ENERGY SPECTRUM IMPROVEMENT WITH THE HEM-11 MODE

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As a result of both theoretical and experimental work on the HEM-11 mode in the disc loaded waveguide it is apparent that it would be practical to use this structure for improvement of the energy spectrum of the electron linear accelerator. A proposed scheme is shown in Fig. 1. The nearly parallel beam is transmitted through a dispersive isochronous momentum (energy) analyzer in such a manner that a parallel beam is formed, with the particles arranged transversely as a function of energy. If the bunches of the beam are synchronized with the longitudinal electric field of the HEM-11 mode of the waveguide particles passing down the axis will be unaffected while off-axis particles will be accelerated (or retarded) proportionally to their displacement. Adjustment of the power level will form the emerging beam into a nearly monoenergetic slit or ribbon beam which can then be focussed onto the target by a quadrupole lens. Synthesis of an appropriate magnet system is not unique and further discussion is omitted here (1).

The proposed waveguide structure is the disc-loaded cylindrical waveguide supporting the HEM-11 (or TM_{11}-like) mode (Fig. 2) for which the field components in the aperture ($r < a$) for the fundamental space harmonic at the velocity of light are, neglecting attenuation (2),

$$E_z = E_o kr \cos\varphi \; \cos(\omega t - \beta z)$$

$$E_r = E_o\left[\left(\frac{kr}{2}\right)^2 + \left(\frac{ka}{2}\right)^2\right]\cos\varphi \; \sin(\omega t - \beta z)$$

$$E_\varphi = E_o\left[\left(\frac{kr}{2}\right)^2 - \left(\frac{ka}{2}\right)^2\right]\sin\varphi \; \sin(\omega t - \beta z)$$

$$\eta H_z = -E_o kr \sin\varphi \; \cos(\omega t - \beta z)$$

$$\eta H_r = -E_o\left[\left(\frac{kr}{2}\right)^2 - \left(\frac{ka}{2}\right)^2 + 1\right]\sin\varphi \; \sin(\omega t - \beta z)$$

$$\eta H_\varphi = E_o\left[\left(\frac{kr}{2}\right)^2 + \left(\frac{ka}{2}\right)^2 - 1\right]\cos\varphi \; \sin(\omega t - \beta z)$$

The equations of motion in cylindrical coordinates for a charged particle interacting with an electromagnetic field are

$$\frac{d}{dt}(\gamma\dot{z}) = \frac{e}{m_o}\left(E_z + \dot{r}B_\varphi - r\dot{\varphi}B_r\right)$$

$$\frac{d}{dt}(\gamma\dot{r}) - \gamma r\dot{\varphi}^2 = \frac{e}{m_o}\left(E_r + r\dot{\varphi}B_z - \dot{z}B_\varphi\right)$$

$$\frac{1}{r}\frac{d}{dt}(\gamma r^2\dot{\varphi}) = \frac{e}{m_o}\left(E_\varphi + \dot{z}B_r - \dot{r}B_z\right)$$

If the beam bunches are of negligible length compared to the wavelength and are synchronized with the Ez component of the field, we have

$$\frac{d}{dt}(\gamma\dot{z}) = \frac{eE_o}{m_o} kr \cos\varphi$$

$$\frac{d}{dt}(\gamma\dot{r}) - \gamma r\dot{\varphi}^2 = \frac{-eE_o}{m_o c} kr \sin\varphi \; r\dot{\varphi}$$

$$\frac{d}{dt}(\gamma r^2\dot{\varphi}) = \frac{eE_o}{m_o c} kr \sin\varphi \; \dot{r}r$$

Since $\dot{z}^2 + \dot{r}^2 + (r\dot{\varphi})^2 = c^2 \frac{\gamma^2-1}{\gamma^2}$ Eq (3a) is easily shown to be reducible to

$$\frac{d\gamma}{dt} = \frac{eE_o}{m_o c^2} kr \cos\varphi \sqrt{c^2\frac{\gamma^2-1}{\gamma^2} - \left[\dot{r}^2 + (r\dot{\varphi})^2\right]}$$

The transverse velocity of the beam is assumed to be negligible so that for $\gamma \gg 1$, $\varphi = 0$, and putting $z \doteq ct$,

$$\frac{d\gamma}{dt} = \frac{eE_o}{m_o c} kr$$

or,

$$\gamma - \gamma_o = \left(\frac{eE_o\lambda}{m_o c^2}\right) kr \frac{z}{\lambda}$$

A practical structure at 2856 mcs might consist of a ten-foot waveguide with a 2 inch disc aperture, for which, in the $\pi/2$ - mode, the following properties have been determined (3);

Series impedance, $E_o/\sqrt{P} = 2.0\ MV/m/\sqrt{MW}$

Figure of Merit, $Q = 9000$

Group velocity, $v_g/c = 0.015$

Thus for 5 MW input power Eo = 4.5 MeV/m, $(eE_o\lambda/m_o c^2) = 0.92$. At two centimeters off the axis kr = 1.2 and a net energy correction of $\Delta\gamma = 33$ or 16 MeV would be possible. Due to attenuation (I = 0.2 nep/m) this amount would be reduced to about 12 MeV. The total energy variation which can be corrected is of course twice this amount.

