

# MASSIVELY PARALLEL WAKE FIELD COMPUTATIONS IN LONG ACCELERATOR STRUCTURES\*

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## Abstract

A new approach for the computation of short range wake fields for ultra relativistic bunches in linear accelerators is presented. The method is based on the direct numerical solution of Maxwell equations for arbitrary 3D-geometry using a specialized split-operator scheme. This approach guarantees exact propagation of longitudinal waves. In addition, it enables the application of a moving computational window. These ideas have been realized in the development of PBCI; a highly efficient, parallelized wake field code. Detailed simulation results for several accelerator components, including the TESLA 9-cell structure and a rectangular collimator are given.

## INTRODUCTION

The X-FEL and the ILC projects require high quality beams with ultra short electron bunches. In order to predict the energy spread and emittance growth of such bunches, an accurate knowledge of the short range wake fields induced in the different accelerator components is necessary. Due to the geometrical complexity involved, however, the computation of wake fields and potentials for long accelerator structures is generally accessible only to numerical simulations.

In the course of the past 20 years, several wake field simulation codes for rotationally symmetric structures have been developed and used with considerable success in the design of linear accelerators [1,2]. The use of a ‘moving window’ in the simulation of ultra relativistic bunches (Bane, Weiland [3]) and the indirect path wake potential integration (Napoly et al [4]) represent, thereby two important milestones in this development. It is however surprising to note that only very recently, the issue of generalizing these two approaches for simulations in 3D-geometry was addressed. In [5] a general procedure for the indirect integration of wake fields in 3D-structures with incoming and outgoing beam pipes is given. In [6] a semi-implicit 3D-discretization technique for Maxwell equations with no dispersion in the longitudinal direction is proposed. The latter is prerequisite for a moving window implementation since in this case, the numerical phase velocity of longitudinal waves must exactly match the speed of light in vacuum.

In this work, we present simulation results obtained with the newly developed code Parallel Beam Cavity Interaction (PBCI) which is designed for massively

parallel wake field simulations in arbitrary 3D-geometry. The algorithms used include a purely explicit and (quasi) dispersionless split-operator scheme as well as a domain decomposition approach for highly balanced parallel computations. A brief description of these algorithms is given in the following section. The rest of the paper is dedicated to the numerical results obtained for some relevant accelerator components in the context of the X-FEL and ILC projects.

## NUMERICAL METHOD

The general framework used in this paper for the simulation of bunch induced electromagnetic wakes is the Finite Integration Technique (FIT) [7]. The time-discrete update equation of FIT can be written compactly as

$$\begin{pmatrix} \hat{\mathbf{e}}^{n+1} \\ \hat{\mathbf{h}}^{n+1/2} \end{pmatrix} = \mathbf{G}(\Delta t) \cdot \begin{pmatrix} \hat{\mathbf{e}}^n \\ \hat{\mathbf{h}}^{n-1/2} \end{pmatrix} \quad (1)$$

where the discrete fields  $\hat{\mathbf{e}}$  and  $\hat{\mathbf{h}}$  are interpreted as electric and magnetic voltages along the edges of a staggered discretization grid. The propagation matrix  $\mathbf{G}(\Delta t)$  represents a discrete operator defined by the specific time stepping scheme employed. The detailed form of  $\mathbf{G}(\Delta t)$  for the commonly used leap-frog integrator is given, e.g., in [8].

### The Split-Operator Scheme

The idea of split-operator methods is to modify the time update equations, such that certain, preferred spatial directions are handled separately. The split-operator scheme used in PBCI is obtained by decomposing the propagation matrix in Eq. 1 into longitudinal and transversal parts. The resulting scheme reads

$$\begin{pmatrix} \hat{\mathbf{e}} \\ \hat{\mathbf{h}} \end{pmatrix}^{n+1} = \mathbf{G}_T\left(\frac{\Delta t}{2}\right) \cdot \mathbf{G}_L(\Delta t) \cdot \mathbf{G}_T\left(\frac{\Delta t}{2}\right) \cdot \begin{pmatrix} \hat{\mathbf{e}} \\ \hat{\mathbf{h}} \end{pmatrix}^n \quad (2)$$

It is shown in [9] that for waves propagating in the longitudinal direction Eq. 2 exactly reproduces the analytical solution of Maxwell equations. Thus, the effect of the above splitting consists in rotating the optimum dispersion direction of the numerical scheme in the longitudinal direction. Since, for typical wake field problems, the longitudinal waves dominate the high frequency spectrum of electromagnetic fields, the method minimizes the unphysical oscillations (numerical noise) which are often a severe limitation of traditional codes.

The exact propagation of longitudinal waves allows for a *moving window* implementation [3]. This enables, in particular, wake field calculations for long structures,

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since the discretization window encloses the bunch region only. Additionally, Eq. 2 is purely explicit in time which makes the time stepping algorithm highly efficient.

