IMPACT OF A RF FREQUENCY CHANGE ON THE LONGITUDINAL BEAM DYNAMICS

R. Duperrier*‡, N. Pichoff†, D. Uriot‡,
‡ CEN Saclay, 91191 Gif sur Yvette,
† CEN Bruyères le Châtel, BP12, 91680, France.

Abstract

Frequency jumps in an ion linac use to be made in order to provide a large transverse acceptance in the low energy part and a very high accelerating gradient in the high energy part. This frequency jump may induce a discontinuity in the average longitudinal force per focusing period and shrink the longitudinal acceptance of the linac if this transition is not performed carefully. In this paper, three techniques are developed which produce a "certain" continuity of the channel at the transition. The continuity type is discussed. It is demonstrated that the longitudinal acceptance can be preserved whatever the frequencies of the cavities in the linac. This point is very important when comparisons between different cavity types are made (spoke and elliptical cavities for instance). A few examples are shown to illustrate the performances of the three techniques.

INTRODUCTION

Frequency jumps in ion linac use to be made in order to provide large transverse acceptance in the low energy part and very high accelerating gradient in the high energy part. The minimization of the size of the cavities in the high energy section is also interesting to reduce the cost because these cavities are the most numerous in the accelerator. This frequency jump may induce a discontinuity in the average longitudinal force per focusing period and shrink the longitudinal acceptance of the linac if this transition is not performed carefully. During the European Spallation Source studies in 2000 [1], we developed a first technique to keep constant the confinement potential shape at the frequency jump. The goal was to maintain the beam in the achieved equilibrium state. Later, we proposed a different approach based on the continuity of the acceptance of the system. More recently, a third technique which is a mix of the two first has been proposed. In this paper, these three techniques are developed and compared to the classical method which is a matching at the transition keeping an high accelerating efficiency.

* rduperrier@cea.fr

CONSTANT POTENTIAL SHAPE

The superconducting option for the European Spallation Source linac is made with different sections which operate at different frequencies (175, 350 and 700 MHz) [2]. To render transparent for the beam these frequency jumps, a first technique to keep constant the confinement potential shape at the transition has been developed [1]. This technique aims to maintain the beam in the achieved equilibrium state. This point is especially relevant when a very intense beam has to be transported. From the equation of the longitudinal motion, it is shown in [3] that the potential well can be written:

$$V(\delta\phi) = -\frac{2\pi}{m(\beta\gamma c)^3 f_{rf}} \cdot qE_0T \cdot [cos\phi_s\,(sin\delta\phi - \delta\phi) + sin\phi_s \cdot (cos\delta\phi - 1)] \quad (1)$$

with $\delta\phi$ the phase shift of the particle in respect to the synchronous phase ϕ_s, f_{rf} the operating frequency, β the reduced speed, γ the Lorentz factor, E_0T the average field per focusing period taking into account the transit time factor, c the speed of light, m the mass of the particle and q its charge. Writing $\delta\phi = 2\pi f_{rf}\delta t$ and developping at third order in δt, the expression 1 can be rewritten as a function of δt. At a frequency change, one should try to keep this potential well as continuous as possible. This can be made at third order by changing both the synchronous phase and the accelerating field. One has to solve the equation system:

$$\begin{cases} E_0T \cdot f_{rf}^2 \cdot cos\phi_s = cste \\ E_0T \cdot f_{rf} \cdot sin\phi_s = cste \end{cases} \quad (2)$$

If f_1 is the frequency for the section before the transition and f_2 the frequency for the section after, writing $k = f_2/f_1$, this gives the necessary conditions:

$$\begin{cases} tan\,(\phi_s)_2 = k \cdot tan\,(\phi_s)_1 \\ (E_0T)_2 = \frac{(E_0T)_1}{k^2}\sqrt{\frac{1+k^2tan^2(\phi_s)_1}{1+tan^2(\phi_s)_1}} \end{cases} \quad (3)$$

The relative variation of the real estate gradient can be calculated with the following formula:

$$\Re = \frac{1}{k^2} \quad (4)$$

if $f_1 < f_2$, $\Re$ is always lower than 1. This is the main drawback of the method.

