PROPERTIES OF DISC-LOADED CYLINDRICAL WAVEGUIDE PROPAGATING THE TM-02 MODE

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Accelerators designed for very large output beam currents are, from practical considerations, operated in the transient regime. In such cases the energy spectrum is improved when the Q of the structure is high. The following report describes experimentally determined properties of the TM-02 mode for the $2\pi/3$ longitudinal mode at the velocity of light, from which it would appear that this mode is significantly better for such applications.

The so-called 'resonance condition' for the TM-01 mode is given by the determinantal equation (1),

$$\frac{J_0(kb)}{N_0(kb)} = \frac{\frac{k_1 a}{ka} J_0(k_1a)J_1(ka) - J_1(k_1a)J_0(ka)}{\frac{k_1 a}{ka} J_0(k_1a)N_1(ka) - J_1(k_1a)N_0(ka)}$$

where $k_1^2 = k^2 - k_3^2 = k^2 \frac{\beta^2 - 1}{\beta}$. This equation establishes the location of radial zeroes of the longitudinal electric field. Hence the resonance condition for the TM-02 mode of propagation is given by the second solution of the LHS, shown in Figure 1. At the velocity of light ($k_1 = 0$) the above equations assume a particularly simple form

$$\frac{J_0(kb)}{N_0(kb)} = \frac{J_2(ka)}{N_2(ka)}$$

Successive modes are, of course, still given by successive solutions of $J_0(kb)/N_0(kb)$. It is of practical importance to notice that the resonance condition for the TM-01 mode is actually given, closely, by $J_0(kb_1) = 0$, that is $kb_1 \doteq 2.405$. One would anticipate then that second zero would be given by $kb_2 \doteq 5.520$. Hence, the TM-02 mode would occur when $b_2 \doteq 2.295 b_1$.

It is possible to estimate the higher order mode Q of the structure from the Q of the cylindrical TM-ono modes. For the TM-ono modes

$$Q = \frac{\eta}{2R_s} \frac{p_{on}}{1 + \frac{b}{h}}$$

To preserve the analogy we put $h = \lambda/3$ and normalize Q estimations to that of the TM-olo mode, whence

$$\frac{Q(TM_{ono})}{Q(TM_{olo})} = \frac{p_{on}\,(2\pi + 3p_{ol})}{p_{ol}\,(2\pi + 3p_{on})}$$

For the first few modes we may estimate:

n	pon	Q(TM-ono)/Q(TM-olo)
1	2.41	1.00
2	5.52	1.36
3	8.65	1.50
4	11.79	1.59
∞	. . .	1.87

The experimental data shown hereinafter was obtained at 1300 mcs, using well-known microwave measurement techniques for periodic structures (2).

In Figure 2 is shown the dimensions for the 'resonant condition'.

In Figure 3 the group velocity is shown as a function of disc aperture. In every case the disc edge was radiused at half the disc thickness. The disc thickness was 0.476 inches ($t/\lambda = 0.0525$).

In Figure 4 the r/Q of the synchronous space harmonic is shown as a function of disc aperture. In every case the amplitude of the fundamental space harmonic was such as to transport about 0.75 (seventy-five percent) of the power flux.

In Figure 5 the series impedance

$$\frac{E}{\sqrt{P}} = \sqrt{2Ir} = \sqrt{\frac{2\pi}{\lambda}\left(\frac{r}{Q}\right)\frac{1}{v_g/c}}$$

is shown as a function of disc aperture.

It is evident that the above data, Figures 3, 4, and 5 can be scaled using well-known scaling laws; if the structure is scaled to a wavelength λ_2 from a wavelength λ_1 group velocity remains the same, $Q_2/Q_1 = \sqrt{\lambda_2/\lambda_1}$ and $r_2/r_1 = \sqrt{\lambda_1/\lambda_2}$

REFERENCES

1. E. L. Chu and W. W. Hansen, The Theory of Disc-Loaded Wave Guides, JAP 8,996 (1947)

2. W. J. Gallagher, Measurements Techniques for Periodic Structures. Microwave Lab. Rep. No. 767, Stanford University (1960)

