A FOUR-ROD CAVITY RFQ

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Abstract

In the Frankfurt Four-Rod RFQ structure the four modulated rods provide the capacitance and the supporting posts are the inductors, together they make a resonant LC circuit. A variation to this geometry is introduced in which the "U" shaped inductors are replaced with cylindrical cavities. This four-rod cavity structure can be designed for a wider range of frequencies. It can operate at higher frequencies than the regular four-rod structure. Also calculations show that at lower frequencies it is more compact and has a higher Q value. In this paper theoretical calculations for the four-rod cavity RFQ are carried out to obtain simple equations for frequency, quality factor, and shunt impedance of the cavity in terms of its geometric characteristics. The results of these equations are compared with the calculations using the MAFIA code package.[1] Also a cold model was constructed to test the theoretical predictions.

Introduction

The Radio Frequency Quadrupole is being established as the standard device for acceleration of both light and heavy ions at low energies. Among different RFQ structures, the Frankfurt four-rod design[2] has the advantages of being easy to construct and maintain and having a good field quality. By replacing the "U" shaped support with a cylindrical cavity we have been able to achieve higher frequencies for a resonable cavity size. As shown in figure 1, a basic unit cell consists of a cylinder with two end caps which are connected to four rods, one end cap is connected to two opposing rods and the other end cap to the other two rods.

This structure retains many virtues of the regular four-rod structure. For example, the unwanted dipole mode is not a problem since the opposing rods are connected to each other at one end of each cell. Also, since the fields are confined within the structure there is not much current through the joints between the structure and the vacuum vessel. Therefore, it can be constructed and tuned outside and inserted into the vacuum vessel. Although we are considering the four rod cavity structure for a high frequency RFQ, lower frequency designs can be made which are compact in size and have good power requirements. The disadvantages of the four-rod cavity design is that the fields are lower as one goes farther from the center; therefore tuning from the side could be a problem. The Frankfurt design does not have this problem.

Theoretical Analysis of The Four-Rod Cavity RFQ

Consider the structure shown in figure 1. If the cell length ℓ is large enough, the capacitance between the two end caps is negligible with respect to the capacitance between the rods. Then one can regard the structure as an LC circuit where the inductance is mainly from the cylindrical cavities, and the capacitance is mainly from the rods. Because of the symmetry,

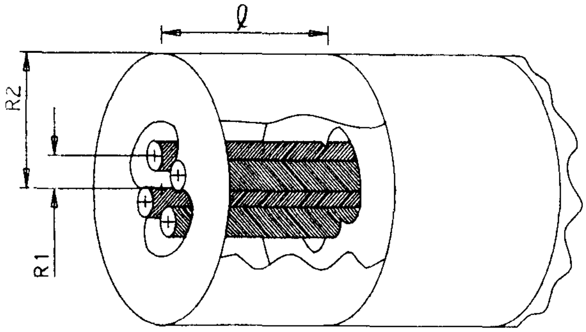

Fig. 1 Four-Rod Cavity RFQ.

the surface current on the rods is zero at the points between the cells. The surface current goes from one set of rods to one end of the cavity and through the cavity to the other two rods. The sum of the currents on the four rods is constant along the cell length and is equal to the current on the inner walls of the cavity going in the opposite direction. This makes a transverse circular $\vec{B}$ field around the four rods.

$$\vec{B} = \frac{\mu_o i}{2\pi r}\hat{\phi} \tag{1}$$

Therefore the inductance is:

$$L_{cell} = \frac{\int \vec{B}\cdot d\vec{a}}{i} = \frac{\mu_o(\ell - T)}{2\pi}\ln\frac{R_2}{R_1} \tag{2}$$

where T is the thickness of the end plates, and R_2 is the radius of the cavity. Obviously R_1 is not well defined because one has to consider the fact that the surface currents on the different rods are not equal except at the middle of each cell and factor in the contribution of this nonuniformity to the cavity inductance. However, taking $R_1 = 0.75(\rho + r_o)$ for circular rods in the calculations gives good approximations with respect to the MAFIA and cold model results (ρ = rod radius and r_o = bore radius. If the full structure consists of n cells then the total inductance is equal to n inductances put in parallel.

$$L_{total} = \frac{1}{n}L_{cell} = \frac{\mu_o(L - nT)}{2\pi n^2}\ln\frac{R_2}{R_1} \tag{3}$$

where L is the total length of the RFQ ($= nl$).

