ACHROMATIC MAGNETIC LENS SYSTEMS FOR HIGH CURRENT ION BEAMS*

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Abstract

Linear transport and focusing of voltage-ramped, high-current, ion beams over distances of several meters for light ion fusion requires an achromatic focusing system. Five types of magnetic lenses are examined, and it is shown that a single lens can be achromatic only if it is a self-field lens with $I \propto \sqrt{V}$ (I = beam current, V = beam voltage). For two lenses with deflection angle $\theta \propto \beta^{-1}$, a theorem is proved that no achromatic system is possible. However, if one lens is a self-field lens, then an achromatic system is possible. One such system, composed of a self-field lens and a solenoid lens, is shown to be achromatic and suitable for the Laboratory Microfusion Facility.

Introduction

Linear transport and focusing of high current ion beams (e.g., 1 MA, 27-30 MeV Li^{+3}) over distances of several meters is required for light ion fusion. A scheme is needed to provide (1) a length for axial bunching of the voltage-ramped beam, (2) a standoff distance between the diode and the fusion target, (3) line-of-sight shadowing of the diode from the target to provide diode survivability, and (4) focusing from a beam radius of 10-20 cm down to a target radius $\lesssim$ 1 cm. Since the voltage pulse is ramped, the system must be achromatic. A novel set of achromatic magnetic lens systems for high current beams is examined in this work.

The magnetic lens types include (1) self-field magnetically insulated lens, (2) Z discharge lens, (3) B_Z solenoid lens, (4) quadrupole lens, and (5) wire-on-axis lens. The type (1) lens is unique to high current beams. It is shown that one lens cannot be achromatic and focusing unless it is of type (1) and the beam current $I \propto \sqrt{V}$. For two lenses with deflection angle $\theta \propto \beta^{-1}$, a theorem is proved that no achromatic system is possible. However, if one lens is a self-field lens, then an achromatic system is possible.

One system, suitable for the LMF (Laboratory Microfusion Facility),[1] consists of a diode (which acts as a lens of type (1)) plus a single B_Z solenoid lens. This system is achromatic to lowest order in α where $V/V_0 = 1 + \alpha$ and V_0 is the starting diode voltage. This system provides axial bunching, standoff, survivability, and focusing. It is concluded that achromatic magnetic lens systems for transport and focusing of high current ion beams do exist, and that these systems are unique to high current beams.

Magnetic Lenses

The magnetic lens types we have considered include (1) self-field magnetically insulated lens, (2) Z pinch lens, (3) B_Z solenoid lens, (4) quadrupole lens, and (5) wire-on-axis lens. Here we briefly discuss each of them.

Self-field magnetically insulated lens.

This is a special lens unique to high current beams, in that the self magnetic field of the net beam current is used to provide a radial focusing force over a short distance (Fig. 1). The ion beam is assumed to be both charge neutralized ($f_i = 1$) and current neutralized ($f_m = 1$) before and after the lens. In the lens region, the beam is assumed to be charge neutralized, but not fully current neutralized ($f_m \neq 1$), so that the net current $I(1 - f_m)$ will produce a focusing magnetic field. For a solid uniform beam, this field is

$$B_\theta = [2I(1 - f_m)/(cr_b)](r/r_b) \qquad 0 \le z \le \ell,\ 0 \le r \le r_b, \tag{1}$$

the bending angle in the lens is

$$\theta = [(Z_\ell B_0 \ell)/(AM_pc^2/e)]\beta^{-1}(r/r_b), \tag{2}$$

and the focal length (measured from the lens center) for a thin lens is

$$f = [(AM_pc^2/e)r_b/(Z_\ell B_0 \ell)]\beta. \tag{3}$$

Here r_b is the beam outer radius, ℓ is the length of the lens, Z_ℓ is the charge state of the ion in the lens, A is the ion mass number, M_p is the proton rest mass, $B_0 \equiv B_\theta(r = r_b, t = 0)$, e is the electron charge, βc is the ion velocity, and c is the velocity of light.

One possible method for constructing a self-field lens is to have a region of space containing a gas at low pressure p_ℓ bounded by thin foils at z = 0 and z = ℓ (Fig. 1b). Outside the lens, the gas pressure p_0 is chosen to provide good charge and current neutralization (p_0 ~ few Torr).[2] Inside the lens, the gas pressure is chosen to provide good charge neutralization but not good current neutralization (p_ℓ ~ 0.1 Torr). Although electrons will be readily drawn from the foils, it is suggested that this configuration should produce $f_m \neq 1$ in the lens region. Alternately, a transverse B field may be used in the lens to impede axial electron flow.

