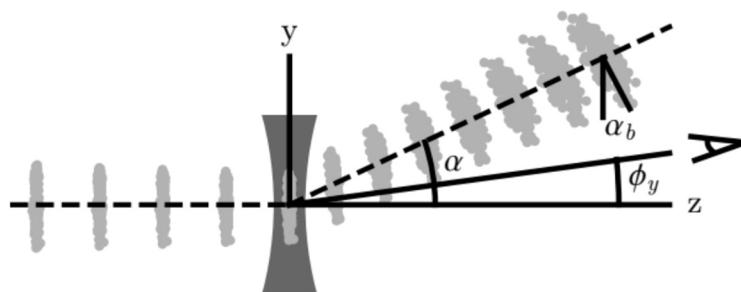


Microbunch Rotation and Coherent Undulator Radiation from a Kicked Electron Beam

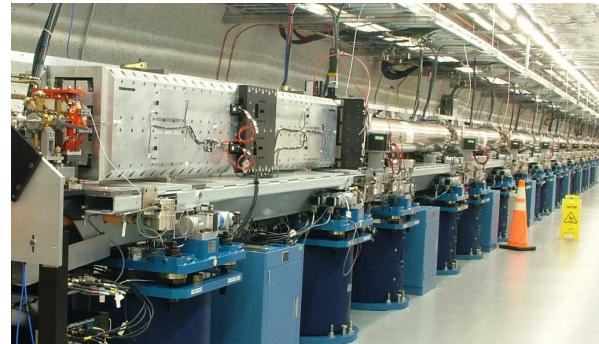


James MacArthur, Alberto Lutman,
Jacek Krzywinski, Zhirong Huang

May 21, 2019

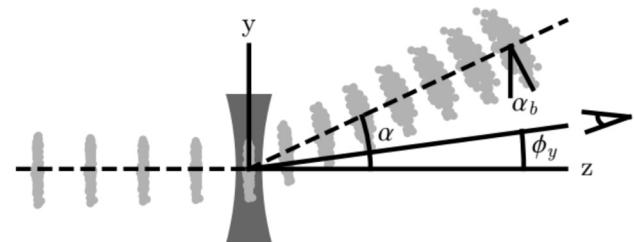
Motivation

- Isolated circular x-ray production at LCLS
- Beam diversion:
unexpectedly successful



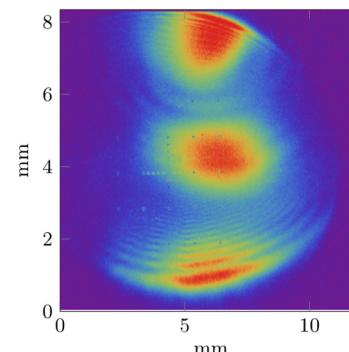
Rotation

- Microbunch dynamics in a FODO lattice
- The theory that matches experiment



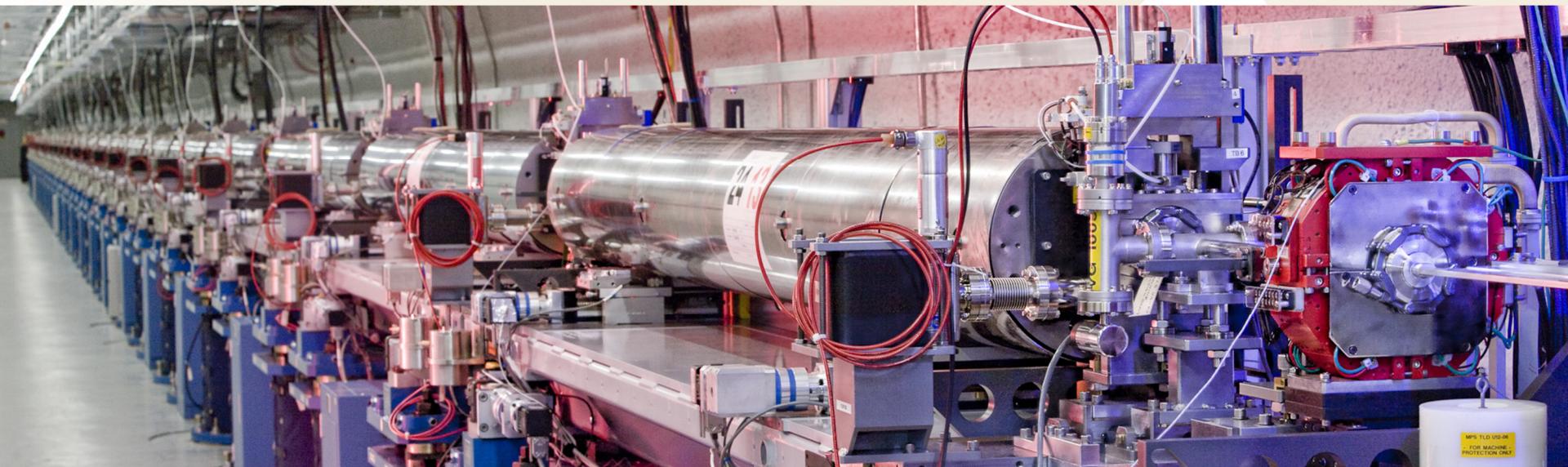
Extensions

- Multiplexing at soft x-rays: 3 beams
- Multiplexing at hard x-rays: 2 beams



LCLS Undulator Magnetic Lattice

SLAC



LCLS Undulator Magnetic Lattice

SLAC

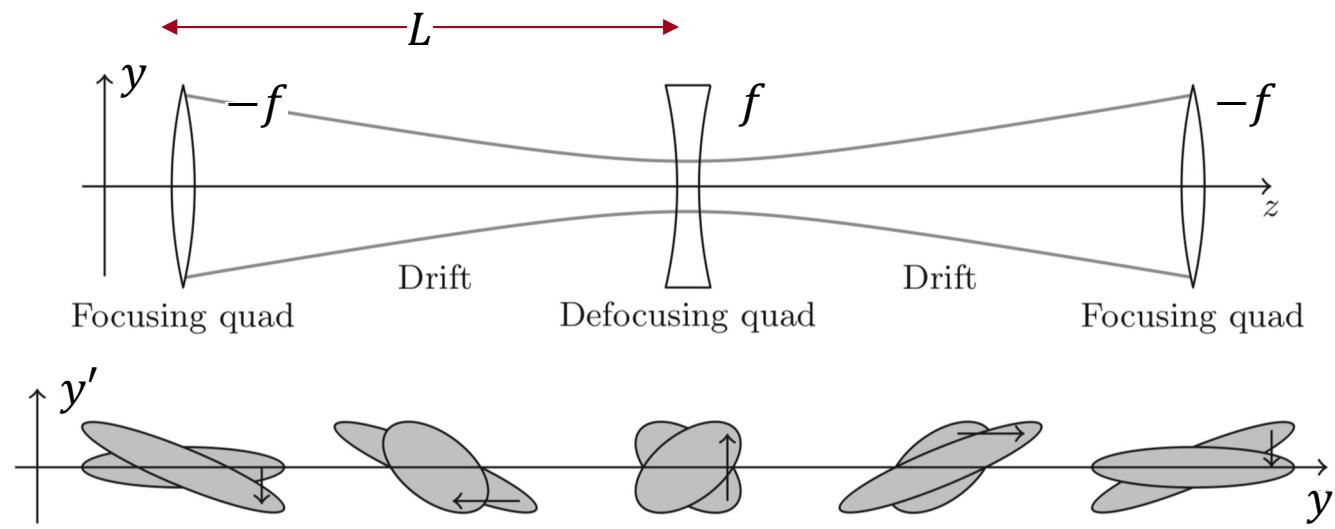
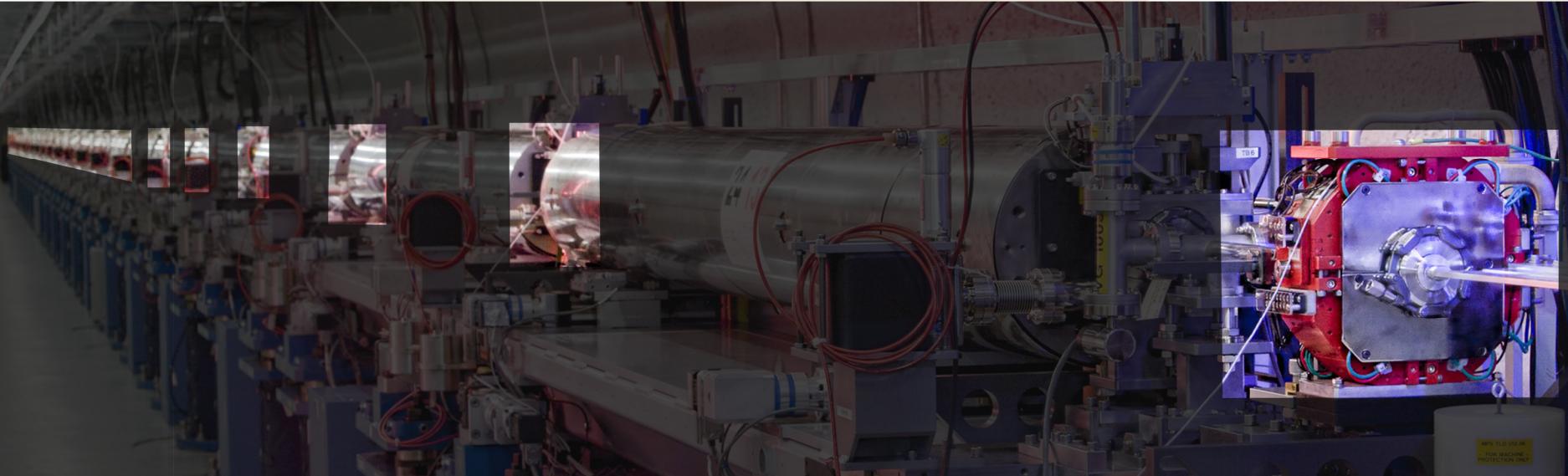


Magnetic Quadrupole & Corrector

- Tunable -40 kG to 40 kG quadrupole
 - Focal length \sim 4 meters for 500 eV operation
- Tunable -6 G·m to +6 G·m dipole
 - Max kick $\sim \pm 55$ uRad for 500 eV operation
- Quadrupole may also be offset to provide additional dipole kick