The principal deficiency of the system appears to arise from poor optics in the accelerator beam. Due to the finite beam diameter, and assuming particles of all energies are distributed throughout the beam cross-section, a 'blurring' in the energy distribution in the cross-section of the ribbon beam will occur. As a result a residual energy spectrum width

$$\Delta\gamma' = \left(\frac{eE_o\lambda}{m_o c^2}\right)\frac{z}{\lambda} kb$$

cannot be removed, where b is the beam diameter. For the above example $\Delta\gamma'$ = 3.3 MeV assuming a two millimeter diameter beam from the accelerator, or an overall decrease of spectrum width of ten times.

References

1. R. Belbeoch, Magnetic Systems for Linear Accelerator Beam Injection, SLAC Report M-292, Stanford University (1962).
2. H. Hahn, Deflecting Mode in Cylindrical Iris-Loaded Waveguides, Rev. Sci. Instr. 34, 1094 (1963).
3. H. Hahn, BNL Internal Report AADD-54 (November 18, 1964)
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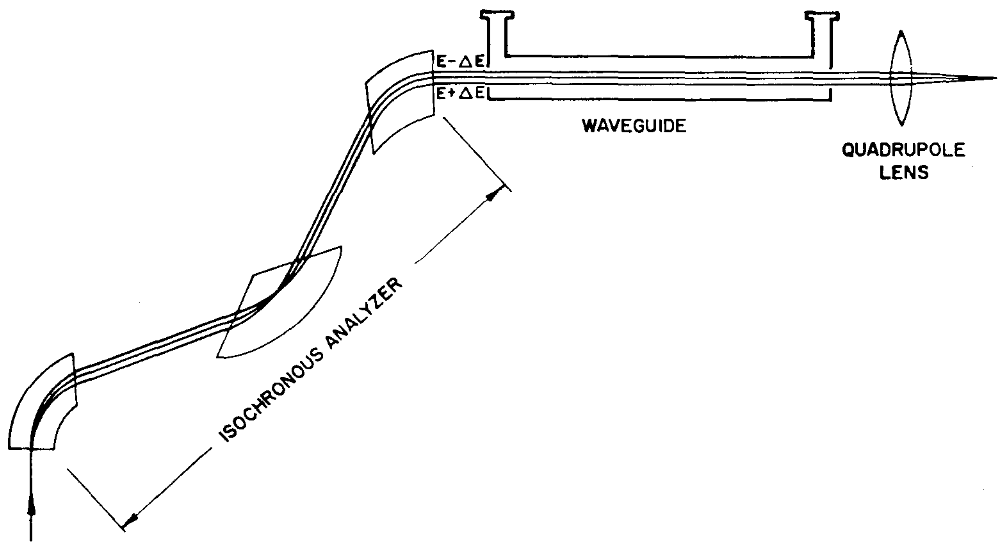

Fig. 1. Horizontal ray (energy) diagram for proposed system

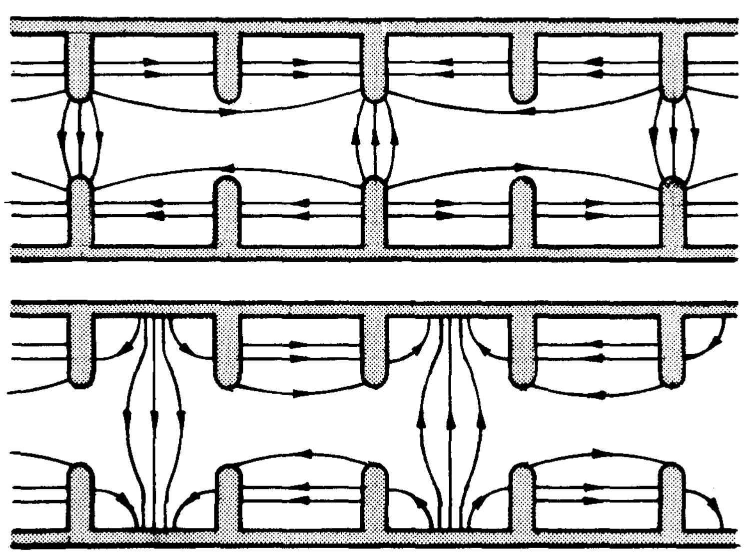

Fig. 2. Electric field configuration for the $\pi/2$-mode (HEM-11)