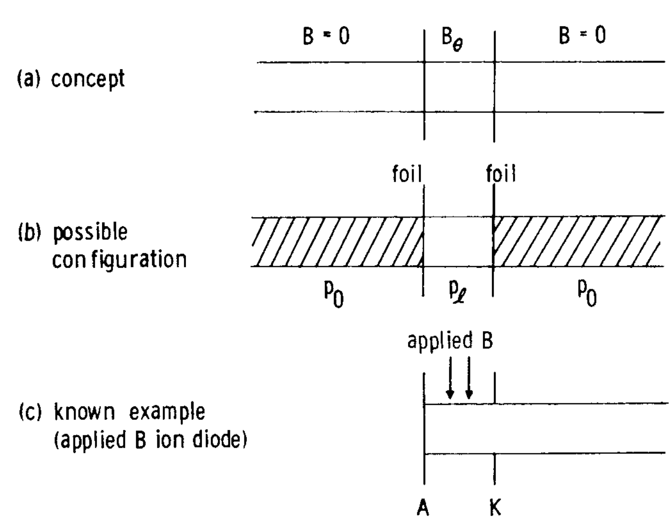

Fig. 1. Self-field magnetically insulated lens.

One known example of a self-field lens is an applied B ion diode (Fig. 1c). Such diodes operate with $f_i = 1$ and $f_m = 0$. The lens effect is commonly known as the "self-field beam sweep" in the diode, and is usually an unwanted effect in that it causes focus sweep for a geometrically focused diode. In the present context, the diode simply behaves as a self-field lens. We will use this effect to advantage in constructing an achromatic focusing system.

Z discharge lens. This lens is a short, current-carrying Z-discharge. Before the ion beam arrives, a discharge is struck in a gas between two axial boundaries. The discharge may be initiated by any of several methods (e.g., plasma injection, laser photoionization, wall discharge, etc.). The plasma parameters are selected so that a uniform current discharge results (μs timescale), and the fields are frozen on the time scale of the ion beam (tens of ns). Good charge and current neutralization is assumed as the ion beam flows through the lens, so the ions see the Z discharge B_θ. For a discharge current I_c inside a radius r_b, this lens has

$$B_\theta = [2I_c/(cr_b)](r/r_b) \qquad 0 \le z \le \ell\,,\ 0 \le r \le r_b\,, \tag{4}$$

with a bending angle in the lens given by (2), and a thin lens focal length given by (3).

B_Z solenoid lens. This is a short, gas-filled, solenoid lens. The gas fill is used to provide charge and current neutralization for the ion beam. For an ideal solenoid, B_Z = CONST and $B_r = 0$ in the region $0 \le z \le \ell$, $0 \le r \le r_b$. Outside this region $B_r \ne 0$. As the incoming ions cross B_r, they attain a velocity component V_θ (Busch's Theorem), which, inside the lens, produces a $V_\theta \times B_Z$ radial focusing force. As the ion exits the lens, the ions cross an oppositely-directed B_r and they lose their V_θ component, but keep their radial focusing V_r component. This lens produces a bend angle

$$\theta = \{(Z_\ell^2 B_Z^2 r_b \ell)/[4A^2(M_p c^2/e)^2]\}\beta^{-2}(r/r_b)\,, \tag{5}$$

and a thin lens focal length of

$$f = \{[4A^2(M_p c^2/e)^2 r_b]/(Z_\ell^2 B_Z^2 \ell r_b)\}\beta^2\,. \tag{6}$$

Quadrupole lens. This is a gas-filled quadrupole triplet lens. It can be shown that a quadrupole triplet lens is the simplest strong-focusing, alternating-gradient magnetic lens that is stigmatic and has equal magnification in both transverse planes,[3] i.e., a quadrupole triplet "thin lens" behaves as the B_θ of a Z-discharge lens or a self-field lens. The gas fill is again used to provide charge and current neutralization for the ion beam. For the present discussion, the key features of a quadrupole triplet are the scalings

$$\left.\begin{aligned} &\theta \propto \beta^{-1} \\ &f \propto \beta \end{aligned}\right\} \tag{7}$$

Wire-on-axis lens. This lens consists of a short, current-carrying, wire-on-axis with appropriate radial end feeds (foils or spider-wire arrangements). The magnetic field inside the lens region is

$$B_\theta = [2I_W/(cr_b)](r_b/r) \qquad 0 \le z \le \ell\,,\ r_W \le r \le r_b\,, \tag{8}$$

and the bending angle in the lens is

$$\theta = \{(Z_\ell B_o \ell)/[A(M_p c^2/e)]\}\beta^{-1}(r_b/r)\,. \tag{9}$$

This lens is not linear in r. However, it can be used with an annular beam aimed toward the axis, and with a negative current ($I_W < 0$), to open the beam and bring it to an approximate focus downstream.