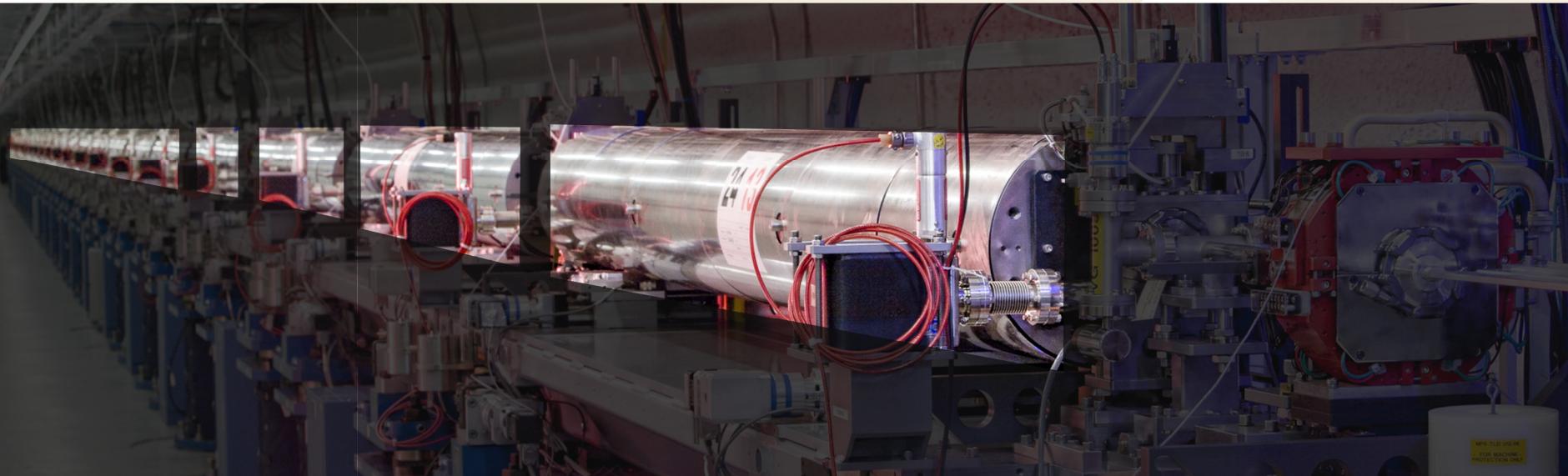
LCLS Undulator Magnetic Lattice

SLAC



LCLS Undulator Line

SLAC

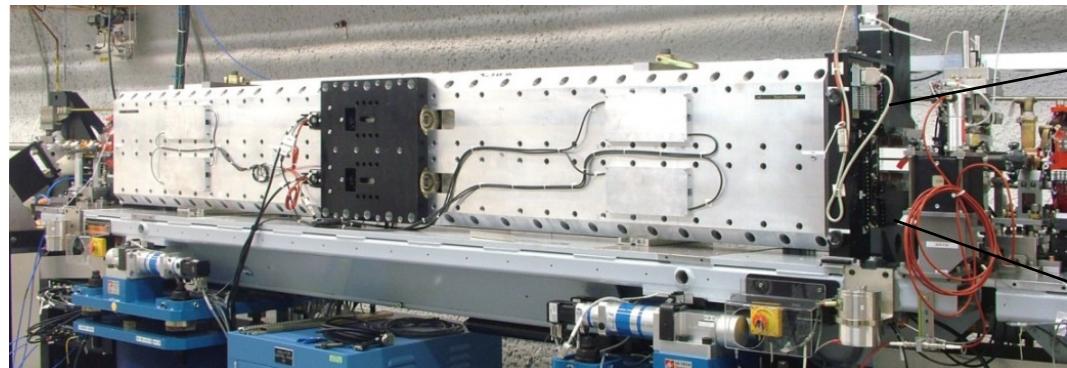


LCLS-I Undulators

- Linear polarization only
- 110 periods, 3 cm period
- Canted poles, $K = 3.44 - 3.5$
 - Note: LCLS-II will have a larger K range via variable gap
- Saturation length = 10's – 100 m

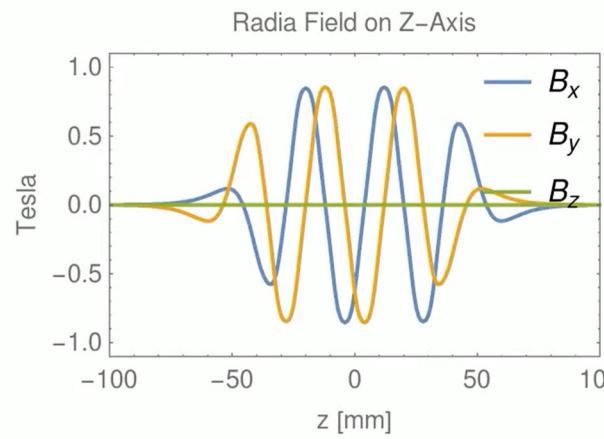
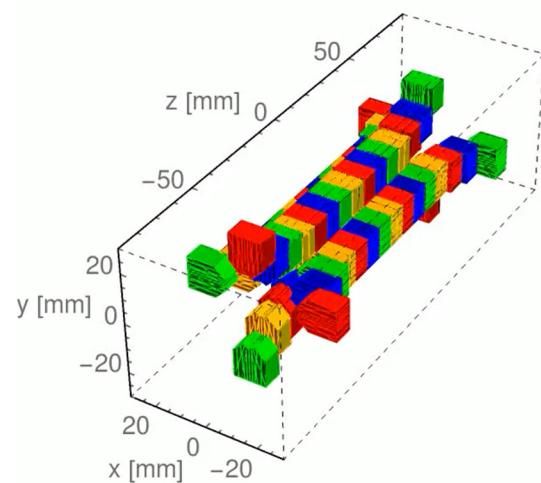
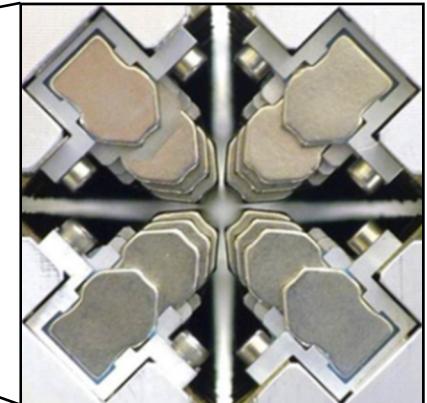
Delta Undulator

SLAC



← 3.2 m, 100 periods →

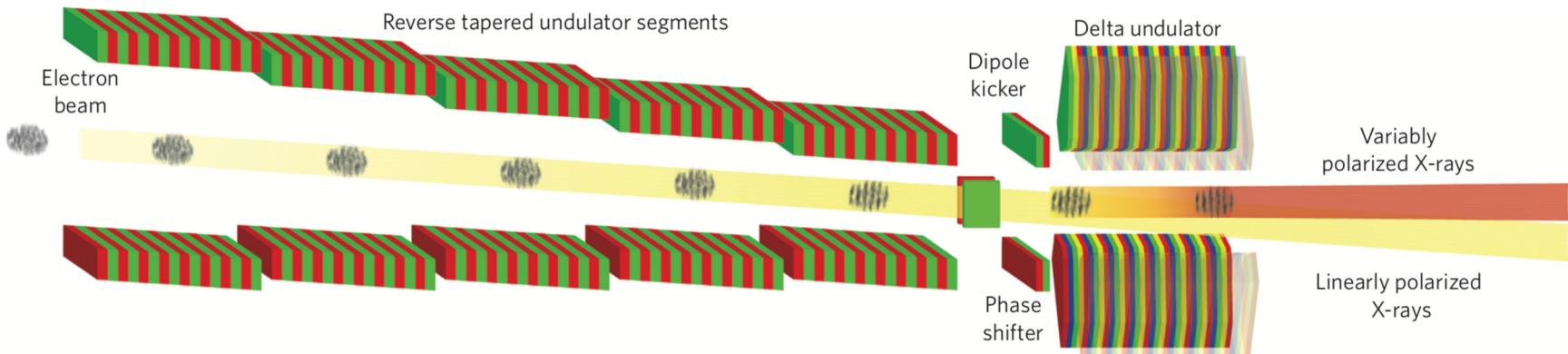
Axial View



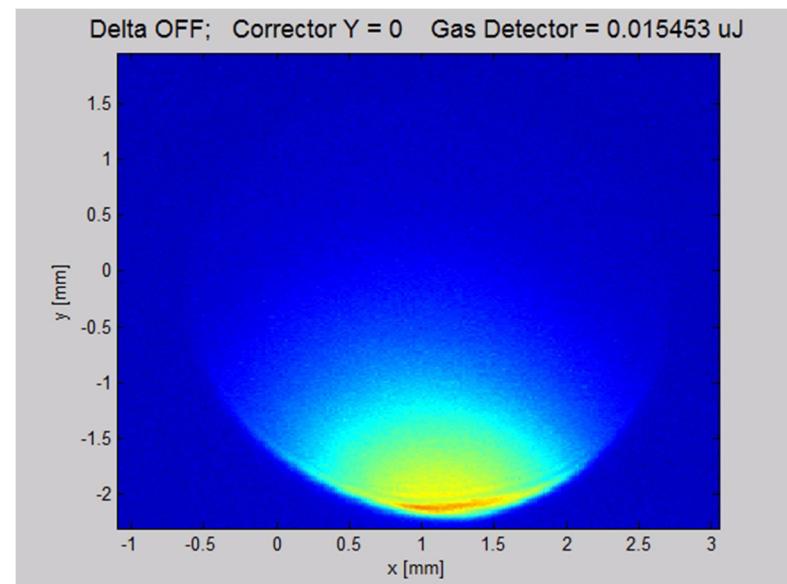
[Radia Simulation]

Isolated Circular Beam

SLAC



- Reverse taper
 - Bunching with minimal field
 - *Schneidmiller & Yurkov, PRSTAB 16, 110702 (2013)*
- Beam diversion
 - Transverse kick before Delta
 - Circular light isolated with jaw
 - Results in *Lutman, MacArthur et al., Nat. Photonics 10 468 (2016)*



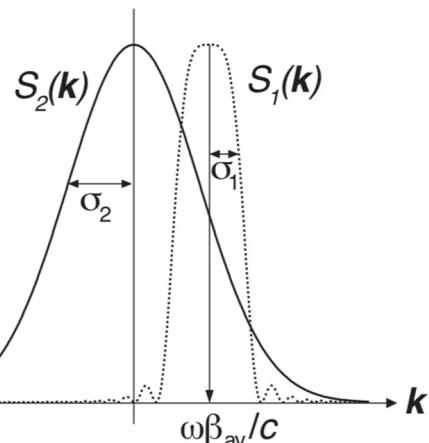
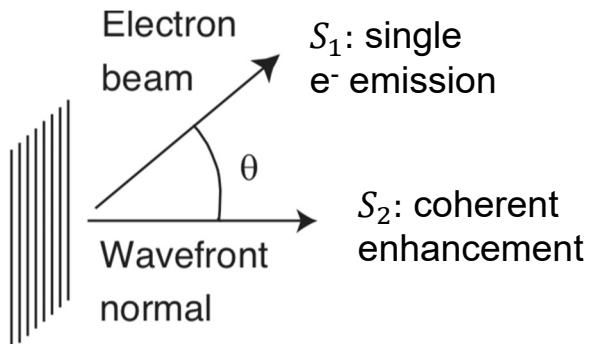
[Lutman 2016]