Now the total capacitance of the rods is

$$C_{total} = C_T L \tag{4}$$

where C_T is capacitance per unit length. (Figure 2 shows a graph of C_T vs. ρ/r_o for circular rods.)

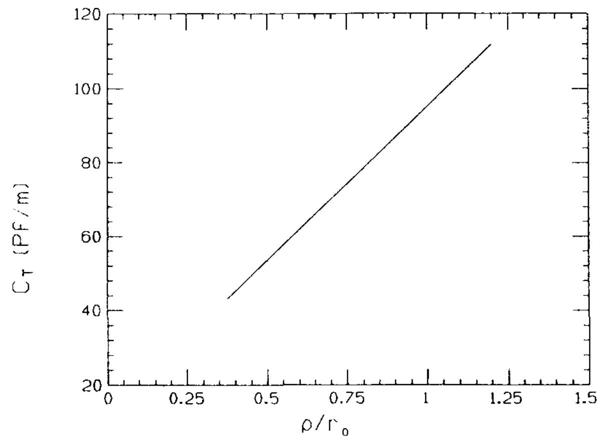

Fig. 2 Plot of Capacitance per unit length vs. ρ/r_o.

At resonance $\omega^{-2} = L_{total}C_{total}$. Therefore,

$$f = \frac{n}{[2\pi\mu_o L(L-nT)C_T \ln\frac{R_2}{R_1}]^{1/2}} \tag{5}$$

or

$$\frac{R_2}{R_1} = \exp\left(\frac{n^2}{2\pi\mu_o L(L-nT)C_T f^2}\right) \tag{6}$$

To find the quality factor Q, we have to first find the resistance for the surface current. We can write the resistance due to the cavity as

$$\begin{aligned} R_{cavity} &= R_s\left(2\int\frac{dr}{2\pi r} + \frac{\ell}{2\pi R_2}\right) \\ &= \frac{R_s}{2\pi}\left(2\ln\frac{R_2}{R_1} + \frac{\ell}{R_2}\right) \end{aligned} \tag{7}$$

where $R_s = (\frac{\mu_o\omega}{2\sigma})^{1/2}$ and σ is the conductivity. For copper $\sigma = 5.75\times10^7(\Omega m)^{-1}$. Resistance due to the four rods is:

$$R_{rods} = R_s\frac{\ell}{S}(\frac{1}{3}) \tag{8}$$

where S is the surface area of one rod per unit length. (The factor $\frac{1}{3}$ is due to the fact that the current on each rod decreases linearly from maximum at one end to zero at the other end of the cell.) Total resistance of each cell is:

$$R_{total} = R_s\left[\frac{1}{2\pi}\left(2\ln\frac{R_2}{R_1} + \frac{\ell}{R_2}\right) + \frac{1}{3}\frac{\ell}{S}\right] \tag{9}$$

The quality factor can now be calculated using

$$Q = \frac{1}{R_{total}\omega C_T\ell} \tag{10}$$

Therefore we can write:

$$Q = (\frac{2\sigma}{\mu_o})^{\frac{1}{2}}\frac{1}{\left[\frac{1}{2\pi}\left(2\ln\frac{R_2}{R_1} + \frac{\ell}{R_2}\right) + \frac{1}{3}\frac{\ell}{S}\right]\omega^{3/2}C_T\ell} \tag{11}$$

