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A NOVEL S-BASED SYMPLECTIC ALGORITHM FOR TRACKING WITH SPACE CHARGE *

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Abstract

Traditional finite-difference particle-in-cell methods for modeling self-consistent space charge introduce non-Hamiltonian effects that make long-term tracking in storage rings unreliable. Foremost of these is so-called grid heating. Particularly for studies where the Hamiltonian invariants are critical for understanding the beam dynamics, such as nonlinear integrable optics, these spurious effects make interpreting simulation results difficult. To remedy this, we present a novel symplectic spectral space charge algorithm that is free of non-Hamiltonian numerical effects and, therefore, suitable for long-term tracking studies. Results presented here include a study of the solver's performance under a range of conditions. First, we show benchmarking and convergence studies for a beam expanding in a drift. Then we present benchmarks for a standard FODO channel comparing with Synergia. Finally we demonstrate the solver's ability to preserve Hamiltonian structure by studying the formation of space-charge driven resonances using both our algorithm and traditional PIC.

INTRODUCTION

High intensity beams are essential for a variety of high energy physics applications, in particular for meeting demands of proton drivers for neutrino and neutron production. To meet these requirements, high current proton synchrotrons and accumulator rings are needed, for which particle loss via beam halo is the chief intensity-limiting factor. One novel idea for suppressing these losses is the nonlinear integrable optics proposed by Danilov and Nagaitsev [1]. By meeting a set of conditions, integrability can be maintained even in the presence of large tune spreads, thereby generating regular orbits while suppressing collective instabilities.

The Integrable Optics Test Accelerator (IOTA), currently under construction at Fermi National Laboratory (Fermilab),will provide a testbed for this and other novel concepts in beam dynamics [2]. However, current simulation techniques present challenges to understanding these dynamics for intense beams on long time scales. Accurate modeling of the dynamics of such beams over many turns is susceptible to the non-symplectic nature of traditional PIC codes.

In this paper we present benchmarking and simulation studies of a, symplectic, s-based algorithm for tracking intense beams with self-consistent space charge. This algorithm overcomes the shortcomings of traditional finite-difference codes because it preserves the Hamiltonian structure, which eliminates grid heating [3]. We demonstrate the application of this algorithm using a gridless spectral solver for self-consistent dynamics akin to what was developed in [4]. We present benchmarks against traditional finite-difference codes and present results of nonlinear beam dynamics compared with traditional PIC.

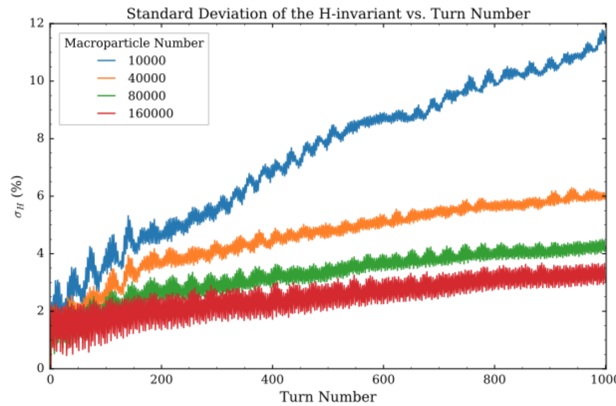

Figure 1: Nonlinear dynamics in the IOTA ring are highly susceptible to noise induced by traditional finite-difference PIC codes. Above, the longterm deviation from integrability scale strongly with particle number.

ALGORITHM

Here we present an overview of the algorithm and underlying assumptions. For a more detailed discussion concerning the derivation of these equations see [5].

For a s based tracking code we need to transform the independent variable from t to s. We then make the following change of coordinate: $(x, y, s) \rightarrow (x, y, z - \beta_0 ct)$, where $\xi = z - \beta_0 ct$. In most cases, it will be convenient to define β_0 to be the beam beta, but this is not strictly required. We then may define the canonical momentum p_ξ according to $p_\xi = \frac{p_\tau}{\beta_0} = \frac{\gamma mc}{\beta_0}$, where γ is the particle Lorentz factor in the lab frame. Note that p_ξ is related to p_τ in a simple way, and like p_τ it is always greater than mc. We also note that the coordinate transform to ξ is ill-defined at $\beta_0 = 0$, hence there is no concern with the appearance of infinite momenta for a stationary beam.

Four critical assumptions have been made in deriving the algorithm, which we outline here for convenience. First, we make the "beam approximation," that $d\xi/ds \ll 1$. Next, we assume that there are no electrostatic elements in our system, and so the only electrostatic scalar potential follows from the beam space charge. Following this, we assume that $d\mathbf{x}_\perp/ds \cdot$

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$\mathbf{A}_\perp \approx 0$, which holds if the beam motion is predominantly in the s direction and radiation is neglected. This permits us to ignore the contribution of the self-consistent transverse vector potential components from the beam. Finally, we extended this condition to be $A_s = A_{\text{ext}} + \mathcal{A}$, where $\mathcal{A}$ is the self consistent vector potential and A_{ext} arises from external fields. This neglects contributions due to fringe fields.