Search for an Achromatic Magnetic Lens System

We wish to find a lens system that will focus the intense ion beam onto the target, and be achromatic so that as the ion energy changes, the beam will remain on target. We consider first a single lens, and then two-lens systems.

For a single lens, we find that the ion trajectory bend angle scales as follows:

(1)	self field	$\theta \sim I/\beta$
(2)	Z discharge	$\theta \sim \beta^{-1}$
(3)	B_Z solenoid	$\theta \sim \beta^{-2}$
(4)	quadrupole triplet	$\theta \sim \beta^{-1}$
(5)	wire-on-axis	$\theta \sim \beta^{-1}$

The only possibility of an achromatic lens is a type (1) lens with $I \propto \sqrt{V} \sim \beta$. For this unique case, θ has no dependence on β and a true achromatic lens results. The usual impedance scaling for a single-stage intense ion diode has $I \sim V^k$ with $k \approx 1.5$-2.2. However, with a two-stage diode, it should be possible to program the impedance so $I \propto \sqrt{V}$; then a single self-field lens would be achromatic.

For a two-lens system, for visible light optics, it is well known that two lenses with different refraction characteristics (e.g., crown glass and flint glass) can be placed in contact to make an achromatic focusing lens. If there is a separation between the lenses, the system is called a dialyte and it can still be achromatic. If the lenses are separated, and both are of the same glass, the system is known as a Schupmann dialyte. Unfortunately, for a Schupmann dialyte positive lens system, the image is located between the lenses.[4] Apparently there is no known achromatic system of two separated lenses of the same glass that can focus a parallel light beam to a single point beyond the last lens element.

Similar problems arise in trying to construct an achromatic-magnetic lens system that is focusing and has the image beyond the last magnet element. In fact, for magnetic lenses which have $\theta \propto \beta^{-1}$ (Z discharge, quadrupole, wire-on-axis), we have developed a theorem that shows that there is no achromatic two-lens system possible that produces an image beyond the last lens element. The proof is rather lengthy, so we will only sketch the key steps here.

We consider two lenses with focal lengths f_1 and f_2 separated by a distance d, with an object to lens (1) distance s (>0) and a lens (2) to image distance ℓ (Fig. 2). The lens equations are

$$\left.\begin{aligned} &s^{-1} + x^{-1} = f_1^{-1} \\ &-(x-d)^{-1} + \ell^{-1} = f_2^{-1} \end{aligned}\right\}\,, \tag{10}$$

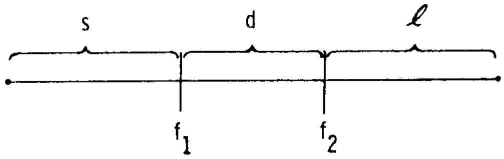

Fig. 2. Geometry of two-lens system.

where $f_1 = K_1\beta$ and $f_2 = K_2\beta$. Solving (10) for ℓ, we find

$$\ell = \frac{f_2[d(s - f_1) - sf_1]}{d(s - f_1) - sf_1 - f_2(s - f_1)} \quad . \tag{11}$$

We then calculate $\partial\ell/\partial\beta$, set the result equal to zero, and solve for d. The resultant expression for d is then used in (11) to give ℓ. At this stage, the results for d and ℓ are

$$d = \frac{f_1 \pm \sqrt{-f_1 f_2}}{1 - (f_1/s)} \quad , \tag{12}$$

$$\ell = \frac{\mp f_2\sqrt{-f_1 f_2}}{\mp\sqrt{-f_1 f_2} + f_2[1 - (f_1/s)]} \quad , \tag{13}$$

where the upper signs must be used together, or the lower signs must be used together. These equations describe the desired achromat (if it exists), if we can have $d > 0$ and $\ell > 0$. From (12)-(13) we see we need $f_1 f_2 < 0$. Four possible cases result:

$f_1 > 0,\ f_2 < 0$	upper signs in d, ℓ
$f_1 > 0,\ f_2 < 0$	lower signs in d, ℓ
$f_1 < 0,\ f_2 > 0$	upper signs in d, ℓ
$f_1 < 0,\ f_2 > 0$	lower signs in d, ℓ

For each case we examine d and ℓ and find explicitly that there is no solution. For example, for the first case, $d > 0$ and $\ell > 0$ require

$$\left.\begin{array}{l} f_1/s < 1 \\ f_1/s > 1 + \sqrt{f_1/|f_2|} \end{array}\right\} \quad , \tag{14}$$

which is impossible. Lack of a solution in all four cases demonstrates that there is no two-lens system that is achromatic to lowest order ($\partial\ell/\partial\beta = 0$), for $f \propto \beta$, and that focuses to an image beyond the last magnet element.

Fortunately, the desired achromatic system is possible if one of the lenses is a self-field lens. We have considered combinations of a self-field lens with each of the other four types of lenses. In the next section we will discuss one such system that is achromatic and is suitable for LMF.

Achromatic Magnetic Lens System

This system consists of two lenses -- (1) an ion diode which acts as a self-field lens, and (2) a B_z solenoid lens (Fig. 3). In the diode $f_i = 1$, $f_m = 0$ and the self B_θ of the beam bends the ion trajectories. The region from the diode to the target is filled with a gas to provide charge and current neutralization. The beam drifts ballistically from the diode to the solenoid lens with all trajectories about parallel to the Z axis. In the B_Z solenoid, the ions are focused toward the target. The ions then drift ballistically from the solenoid to the target.

We consider a diode with

$$V/V_0 = 1 + \alpha \qquad (\alpha << 1)\ , \tag{15}$$

$$I/I_0 = (V/V_0)^k \qquad (k > 0.5)\ , \tag{16}$$

where V_0 and I_0 are the diode voltage and current at the start of the usable pulse, α is a function of time, $|\alpha| \leq 0.1$ for LMF, and typically $k \approx 2.2$. The outer ion trajectory is shown in Fig. 3. The deflection angle in the diode for a uniform solid beam is given by

$$\theta_d = [(Id)/(cr_b)][(2Z_d)/(VAM_pc^2/e)]^{1/2}\ (r/r_b)\ , \tag{17}$$

where d is the diode gap width (or the distance from the anode to the gas bag where current neutralization starts, if the gas bag boundary is beyond the anode) and Z_d is the ion charge state. The diode aiming angle is adjusted so that when the diode voltage reaches V_0 the beam is exactly parallel to the Z axis. Then as V increases above V_0, the beam will deflect toward the axis. The relative deflection angle $\Delta\theta_d = \theta_d(V) - \theta_d(V_0)$ is

$$\Delta\theta_d = K_d\alpha(r/r_b)\ , \tag{18}$$

$$K_d = [(I_0d)/(cr_b)][(2Z_d)/(V_0AM_pc^2/e)]^{1/2}(k - \tfrac{1}{2})\ . \tag{19}$$

The solenoid deflection angle is

$$\theta_\ell = K_\ell\ (1 - \alpha)(r/r_b)\ , \tag{20}$$

$$K_\ell = (Z_\ell^2 B_Z^2 r_b\, \ell)/(8AZ_dV_0M_pc^2/e)\ . \tag{21}$$

To make this two-lens system achromatic, we require $K_d = K_\ell$. To achieve this for typical parameters requires that the gas bag boundary be slightly beyond the cathode.

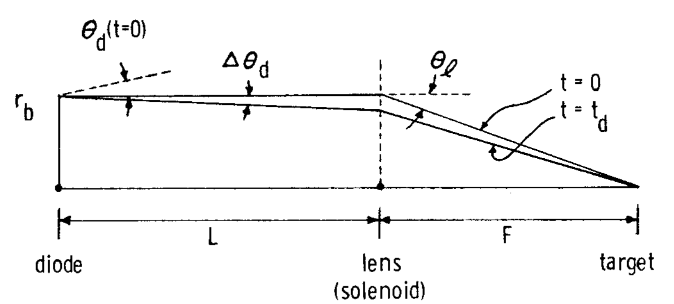

Fig. 3. Achromatic magnetic lens system consisting of a self-field lens (ion diode) plus a B_Z solenoid lens.