The shunt impedance for unit length is:

$$Z_T = \frac{2Q}{\omega C_T} \tag{12}$$

Cold Model and MAFIA Results

To test the validity of our theoretical equations, a four-rod cavity cold model was constructed and its frequency and quality factor was measured. The cavity was modeled and calculated using MAFIA code. The results are:

For

ρ=4.76 mm,	r_o=4.05 mm,	C_T=108. pf/m,
S = 30. mm,	L=155. mm,	T=1.0 mm,
n=3,	R_2=100. mm,	R_1=0.75($\rho + r_o$),
MAFIA gives	f=406.7 MHz,	and Q=8100
Theoretical Equations give		
	f=406 MHz,	and Q=8700.

The cold model had measured f=407.5 MHz and Q=4000. However, we should be able to make better rf joints which would give us a higher Q value.

Several other cavities at different frequencies were modeled by MAFIA the results are as follows:

Table 1

	Model 1	Model 2	Model 3
ρ (mm)	4.76	6.35	6.35
r_o (mm)	4.05	9.0	9.0
C_T (pf/m)	108.	69.1	69.1
S (mm)	30.	40.	40.
L (mm)	50.	130.	240.
T (mm)	2.5	5.	10.
n	1	1	1
R_2 (mm)	100.	180.	180.
MAFIA:			
f (MHz)	423.	198.	111.
Q	8300	12000	11000
Theoretical Equations:			
f (MHz)	426.	202.5	110.
Q	8500	11300	10000

A Comparison of The Four-Rod Cavity structure With The Frankfurt Four-Rod design

For Comparison, consider the 104 MHz cold model tested at Chalk River, Canada[3]. The basic geometry is shown in figure 3. ρ = 6.35 mm, and r_o=8.64 mm, gives $C_T \approx$ 74 pf/m, and H=220. mm, h=63.5 mm, W=102. mm and 2 × ℓ=940 mm, and the calculated properties were f=104. MHz, Q=7970, and Z_T=0.33 $M\Omega\cdot m$. (Note that in this calculation the power loss on the rods were ignored.)

Now using the same rod dimensions, and designing a four-rod cavity RFQ with n=4, R_1 = 0.75($\rho + r_o$)=11.24 mm, R_2=19.73, from equations 5 and 11, we get f=104. MHz, Q=10760, and Z_T=0.45 $M\Omega\cdot m$. Therefore in this example the theoretical Q is higher for the four-rod cavity and therefore the power requirements are lower.

Conclusion

The cold model results show that we can predict the frequency and Q to with a good accuracy. The equations 5 and 11 can tell us how different characteristics of the structure relate to each other, however, the MAFIA calculation with fine mesh size should produce more accurate results. MAFIA generated field plots confirm our theory. A cut in the xy plane at the center of the cavity (Fig. 4.b) shows that the $\vec{B}$ field is transverse and points in ϕ direction and decreases as $1/r$. The same cut for the $\vec{E}$ shows that the $\vec{E}$ field is mainly between the rods (Fig. 4.c). In the x=0. cut view (Fig. 4.d) the $\vec{B}$ field on the back of the rods is decreasing from maximum at the end of the cavity which is attached to the rod to almost zero at the other end confirming our prediction that the current decreases from max to min from one end to the other in the cavity.

The fact that MAFIA gives such a good answer for the frequency of the cold model without having perfect cylindrical geometry, suggests that the structure forgives geometric imperfections on the outer cylinder. This eases the demand on tolerances when manufacturing the cavity. But at the same time can be a problem for tuning the cavity, since the tuning is usually achieved by introducing geometric imperfections to the cavity.