CONTINUITY OF THE ACCEPTANCE

We developed a second approach to calculate the field and the phase at the transition instead of fitting the potential shape, we set that the longitudinal acceptance has to be kept constant. Taking into account that the phase is not a canonical coordinate, this statement implies that ΔW_{max} and ΔZ_{max} have to be maintained at the transition. To compute the expression for ΔZ_{max}, we use the following approximation for the acceptance in phase:

$$\Delta\phi \sim 3\phi_s \tag{5}$$

where ϕ_s is the synchronous phase. It is valid when ϕ_s is close to the crest (strong acceleration) which is the case for our problem. For ΔW_{max} conservation, we will use the analytical formula [3]:

$$\Delta W_{max} = \pm 2\left[\frac{qmc^3\beta^3\gamma^3 E_0T\left(\phi_s cos\phi_s - sin\phi_s\right)}{f_{rf}}\right]^{1/2} \tag{6}$$

This formula is derived from the integration of the motion with the assumption that the speed is quasi constant during the acceleration. To keep constant ΔZ_{max} and ΔW_{max} at the frequency jump, it is required that:

$$\left\{\begin{array}{lcl} \frac{\phi_s}{f_{rf}} & = & cste \\ \frac{E_0T(\phi_s cos\phi_s - sin\phi_s)}{f_{rf}} & = & cste \end{array}\right. \tag{7}$$

always with $k = f_2/f_1$, the first condition implies:

$$\left(\phi_s\right)_2 = k\left(\phi_s\right)_1 \tag{8}$$

For the second condition, a Taylorisation of the cosine and sine at second order in the system 7 gives:

$$\left(E_0T\right)_2 = \frac{\left(E_0T\right)_1}{k^2} \tag{9}$$

The relative reduction of the real estate gradient can be calculated with the following formula:

$$\Re = \frac{cos\left[k\left(\Phi_s\right)_1\right]}{k^2 cos\left[\left(\Phi_s\right)_1\right]} \tag{10}$$

if $f_1 < f_2$, $\Re$ is always lower than 1.

CONTINUITY OF THE PHASE ACCEPTANCE AND THE PHASE ADVANCE PER METER

Recently, we developed a third approach to perform the transition which is a mix of the two previous ones. We set that the phase acceptance (but not the energy one) and the phase advance per meter have to be kept constant. It implies that:

$$\left\{\begin{array}{lcl} \frac{\phi_s}{f_{rf}} & = & cste \\ E_0T \cdot f_{rf} \cdot sin\phi_s & = & cste \end{array}\right. \tag{11}$$

Writing again $k = f_2/f_1$, we found from 11 that the phase and E_0T have to follow the relations:

$$\left\{\begin{array}{lcl} \left(\phi_s\right)_2 & = & k \cdot \left(\phi_s\right)_1 \\ \left(E_0T\right)_2 & = & \frac{\left(E_0T\right)_1}{k}\frac{sin\left[\left(\phi_s\right)_1\right]}{sin\left[k\left(\phi_s\right)_1\right]} \end{array}\right. \tag{12}$$

The relative real estate gradient reduction can be calculated with the formula 13:

$$\Re = \frac{tan\left[\left(\Phi_s\right)_1\right]}{tan\left[k\left(\Phi_s\right)_1\right]} \tag{13}$$

EMITTANCE GROWTH

One figure of merit for the beam dynamics is the conservation of the initial emittance in the linac. To test our three techniques, we propose to use a section of accelerating periods starting at 50 MeV. The lattice is FDO. The accelerating voltage is set to 2 MV and the initial synchronous phase is -20 degrees. The length of the section is 80 meters with 22 focusing periods. The operating frequency of the first half part is 352 MHz. For this first subsection, the field and synchronous phase are kept constant. We designed 5 versions for the second part of the linac:

- a copy of the first section to get a reference simulation which can be compared to the versions with a frequency jump,
- a constant field and phase linac with a frequency of 704 MHz and a matching at the transition,
- a linac at 704 MHz designed according to the technique "constant acceptance",
- a linac at 704 MHz designed according to the technique "constant potential shape",
- a linac at 704 MHz designed according to the technique "constant phase acceptance and phase advance per meter".

For each of these 5 linacs, we computed several transports increasing the input longitudinal emittance. For each run, the relative emittance growth has been calculated. The figure 1 shows the behaviour of these emittance growths. An emittance blow up can be noticed for the linac with just a matching at the transition. Whereas the emittance growth behaviours for the three techniques are comparable with the one for the linac which has no frequency jump. A slight loss of efficiency is observed for high input emittance with the technique "constant potential shape".