An Unexpected Success

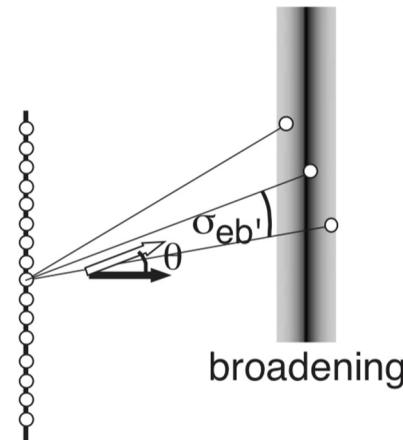
SLAC

In 2004 T. Tanaka identified effects to prevent off-axis lasing
(see T. Tanaka et al., NIMA 528 172 (2004)):

Radiation Suppression



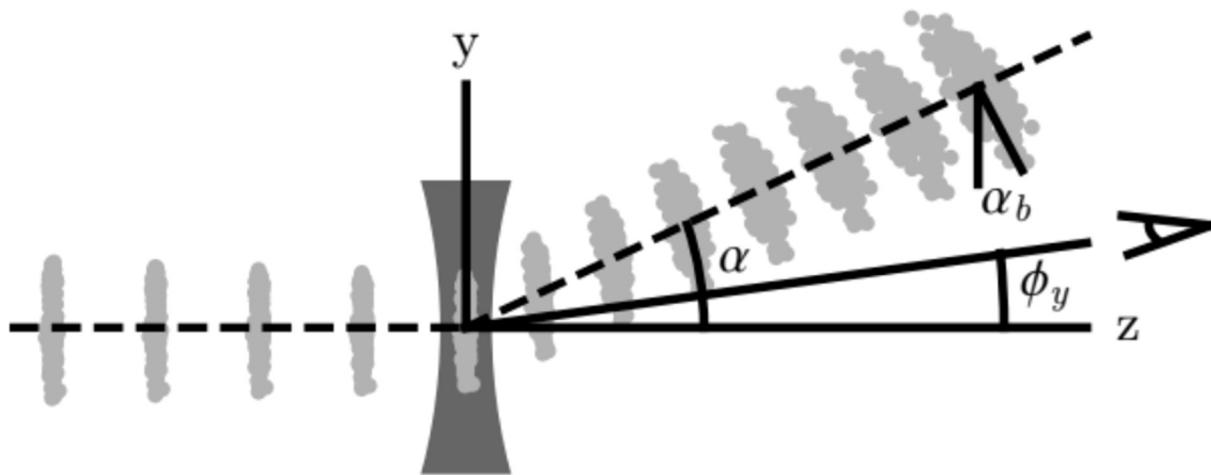
Microbunch Smearing



Kick + angular spread = smearing

New Approach: The Quadrupole Matters

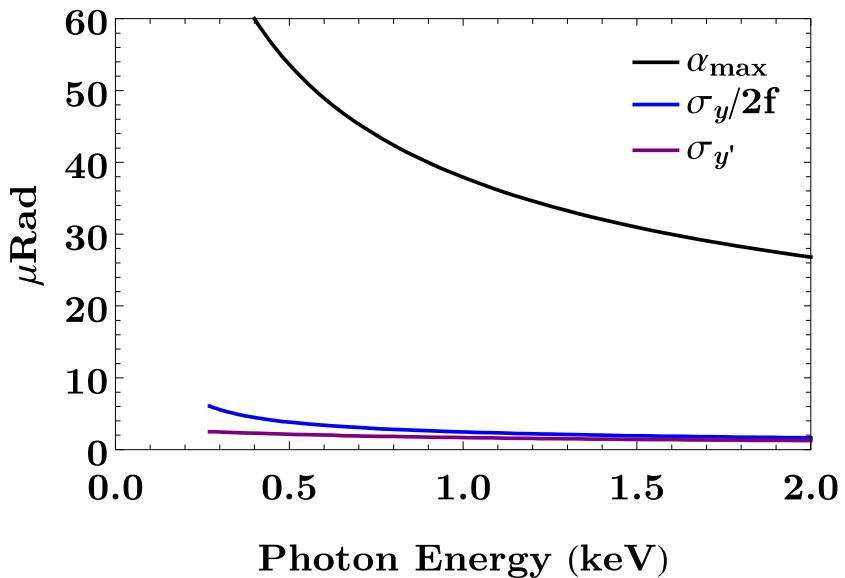
SLAC



- Trajectory ($z=0$ mid-quad):

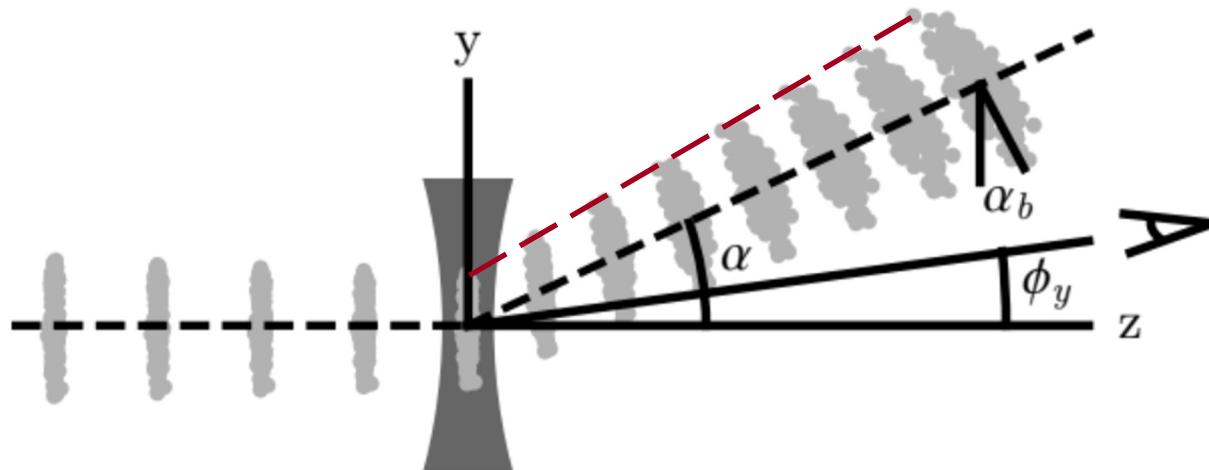
$$y(z) = y_0 + \boxed{y'_0} + \boxed{y_0/(2f)} + \boxed{\alpha} z$$

- The maximum kick at LCLS depends on the photon energy



Intuitive Picture

SLAC

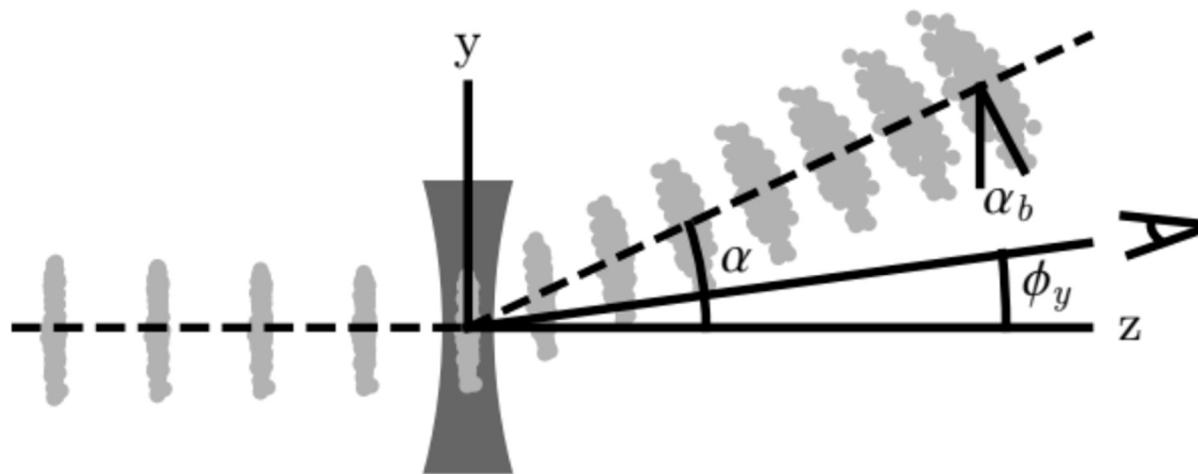


$$\begin{aligned}\Delta s &\approx [z \cos(y'_0 + y_0/(2f) + \alpha)] - [z \cos \alpha] \\ &\approx \underbrace{\frac{\alpha z}{2f} y_0}_{\text{tilt } (\alpha_b)} + \underbrace{\alpha z y'_0}_{\text{smearing}} + \underbrace{\frac{z}{2} \left(\frac{1}{2f} y_0 + y'_0 \right)^2}_{\text{curvature, other}}.\end{aligned}$$

(2nd order beam transport in a drift: $T_{544} = -z/2$)

