INPUT ADMITTANCE MATRIX OF A MULTIPORT DRIVEN DTL

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Abstract

In some applications, tens of megawatts of rf power or higher power is needed to drive the cavity linac. Such high power level necessitates that the linac be fed by a multiport system because of the limitation of the individual rf source device and of the power handling capabilities of the feed structure. The multiport feed system driving a high Q reactive load has made the turn on/tune up an exceedingly difficult task. All ports are strongly coupled. The conventional concept of matching of a single port fed load is no longer applicable. The admittance matrix and the equivalent circuit developed for a waveguide/iris system permit addressing various issues quantitatively. The parameters of the equivalent circuit are obtained from the physical dimensions of the linac and the feed structure. Thus, they can also be used for designing the feed circuit as well as the control system. For a coax/loop system, the input impedance matrix will also be discussed. Although the work addresses in detail only the Alvarez type geometry, basic concepts are applicable to other types of resonant accelerators.

Introduction

Various issues in the rf drive of DT linacs have been studied over the past 30 years.[1] Through a history of theoretical, experimental and empirical investigations a great deal of the understanding of the issues has been achieved, and important devices constructed. However, for devices with significantly high power requirements, because of high beam current and/or energy gradient, multiple drive ports are needed in the design. There is an upper limit to the output power of an individual rf source device, and also to the power handling capability of the feed circuit. The ports are all strongly coupled. Their interactions can cause great difficulties for fast turning-on/tuning-up operations. Many of the issues can be understood, and to some extent dealt with, using an equivalent circuit. Based on earlier work of Potter, Walling[2-4] recently showed the form of the equivalent circuit of a multiple loop driven linac, and the procedure of measuring the coupling factors (β's). She also gave a procedure for tuning the system.

In the present paper we investigate a waveguide-slot driven system. The input admittance matrix and the complete equivalent circuit are obtained analytically. Thus the results can provide information to a first cut design of the rf drive system and deal with various issues on a quantitative basis prior to the construction. Analogy to the dual coax-loop drive system is made.

The Input Admittance Matrix

The input admittance matrix to a high-Q resonant cavity is well known.[5] Figure 1 shows schematically a linac cavity with several feeding waveguide, a, b....c, coupled to the cavity through slots $s_a s_b ... s_c$. The admittance matrix seen at the irises is given by its (a,b)th element:

$$Y_{ab} = \sum_i \frac{s}{s^2 + \frac{\omega_i}{Q_i}s + \omega_i^2} C_{ai}C_{bi} \quad , \tag{1}$$

where s is the Laplace transform variable; ω_i, Q_i are respectively the resonant frequency and quality factor of the i th resonant mode when all the slots are closed, including beam loading and detuning effects if present. C_{ai}'s are coupling factors inversely proportional to the square root of the β's, and given by

$$C_{ai} = \iint_{sa} \hat{n}_a \times \hat{\underline{e}}_a \cdot \underline{H}_i \, ds \quad , \tag{2a}$$

where $\hat{n}_a$ is the unit normal vector inward to the cavity; $\hat{\underline{e}}$ is the modal vector function of the slot and normalized as

$$\iint_{sa} \hat{\underline{e}}_a \cdot \hat{\underline{e}}_a \, ds = 1 \tag{2b}$$

and $\underline{H}_i$ is the magnetic field distribution of the i th mode and normalized as

$$\iiint_{cavity} \mu_o \underline{H}_i \, \underline{H}_i dv = 1. \tag{2c}$$

Thus C_a has the physical dimension of henries to the -1/2 power as expected.

In arriving at (1), one assumes that the actual electric field distribution in the slot is given by $V_a \hat{\underline{e}}_a$, with V_a an arbitrary amplitude factor. If the slot is a small aperture, this is an accurate assumption. For a rectangular slot (Fig. 1.)

$$\hat{\underline{e}}_a = \hat{y}' \sqrt{\frac{2}{a'b'}} \sin \frac{\pi x'}{a'}. \tag{3}$$

Neither taken into consideration in Eq. (1) are the effects of the fringe fields on both sides of the slots. The couplings between the ports are assumed to be through the resonant modal fields $\underline{H}_i$ only. The fringe fields represent extra susceptences connected to the input ports. Thus together with the rest of the feed circuits they will detune the cavity. However, of fundamental importance is that they provide an essential element for matching the cavity to the feed system.