Applying these assumptions and transformations yields a Hamiltonian (Equation 1) and a constraint equation (Equation 2) that are solved in concert.

$$\mathcal{H}_{\text{p-c}} = \sum_j -\sqrt{\left(\beta_0 p_j^{(\xi)}\right)^2 - \left(\mathbf{p}_j^{(\perp)}\right)^2 - (w_j mc)^2} + p_j^{(\xi)} - \frac{w_j q}{\beta_0 c}\psi \tag{1}$$

$$\frac{1}{\gamma^2}\partial_\xi^2\psi + \nabla_\perp^2\psi = \frac{1}{\gamma_0^2}4\pi q n(\vec{x}) \tag{2}$$

We have developed a Python implementation of this algorithm using a spectral solver for Equation 2 and a symplectic integrator for Equation 1. This Python implementation is referred to as respic.

BENCHMARKS AND CONVERGENCE

Before studying the onset of space-charge driven resonances we performed several benchmarking exercises against both analytic solutions and traditional 2-D PIC simulations. The analytic solutions are for two different types of beams (KV and gaussian) with a finite emittance expanding in a drift, and for comparison with PIC we simulated a FODO channel and compared with Synergia [6]. Synergia is a s-based symplectic tracking code that does not currently have a symplectic space-charge solver but utilizes a more traditional PIC solver.

Expansion in a Drift

Figures 2 and 3 show a beam expanding in a drift due to space-charge forces with finite emittance. In each case the analytic solution both with and without space-charge are shown in addition to the respic solution with space-charge. The zero-current solution gives a qualitative indication of how much space-charge is being simulated in the beam.

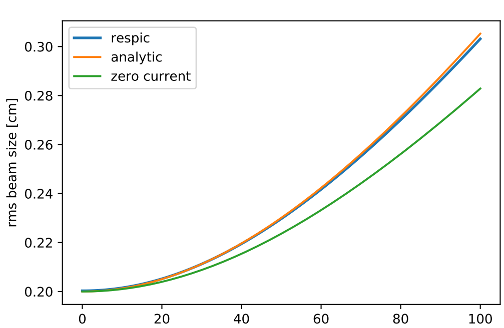

Figure 2: Simulation of a 20mA proton beam at 2.0 MeV with an initial emittance of 4.0 mm-mrad.

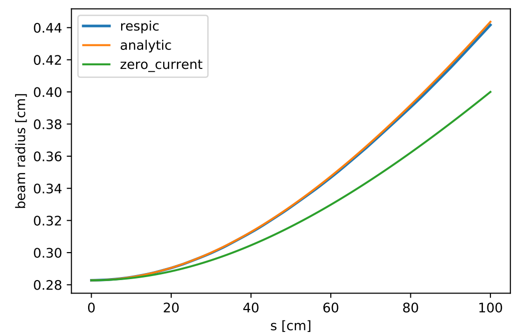

Figure 3: Simulation of a 3A proton beam at 2.0 MeV with an initial emittance of 8.0 mm-mrad.

Here we see quite good agreement between the code and analytic solutions for both types of beam distributions. For the KV beam we saw good convergence for 10k particles and 10 modes for each x and y, for the gaussian beam we saw good convergence for the same number of modes but for 100k particles.

FODO Channel

Next we benchmarked respic against Synergia using a FODO cell. The FODO cell was matched without space charge and then simulated with space-charge in Synergia and respic to simulate the resulting mismatch oscillation. Synergia uses traditional electrostatic PIC and as discussed respic uses a Hamiltonian and constraint equation solved with a spectral basis. Figure 4 shows the rms envelope for Synergia and respic in a focusing channel over 20 turns. For comparison we show the matched beam from an elegant [7] simulation. Here we see that during the first couple of turns all three codes agree quite well. After a few turns the space-charge induced mismatch appears in both respic and synegia.

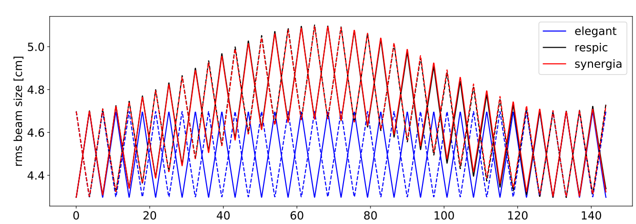

Figure 4: Simulation of fodo channel with space charge using Synergia, opal, and respic.

For this test we see quite good agreement between respic and Synergia.