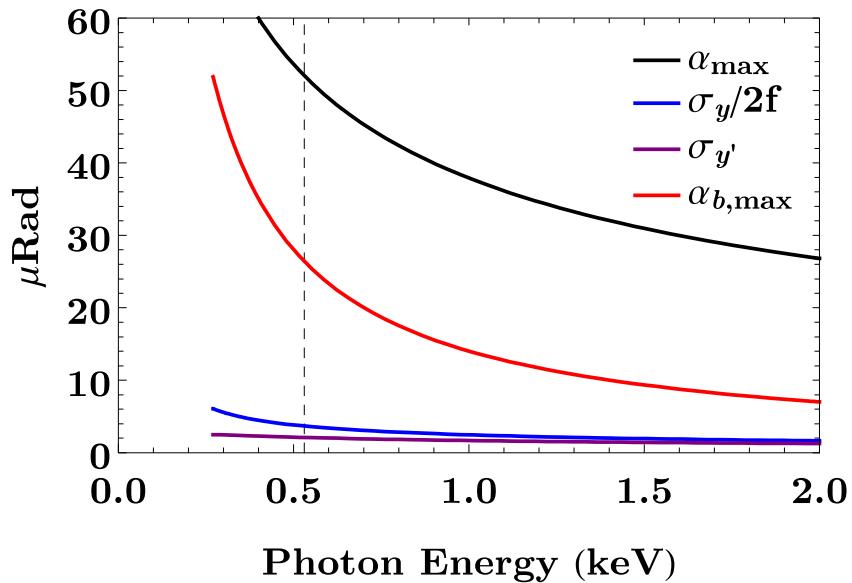
Angle Comparison

SLAC



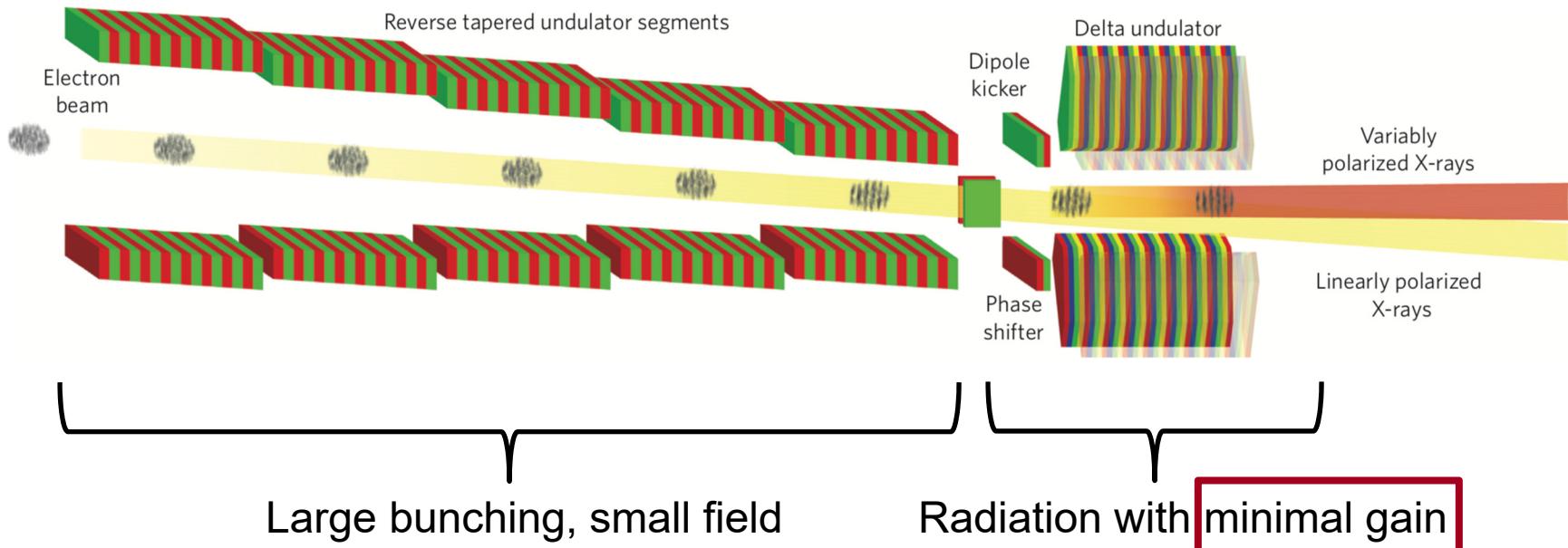
$$y(z) = y_0 + \boxed{y'_0} + \boxed{y_0/(2f)} + \boxed{\alpha} z$$

$$\Delta s \approx \boxed{\frac{\alpha z}{2f}} y_0 + \dots$$



A More Careful Approach

SLAC



Field growth:

$$\left[\frac{\partial}{\partial z} + i\Delta\nu k_u + \frac{ik}{2} \phi^2 \right] \tilde{E}_\nu = -\kappa_1 n_e \int d\mathbf{x}' d\eta \tilde{F}_\nu$$

Microbunch evolution:

$$\left[\frac{d}{dz} + i \left(2\nu\eta k_u - \frac{k}{2} \mathbf{x}'^2 \right) \right] F_\nu = -\chi_1 E_\nu \frac{\partial \bar{F}}{\partial \eta}, \quad \sim 0$$

Result #1: The Bunching Rotation Angle

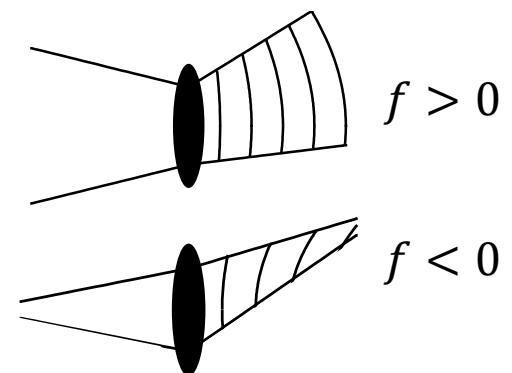
SLAC

With no field interaction and the trajectories given before, the bunching has an analytic expression. It's ugly...

$$\left[\frac{d}{dz} + i \left(2\nu\eta k_u - \frac{k}{2} \mathbf{x}'^2 \right) \right] F_\nu = -\chi_1 E, \quad \frac{\partial \bar{F}}{\partial \eta}, \quad \sim 0 \quad \rightarrow \quad \frac{b_\nu(\phi_x = 0, \phi_y; z)}{b_\nu(\phi_x = 0, \phi_y; 0)} = \frac{e^{-2(k_u \sigma_\eta \nu z)^2 + \frac{|f|k(i\psi + \zeta)}{\sqrt{1-\hat{L}^2} - 2i\hat{\epsilon}|\hat{z}|}}}{i + \frac{2\hat{\epsilon}|\hat{z}|}{\sqrt{1-\hat{L}^2}}}$$

$|b(\phi)|$ is maximized at an angle α_b that matches the geometric prediction ($\hat{z} = z/2f, \hat{L} = L_u/2f, \hat{\epsilon} = k\epsilon$)

$$\hat{\epsilon}^2 \hat{L}^2 \ll 1 - \hat{L}^2 \quad \rightarrow \quad \boxed{\alpha_b(\hat{z} = \hat{L}) \approx \alpha \hat{L} = \frac{\alpha L_u}{2f}}$$

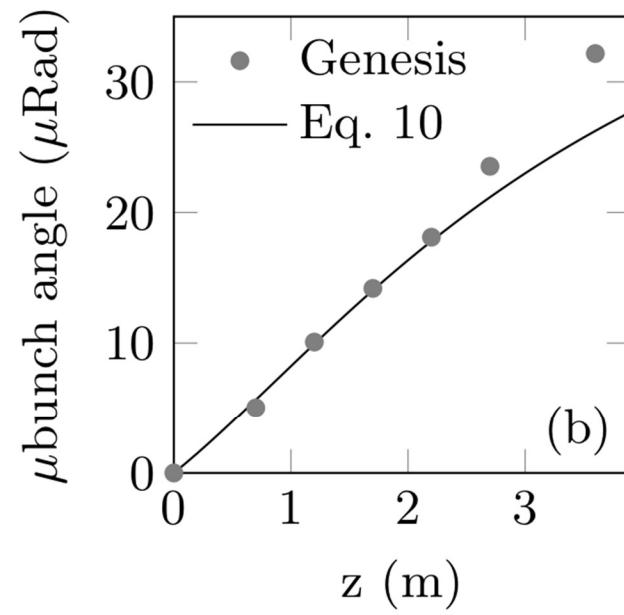
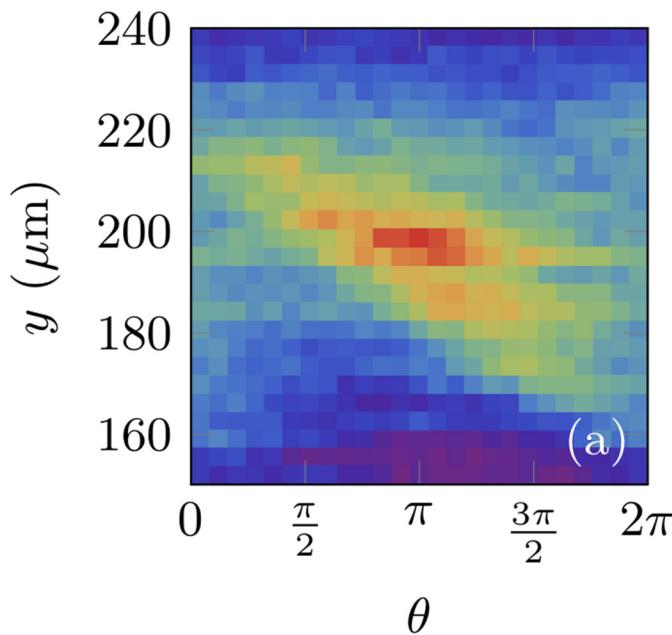


Genesis Comparison

SLAC

The previous result can be compared with Genesis simulations

$$\alpha_b(\hat{z} = \hat{L}) \approx \alpha \hat{L} = \frac{\alpha L_u}{2f}$$

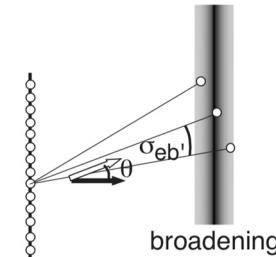


Result #2: A New Critical Angle

SLAC

Tanaka showed that the bunching is suppressed when kicked,

$$|b_\nu(\phi_x = 0, \phi_y = \alpha_b; z)|^2 \propto e^{-\alpha^2/\phi_c^2}$$



A larger critical angle when bunches rotate:

$$\phi_c^2 = \sigma_{y'}^2 + \frac{\sigma_y^2}{\epsilon_y^2 k^2 z^2}$$

Tanaka critical angle



$$\phi_c^2 = s_{y'}^2 + \frac{s_y^2}{\epsilon_y^2 k^2 z^2}$$

New critical angle

The RMS beam size and divergence after a drift z replace the beam size and divergence in the quad:

$$s_y^2 = (1 + z/2f)^2 \sigma_y^2 + \sigma_{y'}^2 z^2$$

$$s_{y'}^2 = \sigma_{y'}^2 + \sigma_y^2/(2f)^2$$

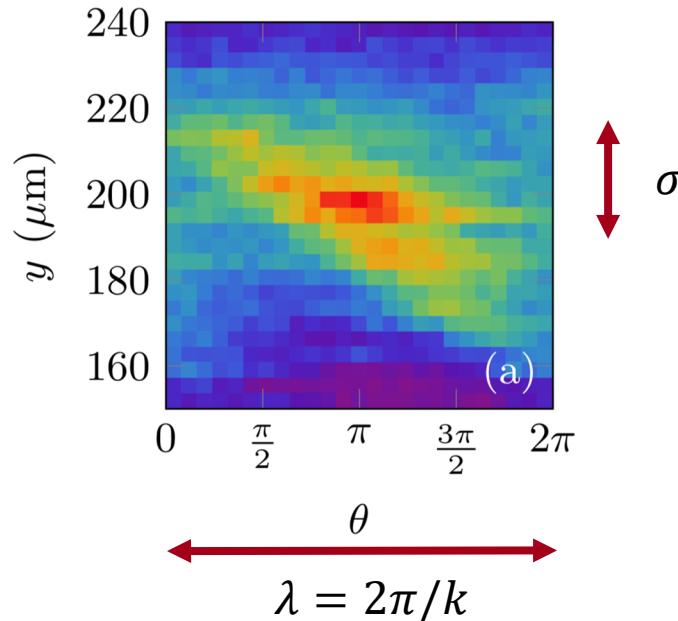
Tilt At The Critical Angle

SLAC

Suppose we kick at the critical angle, how much do the bunches rotate?