### Parallelization Strategy

In 3D simulations involving short bunches (e.g., 300 $\mu$ m for the ILC) and structures of several meters length, huge computational resources in terms of memory and CPU time are needed. Such simulations can only be handled in a parallel computing environment. The parallelization model used in PBCI is based on the distribution of computational tasks among a number of memory independent processes. In this model, however, well balanced workloads should be assigned to each process in order to ensure coherency in parallel program execution.

PBCI uses a geometrical decomposition of the computational domain between the single processes [10]. This approach is shown schematically in Fig. 1 for a three-node cluster of computers. Starting with the complete computational domain, an orthogonal bisection of the domain bounds is recursively applied. The procedure results in a binary tree, whose internal nodes are intermediate sub-domains whereas the leaf nodes correspond to active (computational) sub-domains. This approach allows for an almost ideally balanced distribution of computational workloads. In addition, it can be applied to simulations involving an arbitrary number of processors.

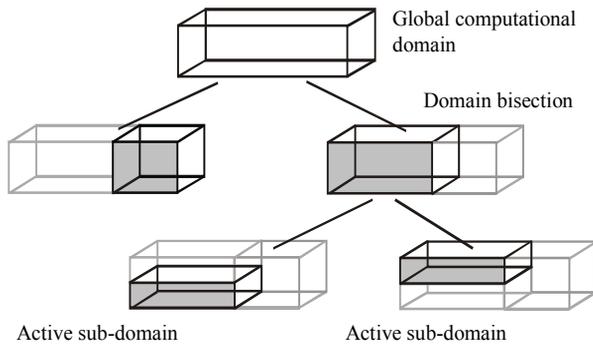


Figure 1: Example decomposition of the computational domain on a three-node cluster.

## SIMULATION EXAMPLES

### TESLA 9-cell Structure

As a first real world example, the wake fields induced by a 1nC bunch of 5mm length in the TESLA 9-cell structure are presented. The simulation was performed on a moving window over a distance of 1.5m. More than 80 million mesh cells were needed for accurately resolving geometry and bunch extension. The total simulation time for this problem size on a 24-node cluster of conventional PCs was slightly higher than 3hrs.

Figure 2 shows a 4-cell section of the structure and the wake fields within the moving window at two different bunch positions. In Fig. 3 the build-up of the longitudinal

wake potential within the structure and in the outgoing beam pipe is shown.

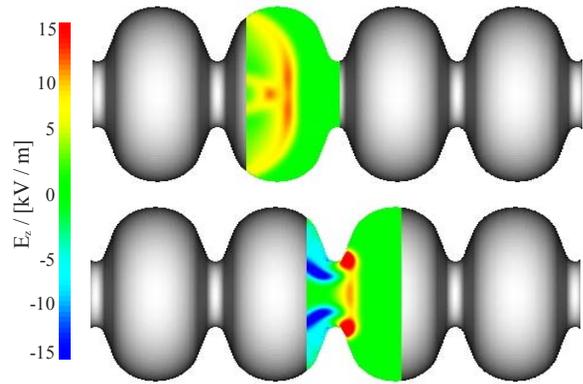


Figure 2: Geometrical view of the TESLA 9-cell structure and wake fields induced at two bunch positions.

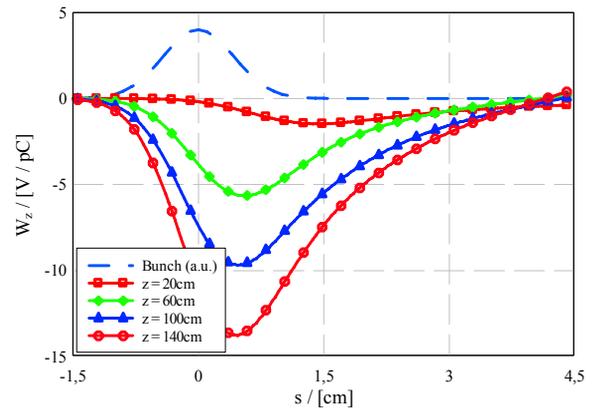


Figure 3: Longitudinal wake potential in the TESLA 9-cell structure at different bunch positions

Since the geometry of the TESLA 9-cell structure is (assumed) rotationally symmetric, a 2.5D simulation could have been performed. The results presented here, however, provide a good test for the performance of a fully 3D code when massive parallelization is used, even compared to traditional 2.5D codes.

### PITZ Diagnostics Double Cross Section

In this application, the wake field effects in the diagnostics double cross of the PITZ injector are considered. This section is the first part in the beam line which breaks the axis symmetry. Thus, a 3D simulation of the structure is absolutely necessary. The geometrical layout of the ten-port vacuum device is shown in Fig. 4.