RESONANCE EXCITATION

We next study the excitation of resonant behavior using our symplectic algorithm and a traditional PIC algorithm. We tuned a FODO channel to be very near a quarter integer resonance without space-charge. We then increase the beam current such that the resulting tune depression shifts the beam towards the quarter integer resonance. Note that for our simulation there is no energy spread and therefore no initial

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tune spread. We expect the nonlinear space charge force in the gaussian distribution to excite an octupolar resonance.

We compared the RMS beam parameters for the respic simulation to the Synergia simulation over the first couple of turns and confirmed their agreement. We then simulated the lattice for 500 turns and examined the resulting phase-space distributions.

Figure 5 shows the initial phase-space distribution for respic and Synergia. Note that the distributions are the same but we require many more particles in Synergia to properly resolve the physics.

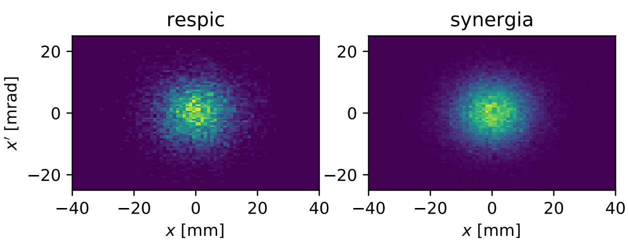

Figure 5: Initial transverse phase-space distribution for Syn-ergia and respic.

Figure 6 shows the phase-space distribution for respic and Synergia after 50 turns. Here we can see very clear filamentation in both simulations with respic appearing more clearly defined.

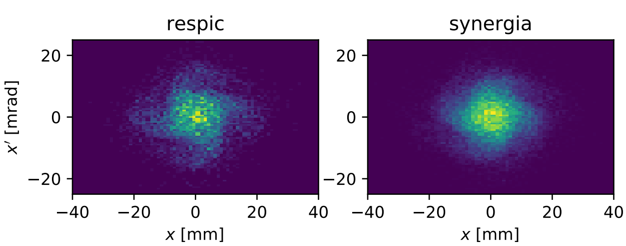

Figure 6: Transverse phase-space distribution for Synergia and respic after 50 turns.

Figure 7 shows the phase-space distribution for respic and Synergia after 500 turns. Here the filamentation in respic has four clearly defined lobes in the phase-space distribution as is characteristic with an octopole resonance. The Synergia simulation however has completely smeared out these phase-space lobes and the distribution resembles very much the initial distribution.

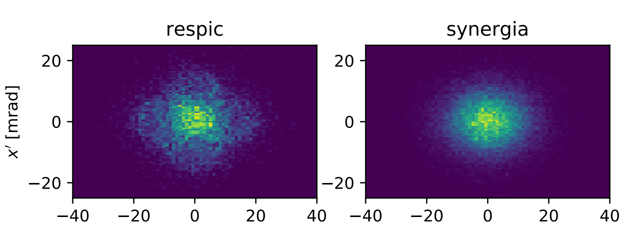

Figure 7: Transverse phase-space distribution for Synergia and respic after 500 turns.

Finally we compare the transverse momentum in both simulations to establish if the difference in these plots is due to non-symplecticity or other discrepancies (resolution etc). Figure 8 shows the total transverse momentum for both simulations. Clearly the variation in respic is much smaller than Synergia.

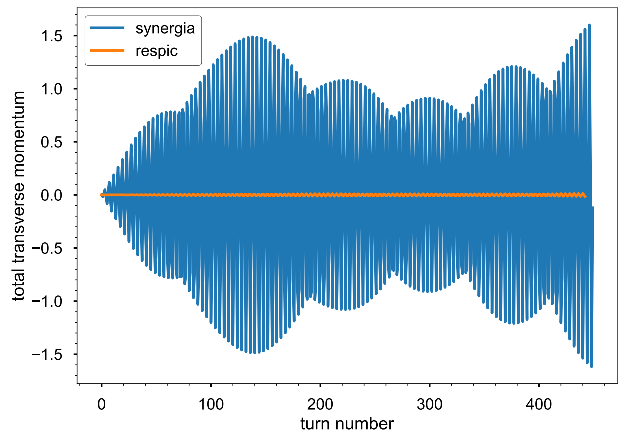

Figure 8: Simulation of fodo channel with space charge using Synergia, opal, and respic.

CONCLUSION

The use of symplectic space charge solvers may provide more insight into complex, long-term dynamics in nonlinear systems by significantly reducing numerical noise. We have developed a novel s-based algorithm for symplectic tracking of beams with self-consistent space charge forces in an accelerator. The algorithm makes use a spectral, gridless, representation of the phase space density to reduce numerical noise and permit exact field propagation. We show excellent agreement between our algorithm and analytic solutions and established community codes for initial benchmarks. Finally we have demonstrated the ability to capture nonlinear optics excited by space-charge with greater detail than existing PIC simulations. We are currently in the process of implementing this algorithm in Synergia.

ACKNOWLEDGMENTS

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