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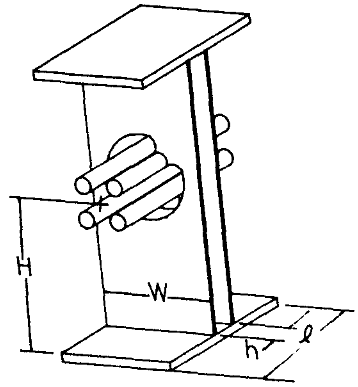
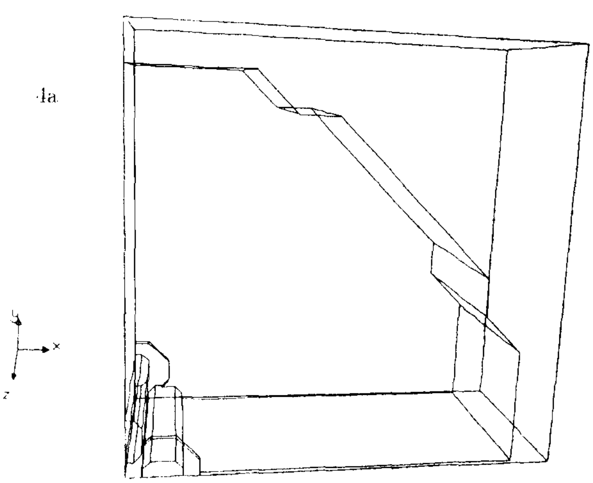
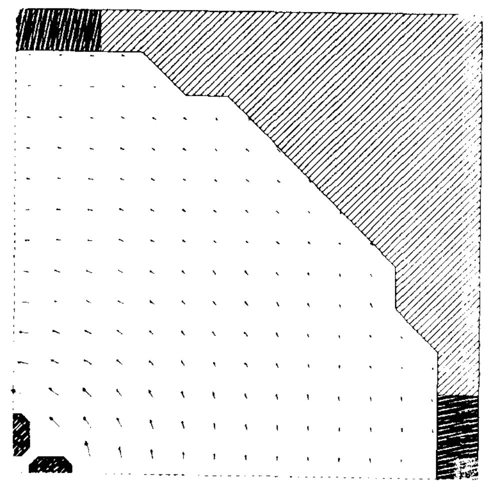


Fig. 3 Basic module of a twin-inductor 4-rod RFQ.

4a



4b



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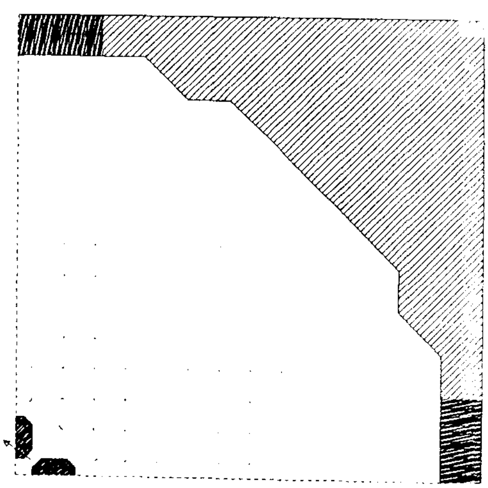

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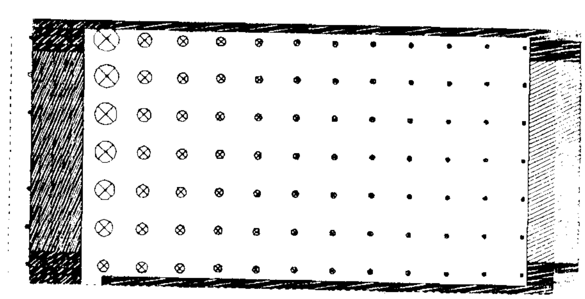

Fig. 4 Graphical results of MAFIA for 406.7 MHz Four-Rod Cavity RFQ Cold Model.

a. 3-D Geometry of one quadrant of a unit cell.
b. $\vec{B}$ plot at z= half cell length.
c. $\vec{E}$ plot at z= half cell length.
d. $\vec{B}$ plot at x=0.