

EMITTANCE EXCHANGE IN MICE

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Abstract

The Muon Ionization Cooling Experiment, MICE, has demonstrated transverse emittance reduction through ionization cooling. Transverse ionization cooling can be used either to prepare a beam for acceleration in a neutrino factory or for the initial stages of beam cooling in a muon collider. Later stages of ionization cooling in the muon collider require the longitudinal emittance to be manipulated using emittance exchange and reverse emittance exchange, where emittance is exchanged from and to longitudinal phase space respectively. A wedge absorber within the MICE cooling channel has been used to experimentally study reverse emittance exchange in ionization cooling. Parameters for this test have been explored in simulation and applied to experimental configurations using a wedge absorber when collecting data in the MICE beam.

MUON IONIZATION COOLING EXPERIMENT

In MICE [1], a proton beam is bombarded against a solid titanium target creating pions, which are subsequently gathered in a quadrupole triplet. These pions then decay to muons in the transport line.

In a muon collider or neutrino factory these muons would occupy a large phase-space volume upon production and have a low phase-space density. To meet the acceptance requirements of a storage ring [2], these muons must be cooled, with ionisation cooling the only viable process to cool the muon beam to the required phase-space density within the lifetime of the muon.

Ionization cooling [3] is achieved by passing a beam through an absorber material reducing the total momentum of the beam, and then through an RF cavity to recover the lost longitudinal momentum.

The MICE muon beam is passed through a series of detectors in the cooling channel (Fig.1). The Time-of-Flight stations, Kloe Light and Electron-Muon Ranger are used to distinguish muons from electrons and pions, while the two tracking detectors either side of the absorber immersed in a uniform multi-Tesla magnetic field measure the position and momentum of each particle. This is done via five stations of scintillating fibres.

Position and momentum measurements made in each tracker allow the comparison of the phase-space density before and after the absorber.

MICE has used Lithium Hydride and liquid Hydrogen as absorbers which have shown an increase in the transverse

phase-space density after the muon beam has passed through the absorber. A Polyethylene wedge has been used to study emittance exchange (Fig.2).

EMITTANCE EXCHANGE

In Emittance Exchange a muon beam is passed through a dipole magnet to create both a position spread and a position-energy correlation in the beam. The beam is then passed through a wedge of specific thickness to eliminate the momentum dispersion (Fig.3). The beam then has an increased longitudinal phase-space density and a reduced transverse phase-space density. Thus, the emittances have been exchanged. The transverse emittance can then be reduced by ionisation cooling resulting in an overall reduction in 6D emittance.

In reverse emittance exchange the beam is first passed through a wedge and then through a magnetic dipole. This allows one to increase the transverse phase-space density at the expense of decreased longitudinal phase-space density. Thus, the emittance exchange has been reversed.

KERNEL DENSITY ESTIMATION

To measure the change in phase-space density, Kernel Density Estimation (KDE) is used [4]. KDE is a non-parametric density estimation technique which makes fewer assumptions about the underlying distribution. This is done by calculating the kernel, a multivariate Gaussian centred on each data point (Fig.4). All of the kernels are then summed to arrive at the KDE. The KDE is then given by

$$f(\vec{x}) = \frac{1}{nh^d \sqrt{2\pi}} \sum_{i=1}^n k\left(\frac{-|\vec{x} - \vec{X}_i|^2}{2h^2}\right) \quad (1)$$

where $k\left(\frac{-|\vec{x} - \vec{X}_i|^2}{2h^2}\right)$ is the kernel written as a function of reference point \vec{x} , i^{th} data point \vec{X}_i and the width of the kernel h . n is the sample size and d is the dimensionality of the dataset.

Figure 5 shows an example of a bimodal distribution. The parametric approach shows its limitation as it assumed a Gaussian distribution. The KDE approach shows its usefulness in comparison to the histogram as it can produce a higher resolution density measurement. The smoothness of the KDE approach is determined by the choice of kernel width, h .