Equivalent Circuit Development

Equation (1) represents the input admittance matrix seen at the irises. When feeding waveguides are connected, the admittances will be transformed. In this section we proceed to construct the complete equivalent circuit beginning from Equation (1). Referring to Fig. 2, the part of the circuit to the right of A represents the Y_{ab} matrix exactly. The fringe fields are shown by the shunt admittances (Y_{sa} and Y_{sa}' etc.). The transmission lines to the left of B represent the dominant mode of the feed waveguides. It is assumed that all higher order modes are cut off. The transformer ratios n_a:1 are the result of projecting the dominant waveguide mode to the slot and will be dealt with later. Each of the R-L-C series circuits at the right of the figure represents one resonant mode. In case the resonant frequencies are separated far apart, one mode could be sufficient for most cases of interest.

In the following we present an analysis leading to the equivalent circuit. For simplicity of notation and algebra, we shall begin with the case that there is only one feed port. We then indicate the straightforward procedure of extending to the case of more than one port, leading to the complete equivalent circuit.

Referring to Fig. 1, we focus on one waveguide through the slots. We express the transverse electric field in terms of a set of ortho-normal modes[6] $\underline{e}_o$ $\underline{e}_1$, $\underline{e}_2 \cdots$, where $\underline{e}_o$ mode is the dominant and the only propagating mode:

$$\underline{E}_t = (V_o e^{-ik_oz} + \Gamma V_o e^{ik_oz})\, \underline{e}_o + \sum_{n=1}^{\infty} V_n e^{\alpha_n z} \underline{e}_n \quad , \tag{4}$$

where V_o is the amplitude of the incident wave from $z = -\infty$. Γ is the reflection coefficient with z = 0 taken at the plane of the slot. The corresponding transverse magnetic field $\underline{H}_t$ is given by

$$-\hat{z} \times \underline{H}_t = Y_o V_o (e^{-jk_oz} - \Gamma V_o e^{jk_oz}) \ldots \sum_{n=1}^{\infty} Y_n V_n e^{\alpha_n z} \underline{e}_n \tag{5}$$

Where Y_o is the characteristic admittance of the dominant mode, while Y_n are those of the cutoff modes and purely imaginary. At z = 0, $\underline{E}_t$ is equal to the field in the slot and zero on the wall. The magnetic field $\underline{H}_t$ is also equal to that in the iris but unknown on the wall. Thus

$$(1+\Gamma)V_o\underline{e}_o + \sum_n^{\infty} V_n\underline{e}_n = \begin{cases} V\underline{\hat{e}} & \text{on s ,} \\ 0 & \text{on wall ,} \end{cases} \qquad (6)$$

$$Y_o(1-\Gamma)V_o\underline{e}_o - \sum_{n=1}^{\infty} Y_nV_n\underline{e}_n = Y V\underline{\hat{e}} \quad \text{on s ,} \qquad (7)$$

where $V\underline{\hat{e}}$ is the $\underline{E}$-field in the slot, and Y is the input admittance at the slot. They are defined by eqs. (1), (2a)-(2c) with subscripts a,b dropped. We define two inner products of two vectors $\underline{A}$ and $\underline{B}$

$$<\underline{A},\underline{B}>_s \equiv \iint_s \underline{A}\cdot\underline{B}\, ds \qquad (8)$$

and

$$<\underline{A},\underline{B}>_w \equiv \iint_{\substack{WG \\ \text{crossection}}} \underline{A}\cdot\underline{B}\, ds \qquad (9)$$

Taking the inner product eq. (9) of both sides of eq. (6) with $\underline{e}_o$ and $\underline{e}_n$ we have

$$(1+\Gamma)V_o = <\underline{\hat{e}},\underline{e}_o>_s \; V \qquad (10)$$

and

$$V_n = <\underline{\hat{e}},\underline{e}_n>_s \; V \qquad (11)$$

Taking the inner product eq. (8) of both sides of eq. (7) with $\underline{e}_o$ and rearranging, we have

$$Y_o(1-\Gamma)<\underline{e}_o,\underline{e}_o>_sV_o = \sum_{n=1} Y_n<\underline{e}_o,\underline{e}_n>_sV_n + Y<\underline{e}_o,\underline{\hat{e}}>_s V \qquad (12)$$