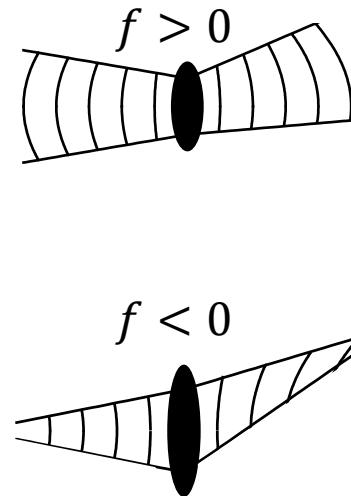
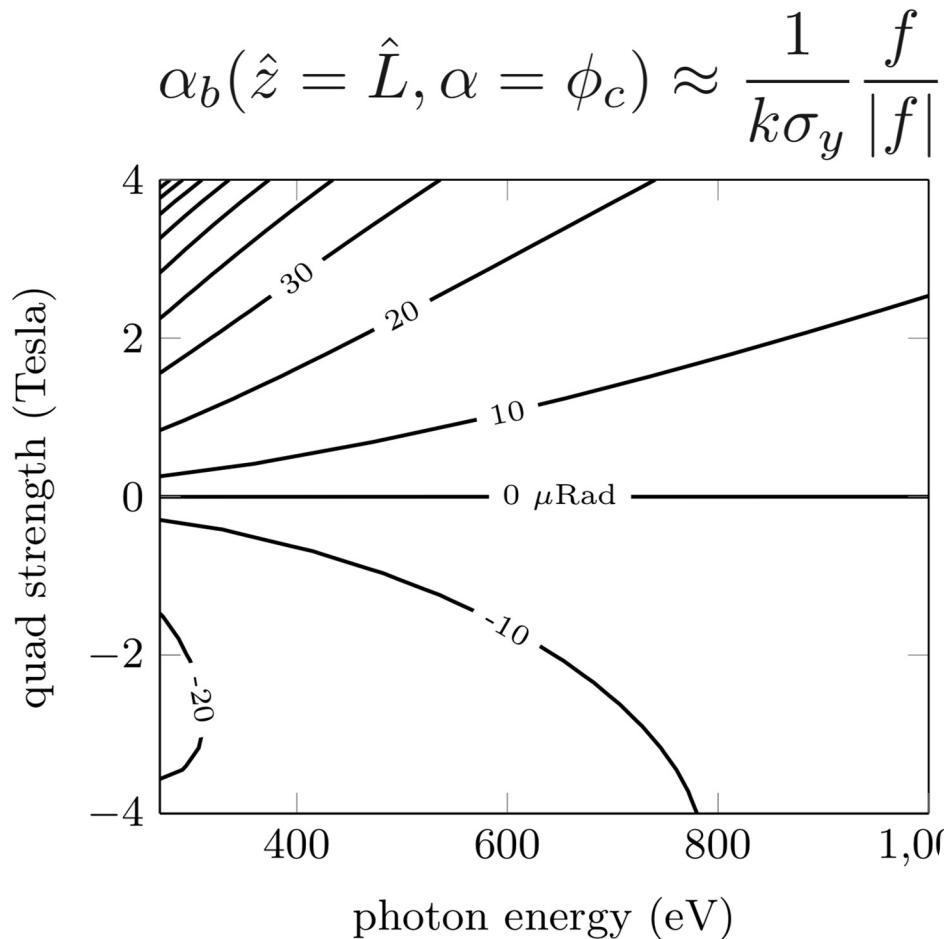
$$\alpha_b(\hat{z} = \hat{L}, \alpha = \phi_c) \approx \frac{1}{k\sigma_y} \frac{f}{|f|}$$

So when the tilt approaches a wavelength over a beam radius, the bunching suffers:



When Does Rotation Matter at LCLS?

SLAC



Result #3: The Field Produced By A Rotating Bunch

SLAC

Plug in expression for microbunch dynamics to field equation

$$\left[\frac{\partial}{\partial z} + i\Delta\nu k_u + \frac{ik}{2}\phi^2 \right] \tilde{E}_\nu = -\kappa_1 n_e \int d\mathbf{x}' d\eta \tilde{F}_\nu$$
$$\left[\frac{d}{dz} + i \left(2\nu\eta k_u - \frac{k}{2}\mathbf{x}'^2 \right) \right] F_\nu = -\chi_1 E_\nu \frac{\partial \bar{F}}{\partial \eta},$$

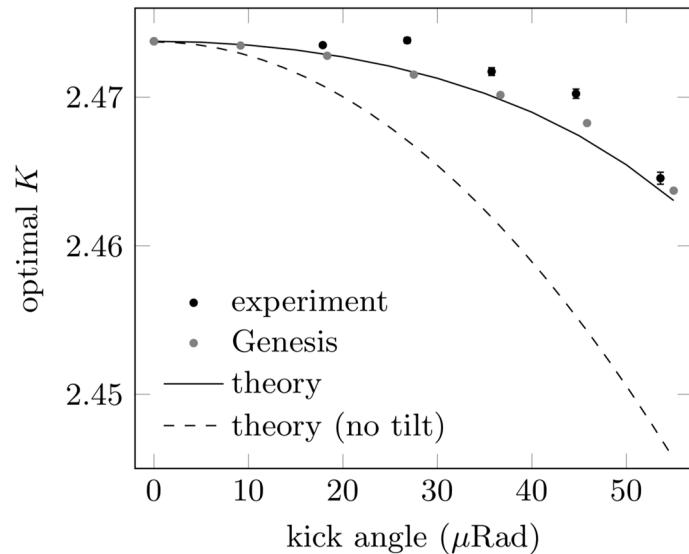
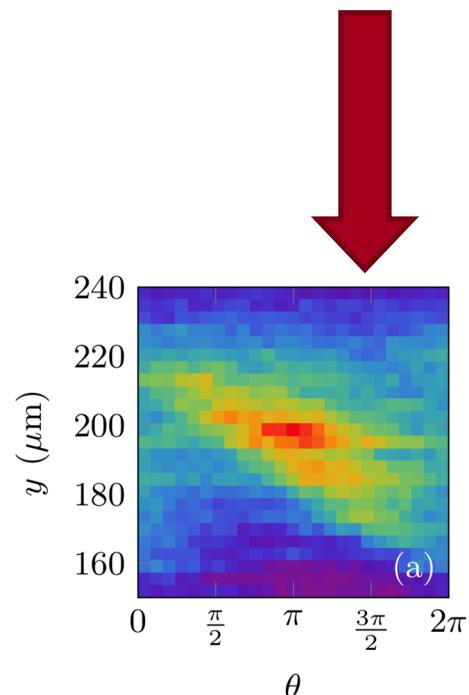
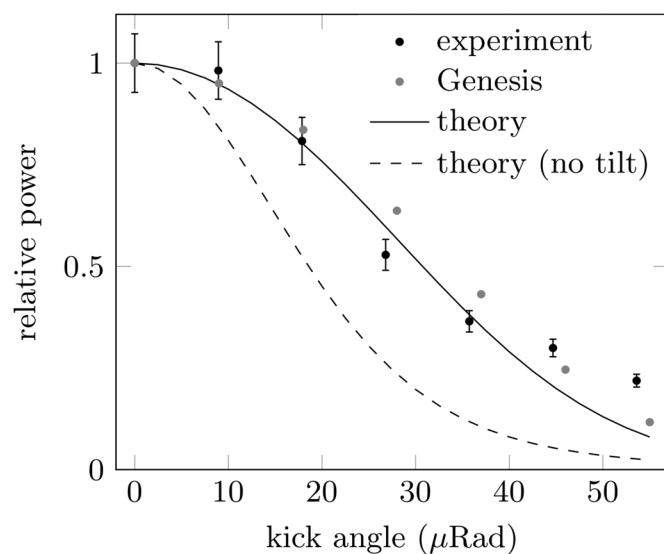
Result: equation showing the field grows with z :

$$\frac{\partial \tilde{E}_\nu}{\partial z} \propto e^{-\frac{\alpha^2}{2\phi_c^2} - \frac{1}{2}(k^2 z \epsilon \phi_c (\phi_y - \alpha_b))^2 + i \left(k_u \Delta\nu + k \frac{(\alpha - \phi_y)^2}{2} \right) z}$$

Result #3: More Radiation Off-Axis, Less Detune

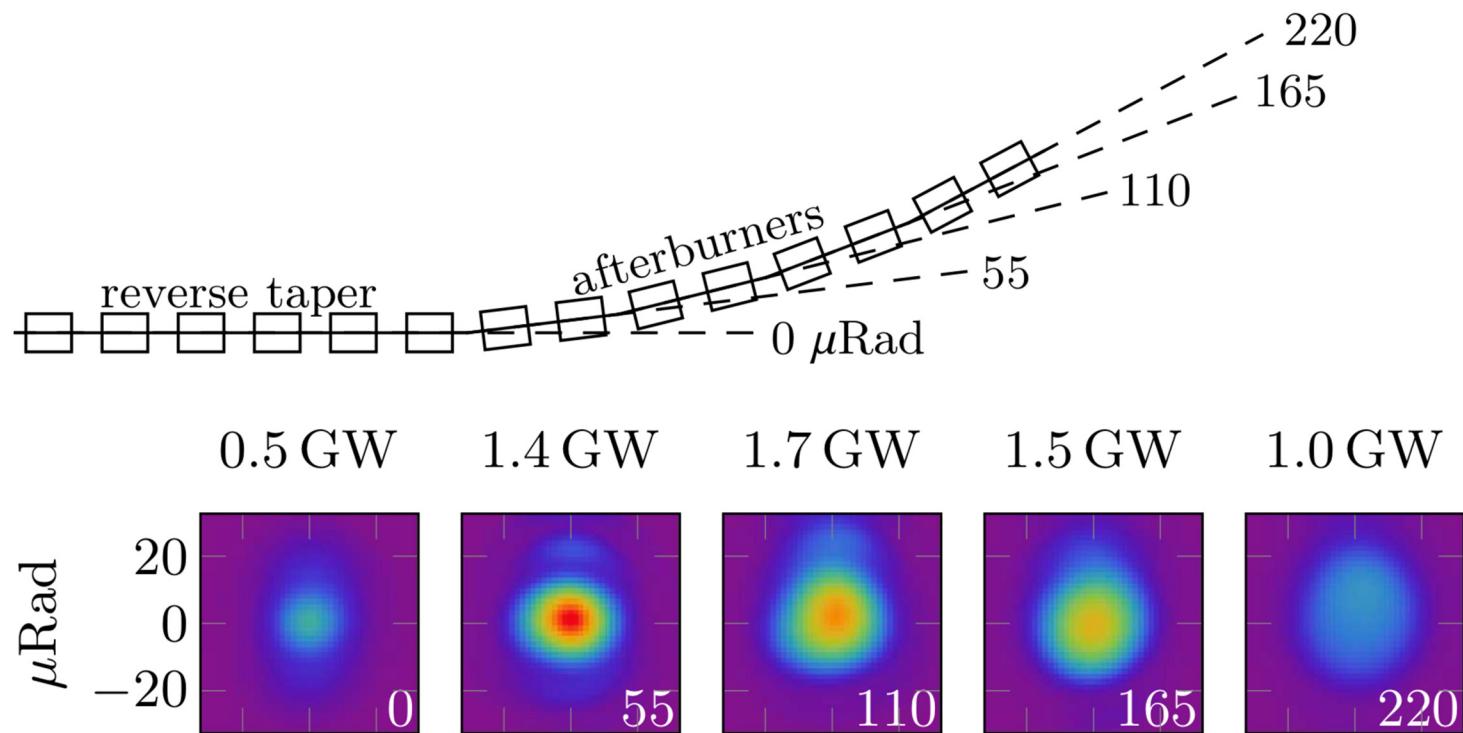
SLAC

$$\frac{\partial \tilde{E}_\nu}{\partial z} \propto e^{-\frac{\alpha^2}{2\phi_c^2} - \frac{1}{2}(k^2 z \epsilon \phi_c (\phi_y - \alpha_b))^2 + i \left(k_u \Delta\nu + k \frac{(\alpha - \phi_y)^2}{2} \right) z}$$