SIMULATION

A simulation has been performed in G4Beamline [5,6], where an initial distribution of particles with the desired

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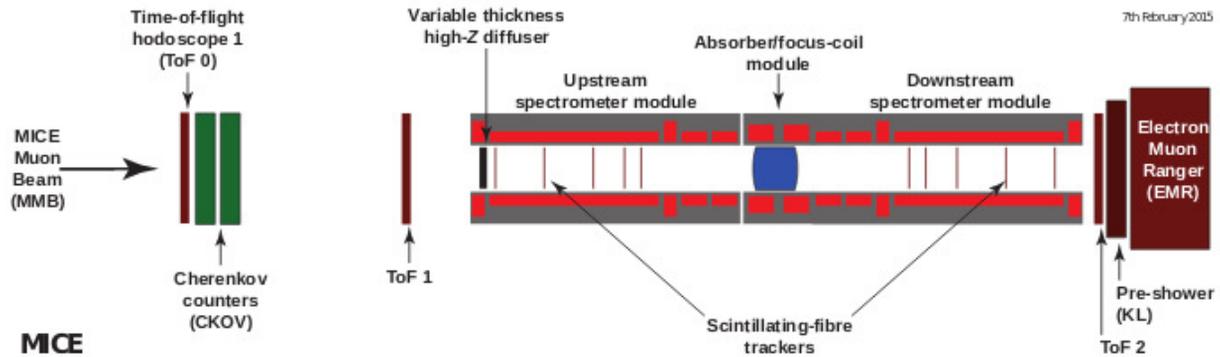


Figure 1: The MICE Beamline. Experimental data was gathered with various absorbers such as liquid Hydrogen, Lithium Hydride and a Polyethylene wedge. The beam was tracked through the spectrometer solenoids at each of the five upstream and downstream scintillating fibre stations.



Figure 2: One half of the MICE wedge.

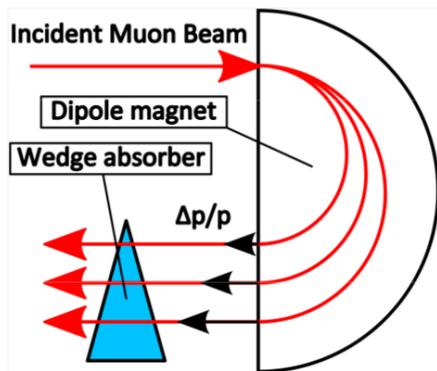


Figure 3: Emittance Exchange between the longitudinal and transverse phase-space.

dispersion at the absorber were propagated backwards to the center of the upstream tracker. The distribution at the upstream tracker was then used to fill a space with 88,000 muons with the same distribution but also with an input emittance of 6mm and 140 MeV/c reference momentum. These muons were then tracked from the center of the upstream tracker through the wedge and into the downstream tracker using the same current settings for the spectrometer solenoids and absorber focus coil as were used during one of the experimental runs.

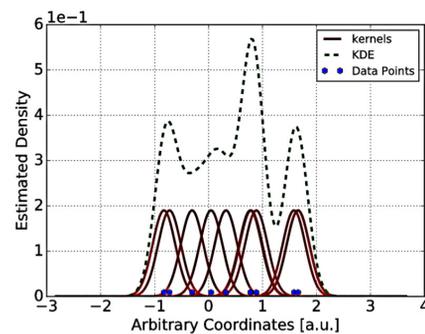


Figure 4: Kernel Density Estimation. Each data point in the distribution is represented by a kernel. Summing over these kernels gives the kernel density estimate.

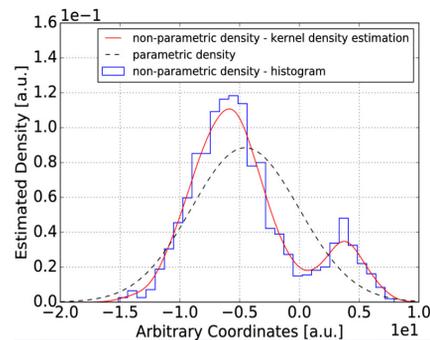


Figure 5: Comparison of three density estimation techniques. KDE produces a higher resolution density estimate and makes no assumptions about the underlying distribution.

The longitudinal (Fig.6) and transverse (Fig.7) phase-space densities for the core of the beam were then calculated and tracked at various points from the last upstream tracker station through the wedge to the first downstream tracker station. An increase in the core longitudinal phase-space density and a slight decrease in the transverse phase-space density is seen when comparing the reference planes of the upstream and downstream trackers.

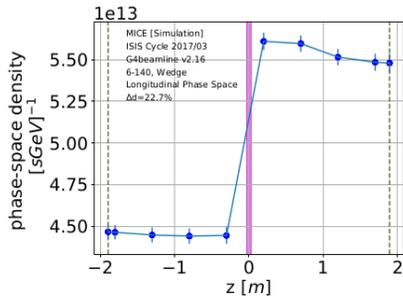


Figure 6: MICE simulation. Increase in the longitudinal phase-space density from the upstream reference plane to the downstream reference plane after passing through the wedge.