Substituting (10) and (11) into (12) and rearranging, we have:

$$Y_L = Y_o\frac{1-\Gamma}{1+\Gamma} = \sum_{n=1}^{\infty} Y_n \frac{<\underline{e}_o,\underline{e}_n>_s <\underline{\hat{e}},\underline{e}_n>_s}{<\underline{e}_o,\underline{\hat{e}}>_s <\underline{e}_o,\underline{e}_o>_s} + \frac{Y}{<\underline{e}_o,\underline{e}_o>_s} \qquad (13)$$

where Y_L comes from $\Gamma = \frac{Y_o - Y_L}{Y_o + Y_L}$, reflection coefficient in a transmission line terminated by Y_L. Thus, with a transformer ratio $<\underline{e}_o,\underline{e}_o>_s$ factored out, (13) shows that the load is the parallel of Y with a shunt susceptence (Y_n imaginary) given by the cutoff modes, the result of fringe fields on the waveguide side. As mentioned earlier, (1) does not contain the effect of fringe fields on the cavity side. If the iris is small, the fringe fields on its two sides should be almost symmetric. Thus, the admittance Y consists of a susceptence approximately equal to the first term of (13) without $<\underline{e}_o,\underline{e}_o>_s$ in addition to the expression of (1), representing the cavity side fringe fields.

To extend the above derivation to more than one port, we first note that: (a) in (4) and (5), the "reflected" or "left traveling" wave term should include couplings from other ports; and (b) the magnetic field in the slot should also contain the effect of other slot voltages. It is not difficult to see that one only needs to repeat the above by properly taking the multiport nature into consideration, i.e.: (a) $\underline{E}_t$, $\underline{H}_t$, v_o, v_n, $\underline{e}_o$, $\underline{e}_n$, and V are column vectors; (b) Γ is the square matrix, commonly known as the scattering matrix; (c) Y is the square matrix of eq. (1) plus a diagonal matrix Y_s' for the cavity side fringe field susceptences; (d) all inner products are diagonal matrices; (e) eq. (13), hence Y_L is a square matrix; and (f) if the feed waveguides are not the same for different ports the exponential factors $e^{\pm jk_oz}$ and $e^{\alpha_n z}$ in (4) and (5) are diagonal matrices premultiplying the column vectors V_o and V_n. Y_o and Y_n are also diagonal matrices. The denominators of (13) should be properly replaced with the corresponding matrix inverses. The equivalent circuit of Fig. 2 is then complete where

$$Y_{sa} = \sum_n Y_{na} \frac{<\underline{e}_{oa},\underline{e}_{na}>_{sa} <\underline{\hat{e}}_a,\underline{e}_{na}>_{sa}}{<\underline{e}_{oa},\underline{\hat{e}}_a>_{sa}} \qquad (14)$$

$$\cong Y_{sa}'$$

and the transformer ratios

$$n_a = \sqrt{<\underline{e}_{oa},\underline{e}_{oa}>_{sa}} \qquad (15)$$

The subscript is now restored to indicate the quantity belongs to port a.

Discussion

The circuit of Fig. 2 describes the various interactions between the linac load and the rf source system. At present we shall limit ourselves only in the issue of matching. For simplicity we shall assume that rectangular waveguides are used and all ports are identical. All undesirable modes are effectively suppressed. Further, the slots are reasonably away from the local fields of stems and post couplers so that we may use the unperturbed TM_{010} modal field distribution to estimate the transformer ratio C_a, eq. (2a). Since all ports are identical and only one mode is of concern, Fig. 2 can be further transformed into a one port where m= number of ports originally.