Multiplexing Possibilities (530 eV Simulation)

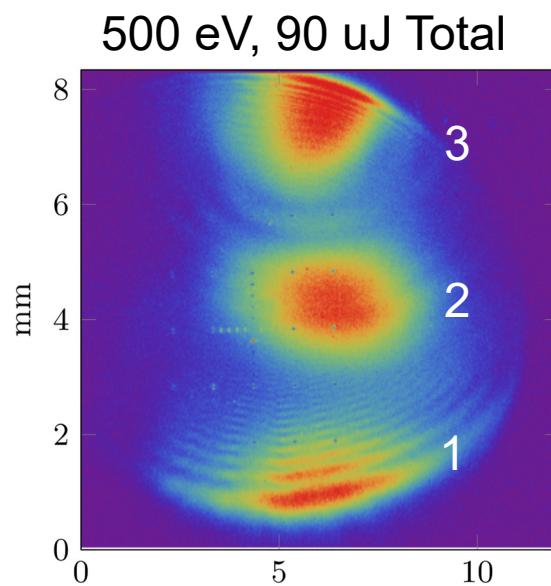
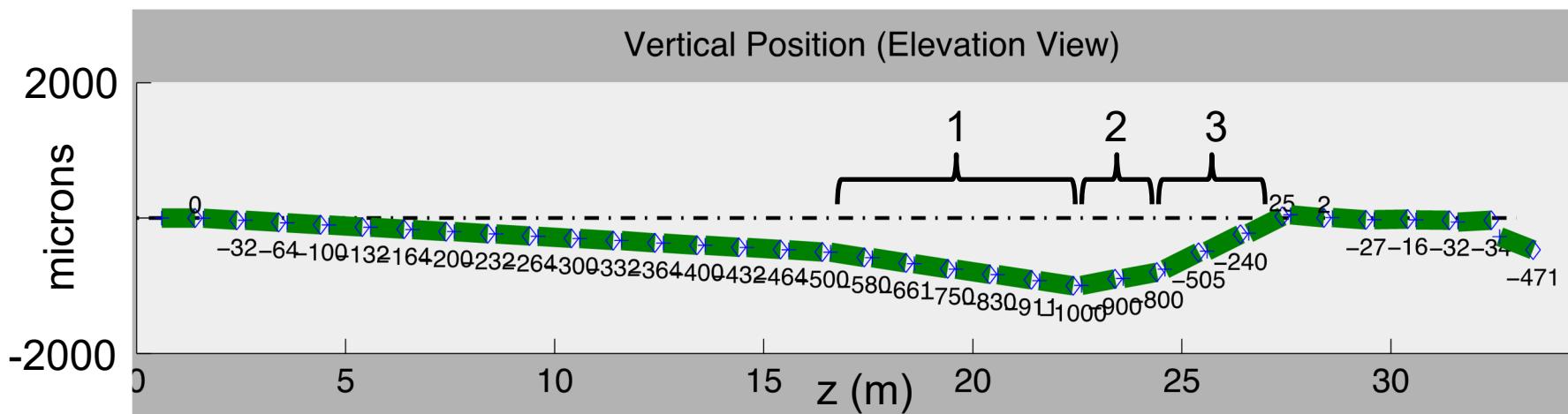
SLAC



Note: critical angle for a single kick is only 25 uRad!

530 eV Multiplexing Experiment

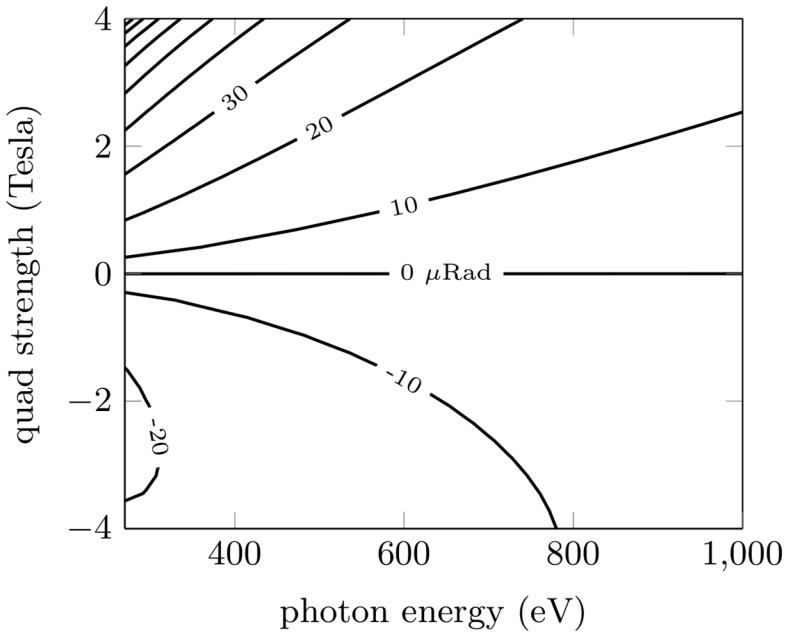
SLAC



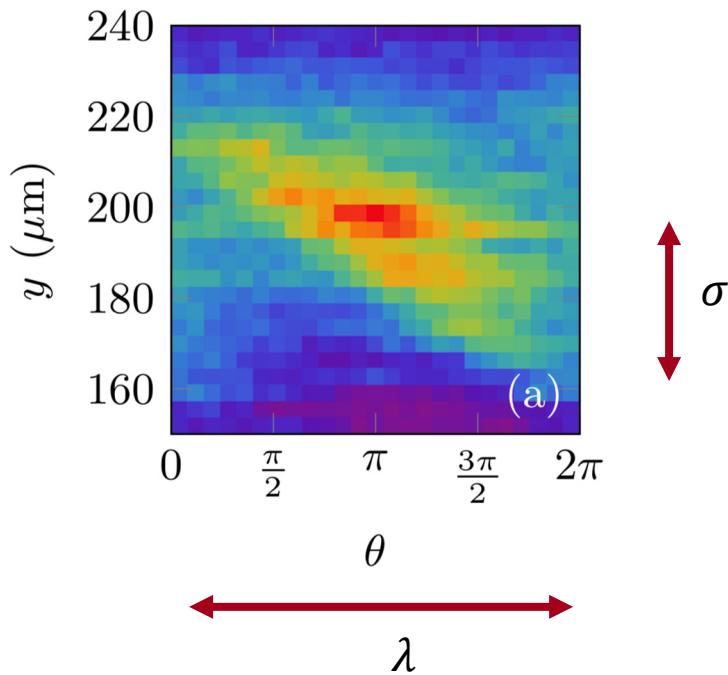
Hard X-Rays: Delicate Microbunches

SLAC

$$\begin{aligned}\Delta s &\approx z \cos(y'_0 + y_0/(2f) + \alpha) - z \cos \alpha \\ &\approx \frac{\alpha z}{2f} y_0 + \alpha z y'_0 + \frac{z}{2} \left(\frac{1}{2f} y_0 + y'_0 \right)^2.\end{aligned}$$

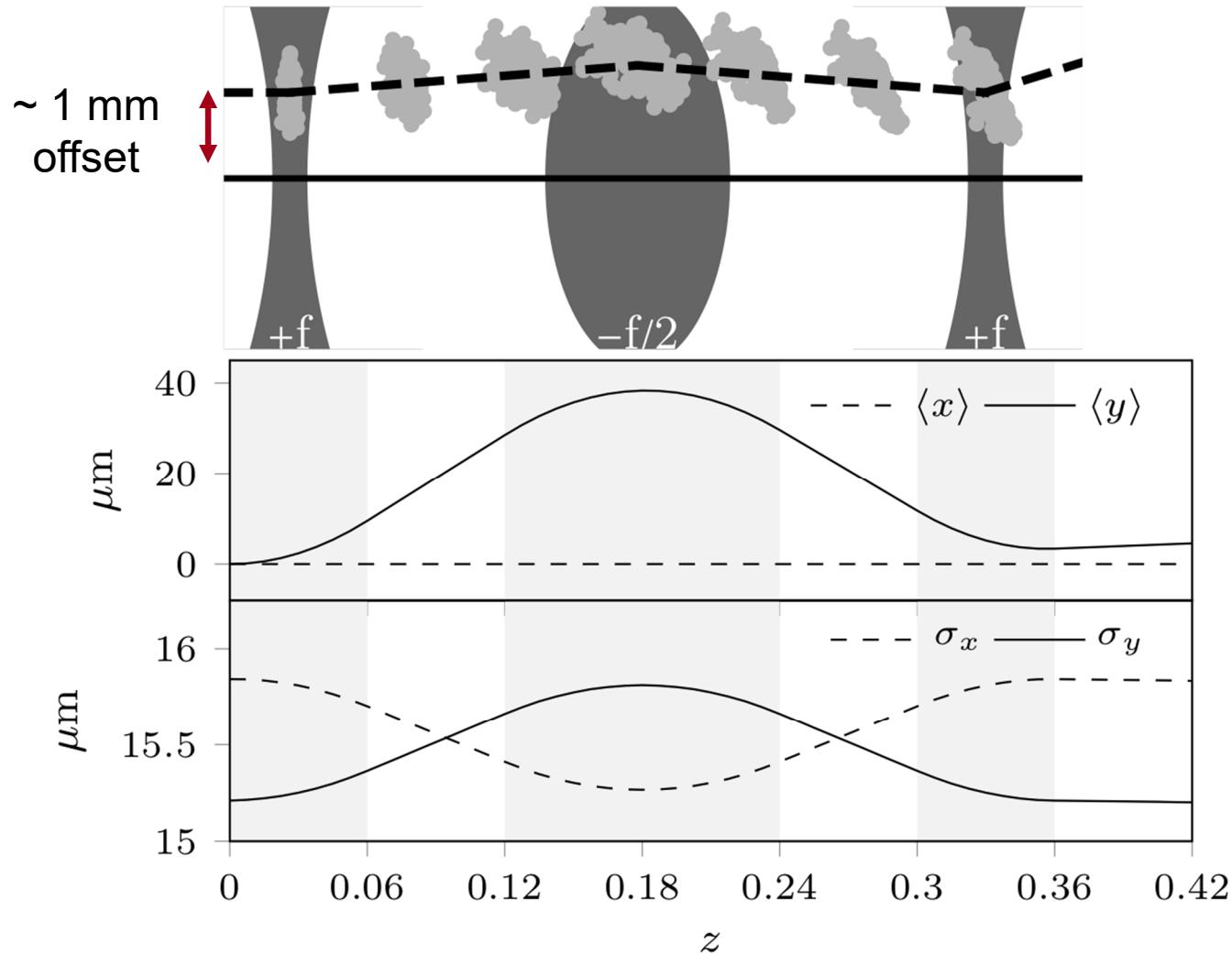


$$\alpha_b(\hat{z} = \hat{L}, \alpha = \phi_c) \approx \frac{1}{k\sigma_y} \frac{f}{|f|}.$$