To demonstrate that this system is achromatic to first order in α, we note from Fig. 3 that at the start of the ion pulse ($\alpha = 0$), for the outer trajectory,

$$K_\ell F = r_b . \qquad (22)$$

Later in the pulse, $\alpha \neq 0$, and the radial miss distance at the target Δr will be

$$\Delta r = r_b - K_d \alpha L - \{K_d \alpha + K_\ell (1 - \alpha)[(r_b - K_d \alpha L)/r_b]\} F . \qquad (23)$$

Since $K_d = K_\ell$, this is

$$\Delta r = -\alpha^2 (L/F) r_b . \qquad (24)$$

This demonstrates that the system is achromatic to first order in α for the outer trajectory, and that the radial miss distance is second order in α. Note that the system is achromatic even if L is changed.

For a uniform beam, the system is achromatic at all r. For an annular beam, the factor (r/r_b) in (18) must be replaced by the factor $r_b(r^2 - r_{in}^2)/[r(r_b^2 - r_{in}^2)]$, where r is the inner radius. For the annular beam, the system is achromatic only at $r = r_b$.

It is important to note that the spot size at the target is given by

$$r_s = \theta_\mu F , \qquad (25)$$

and not by $\theta_\mu(F + L)$, where θ_μ is the ion beam microdivergence at the diode. This occurs, because as the beam translates from the diode to the lens, θ_μ remains the same. The standoff distance from the solenoid lens to the target therefore varies from F = 150 cm to F = 200 cm as θ_μ varies from 6.7mrad to 5 mrad.

If instead of a parallel beam from the diode, the beam is geometrically focused, then the value of θ_μ at the lens will be higher than in the diode, and the standoff length would be reduced. However, if the beam is geometrically defocused, then the value of θ_μ at the lens will be lower than in the diode, and the standoff length could be increased. Alternatively, the standoff length could be held fixed, and θ_μ could increase well beyond r_p/F if the diverging, defocused case is used. Schematic examples of these cases are shown in Fig. 4. Note that in all cases, the diode is shadowed from direct line of sight from the target.

For LMF, typical parameter values are 30 MeV Li ions, $I_o = 1$ MA, $r_b = 10$ cm, $r_t = 1.0$ cm, L = 2.5 m, F = 1.5 m, $B_z = 20$ kG, $\ell = 30$ cm, and $\theta_\mu = 6$ mrad. (Foil thicknesses and gas pressures must be chosen so that scattering is minimal and the total microdivergence is dominated by the diode microdivergence.) Recent particle simulations of this achromatic solenoid system verify the basic theoretical results presented here.[5]

Conclusions

We have examined five different magnetic lenses, and shown that only one (self-field lens for $I \propto \sqrt{V}$) can be achromatic. For a two-lens system with $\theta \propto \beta^{-1}$, we have proven a theorem that shows that there is no positive achromatic system with an image past the last element. A system that is achromatic was proposed, consisting of a self-field lens (ion diode) and a solenoidal lens. This system was shown to be achromatic to first order in α, have a solenoid to target standoff length set by $F = r_p/\theta_\mu$, and for an initially defocused beam have an even greater standoff length.

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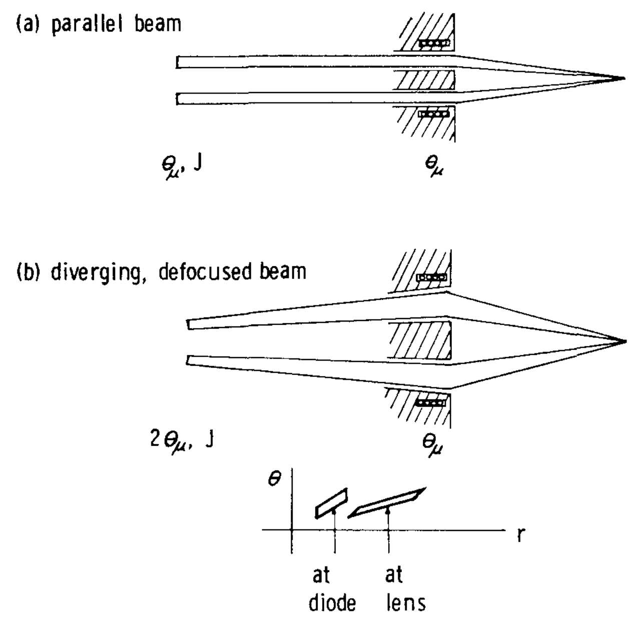

Fig. 4. Containment vessel geometries.

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