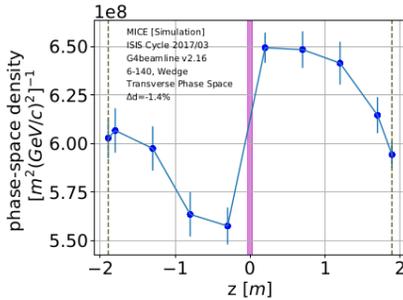


Figure 7: Change in the transverse phase-space density from the upstream reference plane to the downstream reference plane.

BEAM SELECTION

The MICE muon beam has a small natural dispersion, which is not enough to demonstrate emittance exchange. To demonstrate emittance exchange in MICE a beam selection procedure must be applied where a dispersive beam is selected upstream of the wedge and then tracked through the wedge and MICE cooling channel.

As MICE uses a particle by particle measurement system, a muon beam is constructed by assembling all of these individual measurements. A dispersive beam can then be selected from this assembled beam by applying a weighting procedure.

Figures 8 and 9 show an example of a moment based weighting procedure [7]. From an initial two dimensional multivariate Gaussian distribution with normalized emittance $\epsilon_x = 15mm$, $\beta_x = 334mm$ and $\alpha_x = 0$, a beam with normalised emittance $\epsilon_x = 4.2mm$, $\beta_x = 260mm$ and $\alpha_x = -0.75$ was selected using the moment based weighting procedure. Choosing the correct weights to select for a sufficiently dispersive beam from an initial MICE distributed beam and tracking that beam through the cooling channel should allow for the demonstration of emittance exchange. This can be achieved by showing the change in phase-space density from the upstream selection to the downstream measurement.

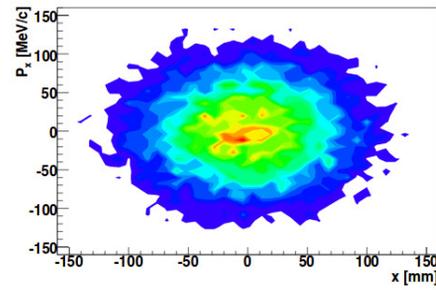


Figure 8: A two-dimensional multivariate Gaussian distribution.

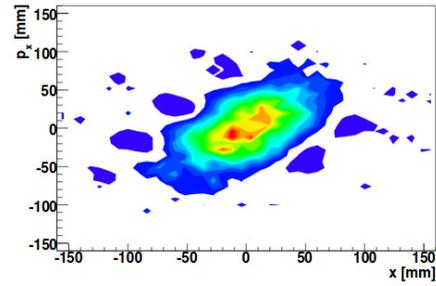


Figure 9: Beam distribution after a moment based re-weighting procedure was applied to Fig. 8.

CONCLUSION

Emittance Exchange in the wedge has been demonstrated in simulation by showing the change in the core longitudinal and transverse phase-space densities through the MICE cooling channel. Using Kernel Density Estimation to analyze beam cooling instead of RMS emittance allows us to minimize the effects of tails in the distributions.

A beam selection procedure to statistically weigh the MICE beam has also been developed, which will allow for the selection of dispersive beams. As the MICE beam only has a small natural dispersion, being able to select for a dispersive beam will allow for the demonstration of emittance exchange.

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REFERENCES

- [1] MICE Collaboration, Adams, D., Adey, D. *et al.*, "First particle-by-particle measurement of emittance in the Muon Ionization Cooling Experiment", *Eur. Phys. J. C* vol. 79: p. 257, 2019
- [2] Neutrino Factory and Muon Collider Collaboration, M. Alsharo'a *et al.*, "Recent progress in neutrino factory and muon collider research within the Muon collaboration", *Phys. Rev. ST Accel. Beams*, vol. 6, p. 081001, 2003.
- [3] Kaplan, D., "Muon Colliders, Neutrino Factories, and Results from the MICE Experiment", in *Proc. of CAARI 2018*, <https://arxiv.org/pdf/1810.10610v1.pdf>
- [4] T. A. Mohayai *et al.*, "A non-parametric density estimation approach to measuring beam cooling in MICE", in *Proc. IPAC'18*, Vancouver, Canada, May 2018, paper TUPML063, pp. 1684-1687.
- [5] T. A. Mohayai *et al.*, "A wedge test in MICE", in *Proc. IPAC'18*, Vancouver, Canada, May 2018, paper TUPML062, pp. 1680-1683.
- [6] T. A. Mohayai, "A Novel Non-Parametric Density Estimation Approach to Measuring Muon Ionization Cooling And Reverse Emittance Exchange in the MICE Experiment", Thesis, Illinois Institute of Technology, 2018.
- [7] C. T. Rogers, "Statistical weighting of the MICE beam", in *Proc. EPAC08*, Genoa, Italy, paper TUPC088, pp. 1260-1262.