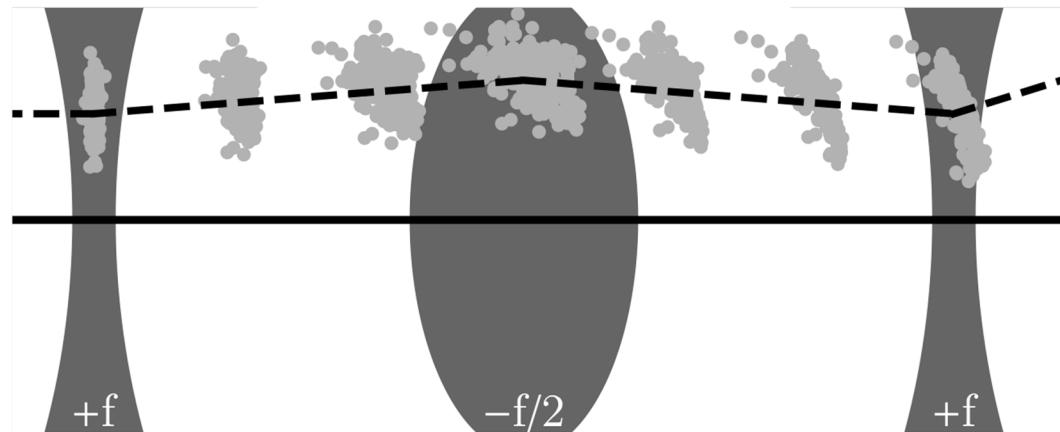
The Offset Triplet

SLAC



Microbunch Recovery

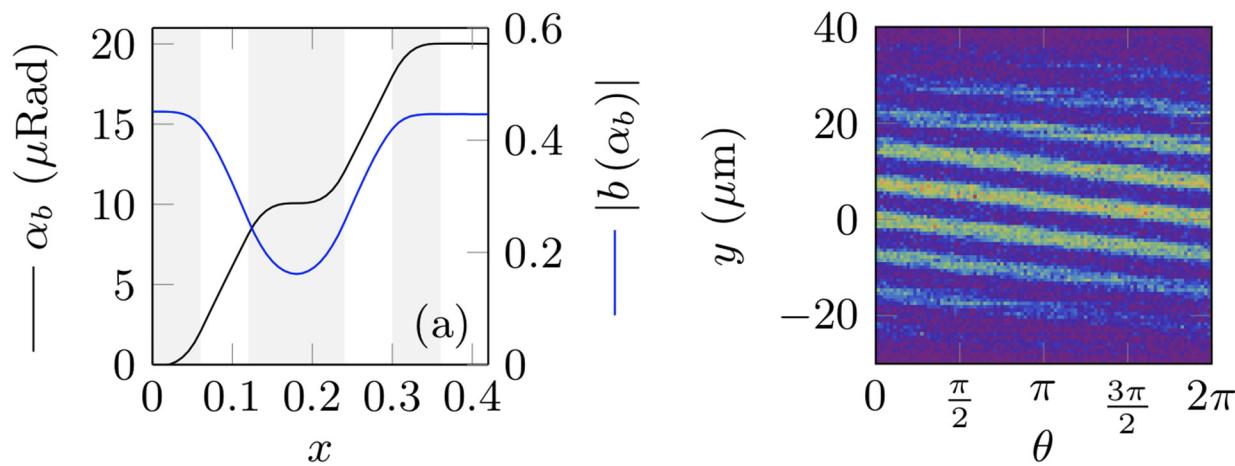
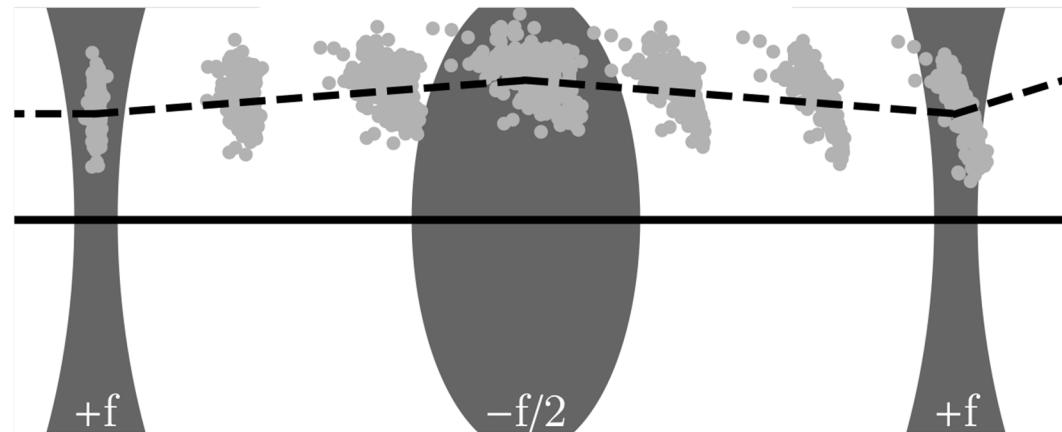
SLAC



$$\Delta s \approx \frac{\alpha z}{2f} y_0 + \alpha z y'_0 + \frac{z}{2} \left(\frac{1}{2f} y_0 + y'_0 \right)^2 .$$

10x Rotation, Minimal Degradation

SLAC



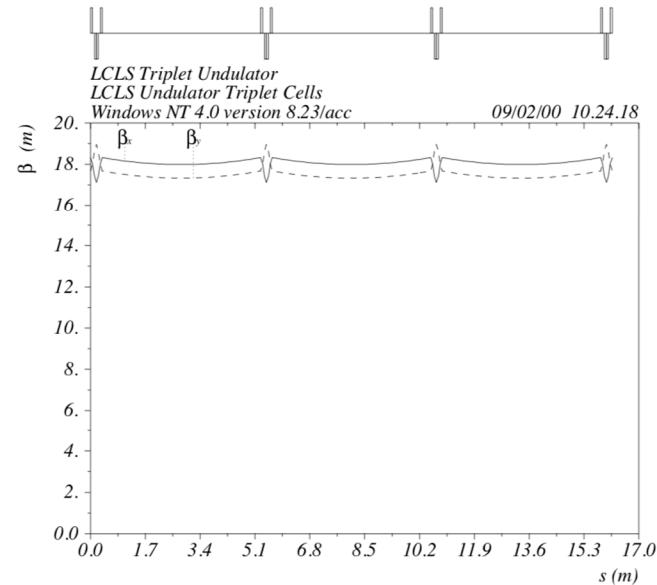
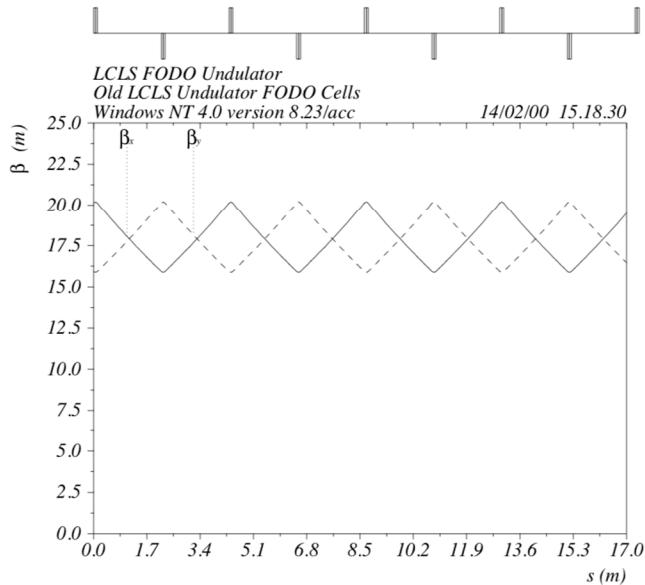
~20 uRad rotation, 10x critical angle

The Magnetic Triplet at LCLS

SLAC

Quadrupole Magnet Error Sensitivities for FODO-Cell and Triplet Lattices in the LCLS Undulator

Paul Emma
Heinz-Dieter Nuhn
SLAC
February 24, 2000

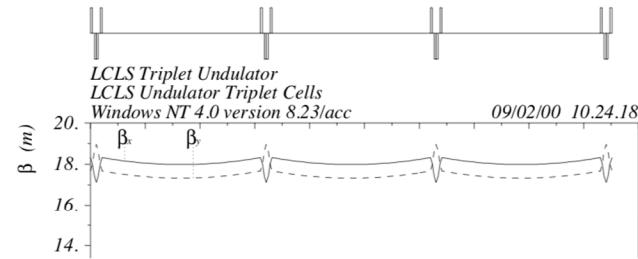
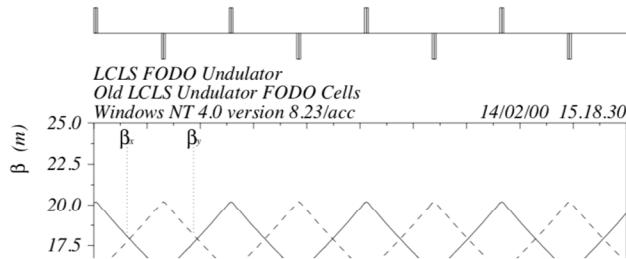


The Magnetic Triplet at LCLS

SLAC

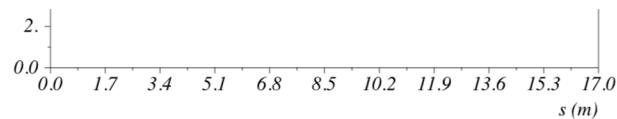
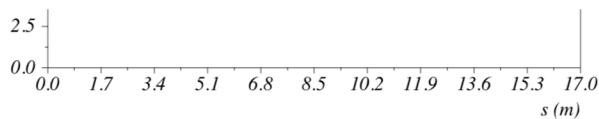
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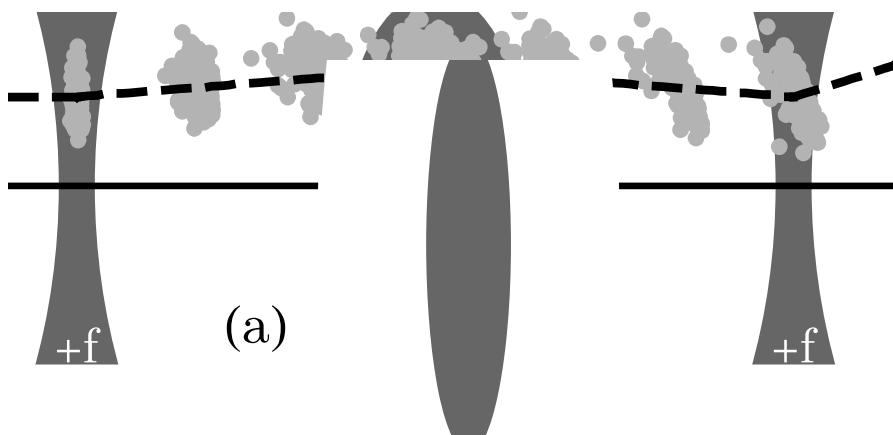
5 Summary

The triplet lattice for the LCLS undulator is a clear prescription for failure.

