for a given voltage amplitude (see Fig. 5b), and evaluate the dynamic aperture of the lattice, defined as here as the largest circle in real space

within which no particles are lost (as shown in Fig. 5a). The tune spread shown is similar to that predicted for IOTA, in [4].
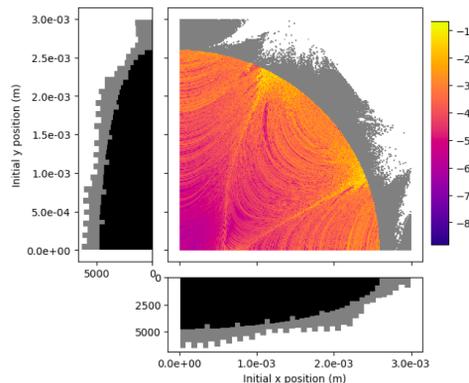
(a) Dynamic aperture
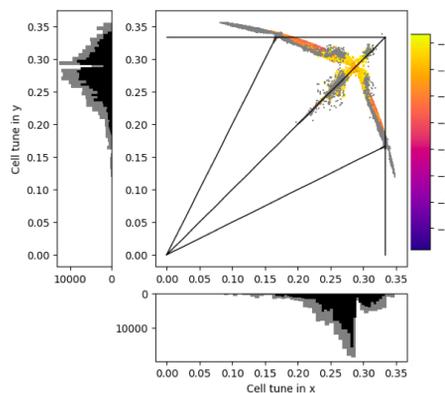
(b) Tune spread calculated using NAFF

Figure 5: The dynamic aperture and tune spread over 1000 periods when a perfect octupole potential is applied in the drift region of the lattice shown in Fig. 2. The T-insert is applied using a single matrix transformation. The colour bar shows the log of the change in tune over the simulation for particles within the dynamic aperture. Grey regions are particles that survived but are outside of the dynamic aperture, which was found to be 2.6 mm.

DISCUSSION

Testing of T-insert lattice shown in Fig. 2 has started using the IBEX Paul trap. Further diagnostics will be required to show that round beams have been achieved, a fast phosphor and camera installed after the MCP are being investigated for this purpose. An octupole trap has been built and is in commissioning at the University of Hiroshima, early results indicate that the added plates do not effect the quadrupole trapping [13], indicating that a nonlinear Paul trap can be used to test quasi-integrable nonlinear optics.

REFERENCES

[1] V. Danilov and S. Nagaitsev, "Nonlinear accelerator lattices with one and two analytic invariants", *Phys. Rev. ST Accel.*

Beams, vol. 13, 084002, 2010.

[2] S. Antipov *et al.*, "IOTA (Integrable Optics Test Accelerator): facility and experimental beam physics program", *Journal of Instrumentation*, vol. 12, T03002, 2017.

[3] S. A. Antipov, S. Nagaitsev and A. Valishev, "Quasi-integrable nonlinear optics experiment at IOTA", in *AIP conference proceedings*, 1777, 100001 (2016). `doi:10.1063/1.4965682`

[4] S. Antipov, S. Nagaitsev and A. Valishev, "Single-particle dynamics in a nonlinear accelerator lattice: attaining a large tune spread with octupoles in IOTA", *Journal of Instrumentation*, vol. 12, P04008, 2017.

[5] S. D. Webb *et al.*, "Suppressing Transverse Beam Halo with Nonlinear Magnetic Fields", in *Proc. 4th Int. Particle Accelerator Conf. (IPAC'13)*, Shanghai, China, May 2013, paper THYB101, pp. 3099–3102.

[6] H. Okamoto *et al.*, "Radio-frequency quadrupole trap as a tool for experimental beam physics", *Nucl. Instr. Meth. Phys. Res. A*, vol. 485, pp. 244-254, 2002.

[7] S.L. Sheehy *et al.*, "Commissioning and first results of the intense beam experiment (IBEX) linear Paul trap", *Journal of Physics Conference Series*, 2017.

[8] K. Ruisard, "Design of a Nonlinear Quasi-integrable lattice for resonance suppression at the university of Maryland electron ring", Ph.D. thesis, Phys. Dept., University of Maryland, 2018.

[9] S. A. Antipov, K. Carlson, A. Valishev, S. J. Wesseln, and R. Castellotti, "Design of Octupole Channel for Integrable Optics Test Accelerator", in *Proc. 7th Int. Particle Accelerator Conf. (IPAC'16)*, Busan, Korea, May 2016, pp. 1731–1733. `doi:10.18429/JACoW-IPAC2016-TUPOR030`

[10] K. Fukushima and H. Okamoto, "Design study of a multipole ion trap for beam physics applications", *Plasma and Fusion Research: Regular Articles*, vol. 4, 2015. `DOI:10.1585/pfr.4.000`

[11] A. Friedman, R.H. Cohen, D.P. Grote, S.M. Lund, W.M. Sharp, J.L. Vay, I. Haber, and R.A. Kishek, "Computational methods in the warp code framework for kinetic simulations of particle beams and plasmas", *IEEE Transactions on Plasma Science*, 42(5):1321-1334, 2014.

[12] Code from Dr. K. Ruisard (2018)

[13] H. Okamoto, private communication, Nov. 2018.

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