ACCEPTANCE

Several hypothesis have been used to obtain the frequency transition rules in the previous sections. For the first technique, it is assumed that the phase shift

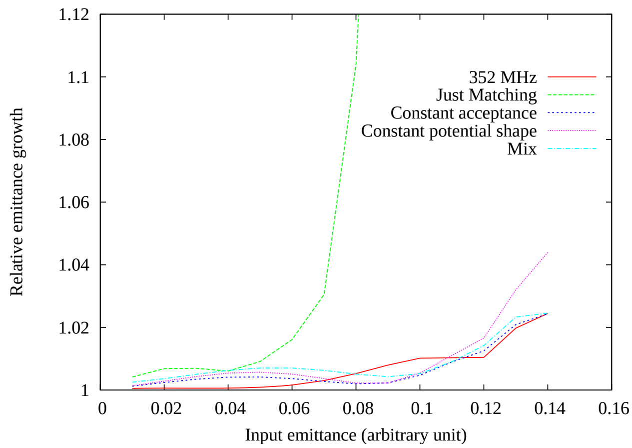

Figure 1: The relative emittance growths for an increasing input longitudinal emittance for the four linacs.

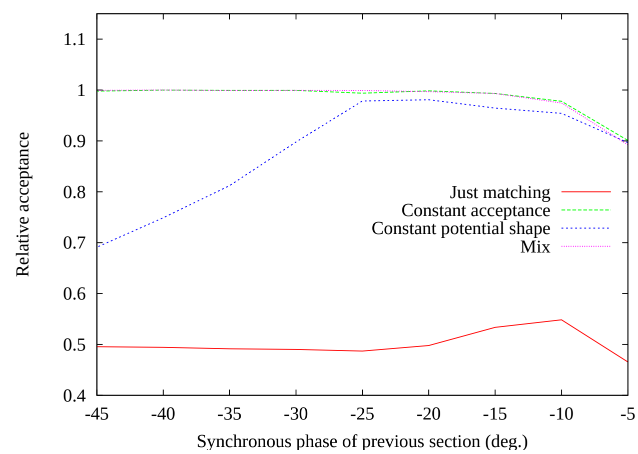

Figure 2: The relative acceptance of the linacs with frequency jumps in respect to the initial synchronous phase. The reference is the linac without frequency jump.

between the particles and the synchronous one is small. For the second technique, the hypothesis that the synchronous phase is close to zero is made. And, for the three techniques, the integration of the motion is performed with the approximation that the speed of the particle is constant in an accelerating gap. To check these approximations, we propose to compute the acceptance of the preceeding linacs with the transport of a huge cloud of particles which are initially uniformly distributed, and, to count the surviving particles to allow a relative comparison of the linac acceptances without approximation. We repeated these simulations for different initial synchronous phase from -45 degrees to -5 degrees to check the validity of the several approximations about the phase. The figure 2 shows the evolution of the relative acceptances for linacs with a frequency jump in respect to the linac with only 352 MHz cavities.

First, it is clear that keeping constant the field and the phase and perform a matching at the transition is not satisfying. Second, the second and third techniques appear to be very efficient for a wide range of synchronous phase. For the technique of the constant potential shape, it is equivalent to the two previous ones in the range beyond -25 degrees to the crest. It can be noticed that the efficiency for the three methods is faded when the phase is close to zero. It has been checked that it is due to the approximation of the constant speed for the integration of the motion which provided the equations 1, 5 and 6.

CONCLUSIONS

We investigated three different techniques to manage a frequency change in a ion linac to render it as transparent as possible: the ”constant potential shape”, the ”constant acceptance” and a mix which is a constant phase acceptance and a constant phase advance per meter. We compared them with the common method which is a simple matching maintaining a high acceleration efficiency. It is shown that the three techniques enhanced significantly the acceptance of the system. To simplify and to provide to linac designers rules which are valid for a wide range of parameters, we recommend to use the following formulae at the transition:

$$\begin{cases} (\phi_s)_2 = k\,(\phi_s)_1 \\ (E_0T)_2 = \frac{(E_0T)_1}{k}\frac{sin\left[(\phi_s)_1\right]}{sin\left[k(\phi_s)_1\right]} \end{cases} \quad (14)$$

with $k = f_2/f_1$ and f_i the frequency, $(E_0T)_i$ the average field taking into account the transit time factor and $(\phi_s)_i$ the synchronous phase of the section i. The main drawback of the three methods is a reduction of the real estate gradient at the transition. But higher is the exit energy of the linac, lower is the impact of this reduced acceleration at the transition as the high gradient of high frequency cavities enhances significantly the accelerating efficiency. It is very important to respect such rules at the frequency transition in the linac when comparisons between different cavities are made from the beam dynamics point of view (spoke and elliptical cavities for instance).

REFERENCES

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[2] The ESS Project, Technical report, Volume III, may 2002.

[3] P. Lapostolle and M. Weiss, Formulae and procedures useful for the design of linear accelerators, CERN-PS-2000-001 (DR), 28 january 2000.