The investigation includes three separate simulations for performing a comparative study of the wakes induced by the different geometrical obstacles within the device. In the first simulation, the geometry was simplified to the beam tube including only the small kink at the entrance of the section. The second simulation included the vacuum vessel without shielding tube. The third simulation considered the full geometry as shown in Fig. 4.

The simulation results for an electron bunch of charge 1nC and rms length 2.5mm are shown in Fig. 5. For resolving the small details of the geometry, a total of 250

million cells were used in the discretization. It was found, in particular, that the kink of 1mm height within the beam tube is responsible for 10-15% of the induced wake fields. The effect of the vacuum vessel inside the cross is about six times higher. The wake field effects are reduced, as expected, when the tube shielding is included.

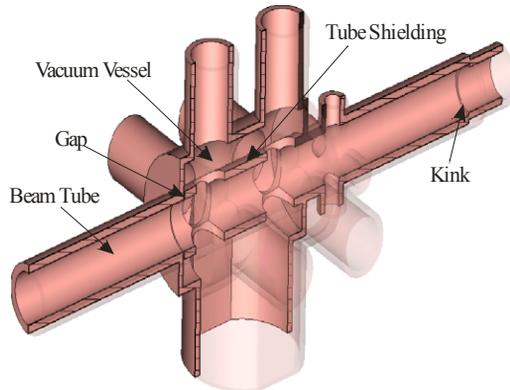


Figure 4: Geometrical view of the diagnostics double cross of the PITZ injector.

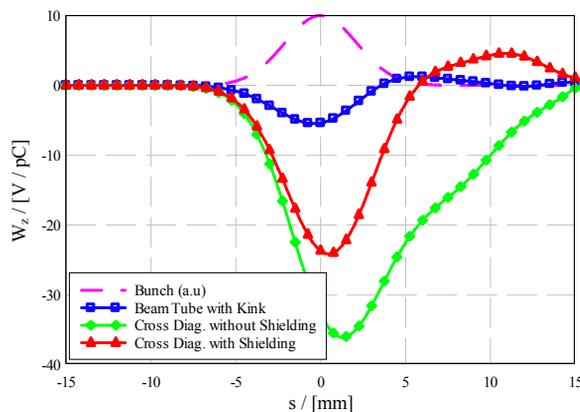


Figure 5: Longitudinal wake potentials induced by the different obstacles within the diagnostics double cross.

### ILC-ESA Rectangular Collimator

The collimator under consideration is part of the ILC-ESA beam test program. A schematic view and the dimensions of the structure are shown in Fig. 6. Again, the rectangular cross-section of the structure makes the application of 2.5D simulation codes impossible.

Due to the extremely short bunch length (300 $\mu$ m) and to the smooth tapering of the collimator slow numerical convergence is expected. Therefore, a fine discretization with a mesh resolution of 37.5 $\mu$ m in the longitudinal direction was used in the simulation. Figure 7 shows the build-up of the longitudinal wake potential at different positions along the collimator axes. Note that here, as in the two previous examples, no indirect path integration for the wake potential at infinity is used. Instead, the wake potentials at fixed positions within and behind the structure are calculated. The indirect wake potential integration for PBCI is still in development and it will be presented in the near future.

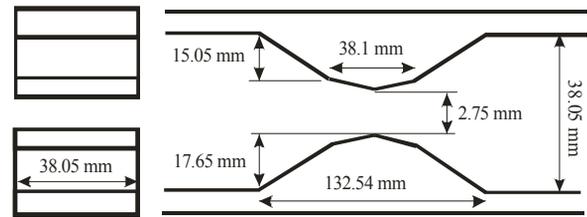


Figure 6: Beam and side views of the collimator #8 for the ILC-ESA beam test experiments.

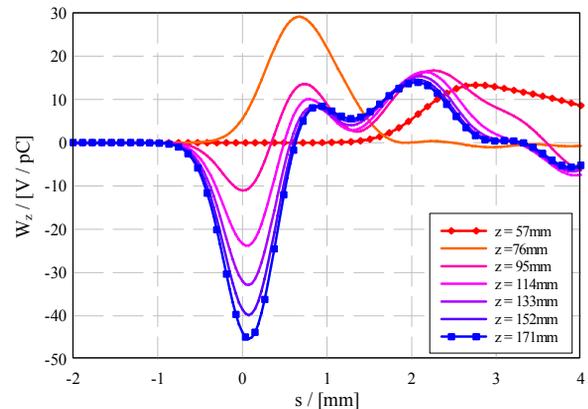


Figure 7: Build-up of the longitudinal wake potential in the ILC-ESA collimator. The two tagged lines show the potentials shortly after the bunch enters the collimator (red) and behind the collimator (blue), respectively.

## CONCLUSIONS

In this paper wake field simulations with the PBCI code for several X-FEL and ILC components are presented. The simulations demonstrate that the wake fields of ultra short bunches in 3D geometry can be efficiently computed when specialized numerical algorithms combined with massive parallelization are employed.

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