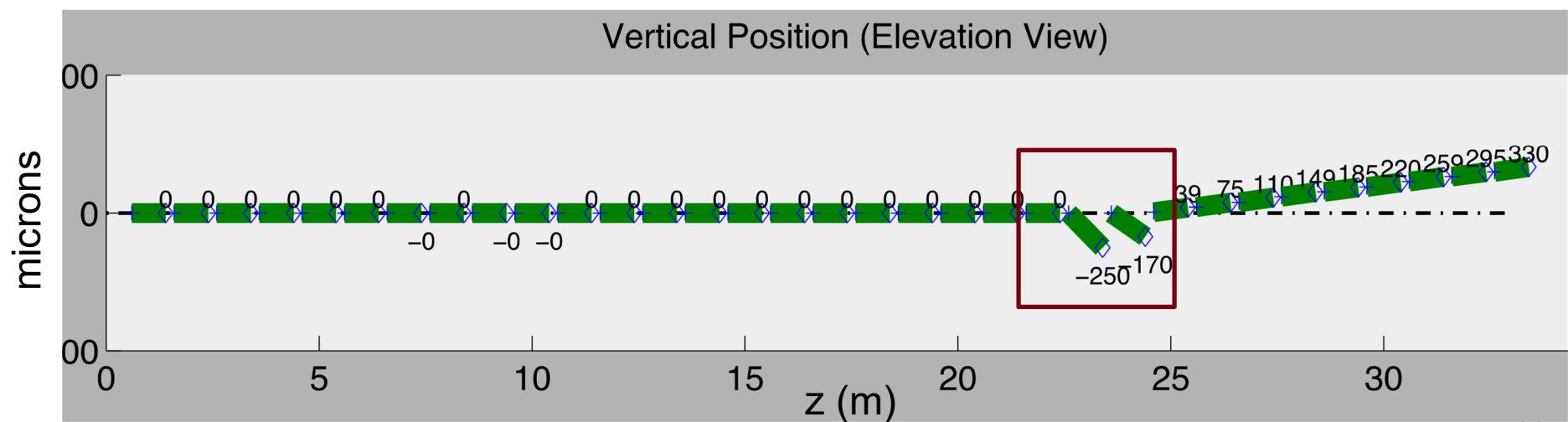


Girder Gymnastics

SLAC



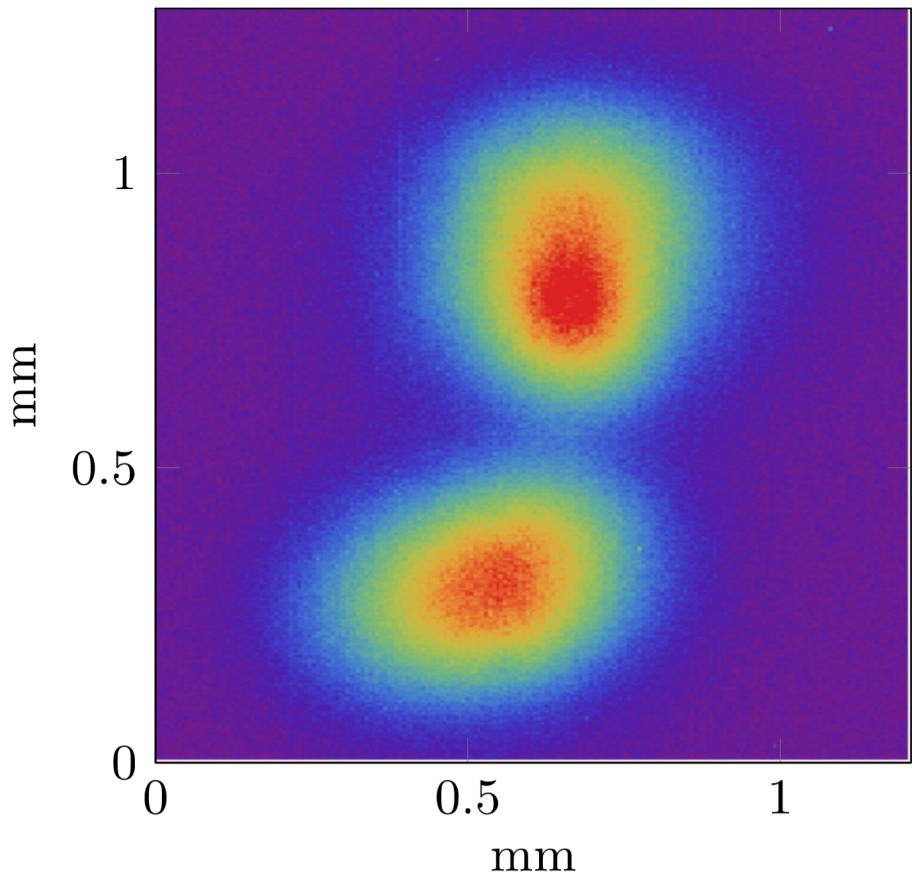
- U22 end:** Beam kicked up with dipole corrector. Beam defocused.
- U23 end:** Beam kicked down with misaligned quad. Beam focused.
- U24 end:** Beam kicked along new u-bunch direction with quad. Beam defocused



Hard X-Ray Microbunch Rotation Experiment

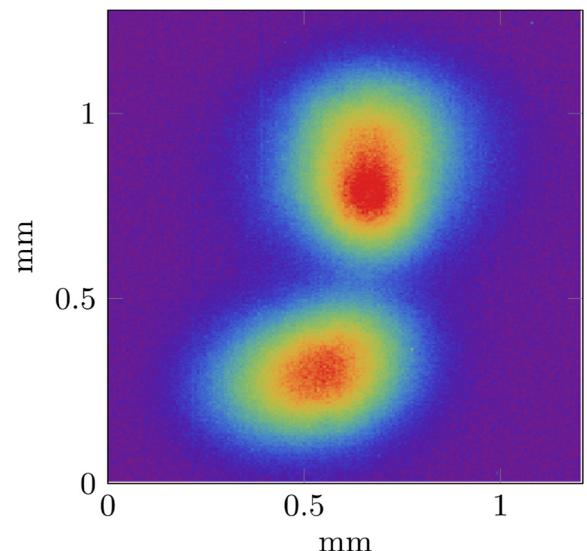
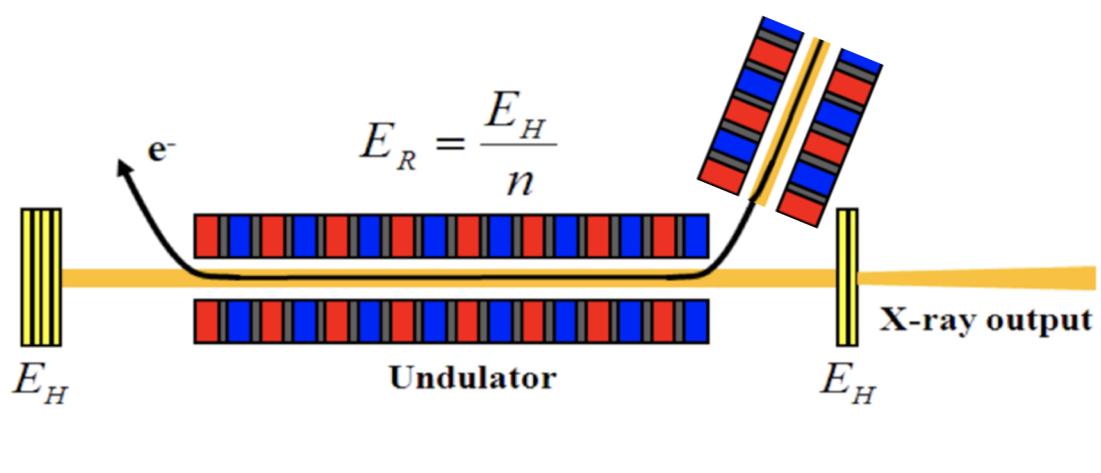
SLAC

- Dec 19, 2018
(last day of LCLS-I)
- Two hard x-ray spots produced
- 850 uJ, split equally between spots
- ~5 uRad separation
- First validation of microbunch rotation at hard x-ray (9.5 keV)
- Analysis & simulation ongoing (R. Margraf)



Oscillator Application

SLAC

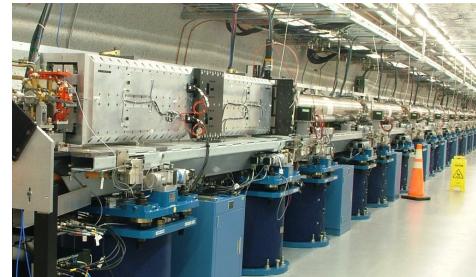


Summary

SLAC

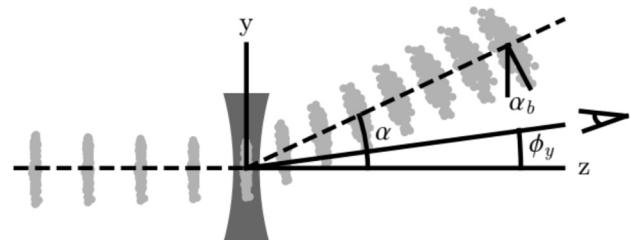
Motivation

- LCLS produces circular beams by beam diversion



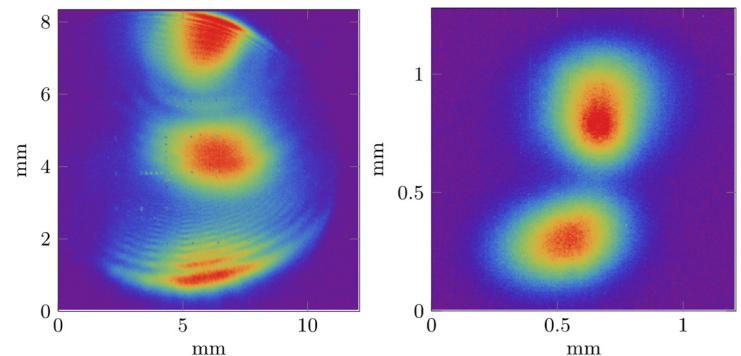
Rotation

- Microbunches rotate in response to a kick
- Consistent with simulation and experiment



Extensions

- Multiplexing at soft x-rays: 3 beams
- Multiplexing at hard x-rays: 2 beams



Questions?

Microbunch rotation and coherent undulator radiation from a kicked electron beam

James P. MacArthur,^{1, 2,*} Alberto A. Lutman,¹ Jacek Krzywinski,¹ and Zhirong Huang^{1, 2}