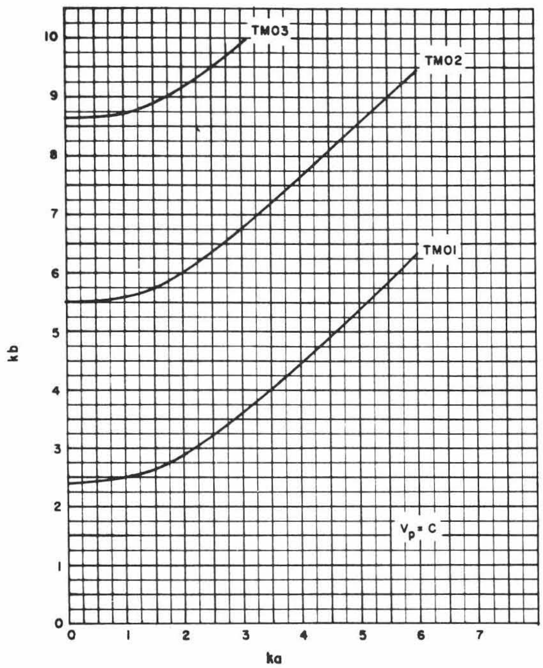

Fig. 1. Solution of determinantal equation for TM-02 mode, $V_p = c$.

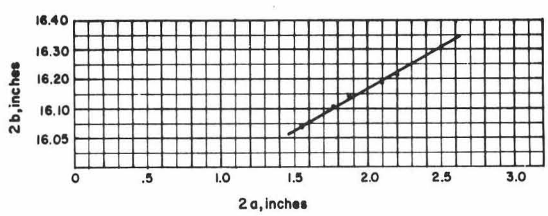

Fig. 2. Cylinder bore as a function of disc aperture, $2\pi/3$ mode.

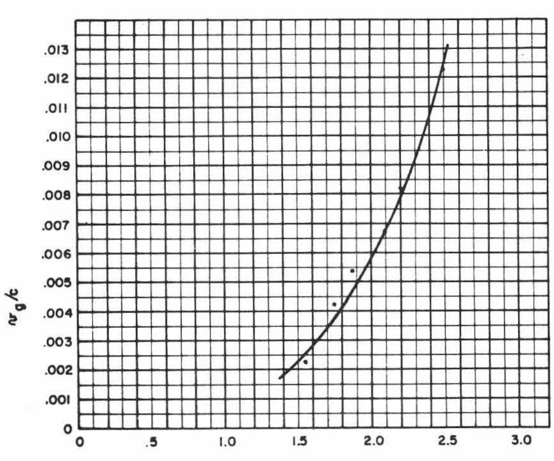

Fig. 3. Group velocity as a function of disc aperture.

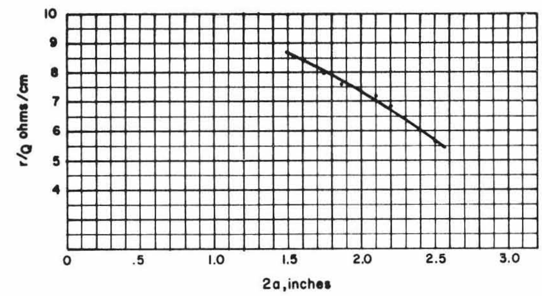

Fig. 4. r/Q as a function of disc aperture.

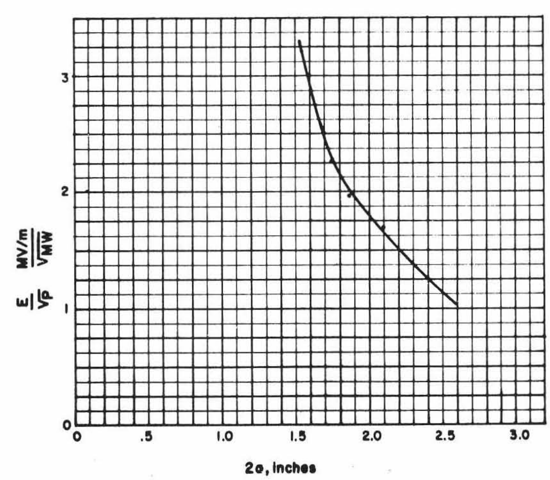

Fig. 5. Series impedance as a function of disc aperture.