Consider typically $\omega_1=2\pi \times 400 \times 10^6$, Q_1 ranges from a few thousand to tens of thousands. The resistor representing the load ω/Q ranges from tens of KΩ to hundreds of KΩ. A typical value of the TE_{10} mode impedance $Z_o(=1/Y_o)$ is about 500 Ω. Without the fringefields admittance Y_s+Y_s', an overall transformer ratio $c_a/\sqrt{<\underline{e}_o,\underline{e}_o>_s} = \sqrt{\omega_1/(Q_1 m z_o)}$ is needed to match the waveguide to the cavity. At $\omega/q \cong 10^5$ a turn ratio of about $15/\sqrt{m}$, is needed, the value of $c_a/\sqrt{<\underline{e}_o,\underline{e}_o>_s}$ can be estimated from eqs. (2a), (2b) and (2c). $|\underline{H}_1| \simeq \sqrt{1/\alpha\mu_o V}$ where V is the volume cavity in M^3; α is a constant less than unity and approximately 0.1-0.2 resulting from the non-uniformity of H_1 distribution with $\mu_o=4\pi \times 10^{-7}$. The magnitude of $|H_1|$ would be in the range of a few hundred. Equation (3) shows that $|\underline{\hat{e}}_a| \sim (a'b')^{-1/2}$. The surface integral (2a) over the slot is therefore proportional to $\sqrt{a'b'}$. On the other hand, $<\underline{e}_o,\underline{e}_o>_s$ is proportional to the ratio of the slot area a'b' to that of the waveguide cross section ab. The square root $\sqrt{<\underline{e}_o,\underline{e}_o>_s}$ is therefore also proportional to $\sqrt{a'b'}$. Thus, the overall transformer ratio $c_a/\sqrt{<\underline{e}_o,\underline{e}_o>_s}$ is essentially independent of the slot dimensions and it is usually too large a ratio. This is precisely why the shunt susceptance Y_s+Y_s' becomes important, and other waveguide tuners are almost always needed.

Since Y_n is inductive for TE modes and capacitive for TM modes, it should be possible to obtain a large range of susceptences to achieve matching. We further notice that C_a may also be reduced by changing the direction of the slot to that of the $\underline{H}_1$ field.

For a coax-loop system, an impedance matrix can be obtained as

$$Z_{Iq} = \sum_i \frac{s\omega_i^2}{s^2+\frac{\omega_i}{Q_i}s+\omega_i^2} C_pC_q \qquad (16a)$$

$$\text{where } C_p = (\mu_o A \underline{H}_i\cdot\underline{\hat{n}})_p \qquad (16b)$$

$$\iiint_{\text{cavity}} \mu_o\underline{H}_i\cdot H_i dv = 1 \qquad (16c)$$

and ω_i, Q_i again are the ith mode resonant frequency and Q-factor with loops open. n̂ is the unit vector to the plane of loop, A is its area, and $\underline{H}_i$ is the normalized ith modal field. The equivalent circuit is shown in Fig. 3 where is the fringe field impedance dominated by the loop self-inductance. A quantitative analysis will show that it is also very difficult to achieve matching without Z_{sp} and other means.

References

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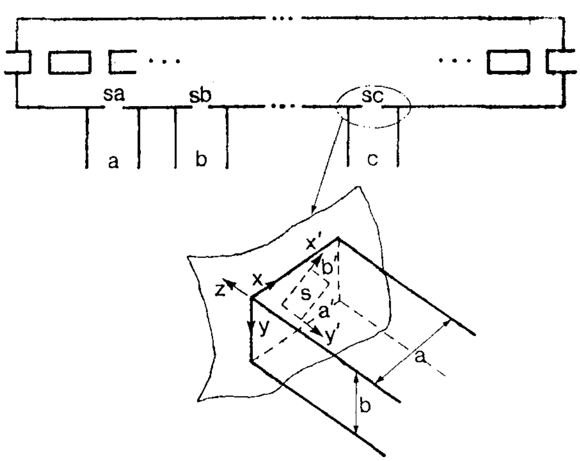

Fig.1

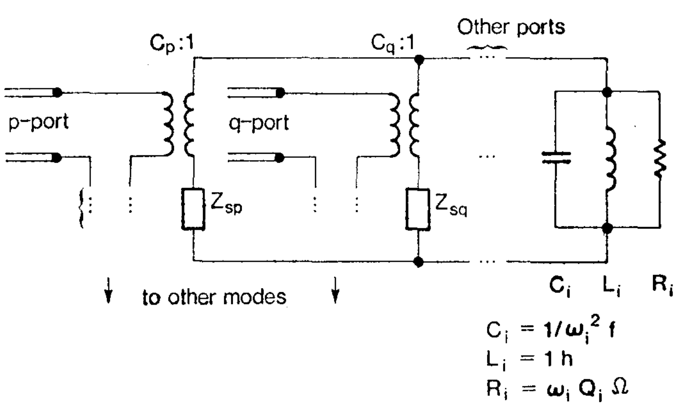

Fig.3

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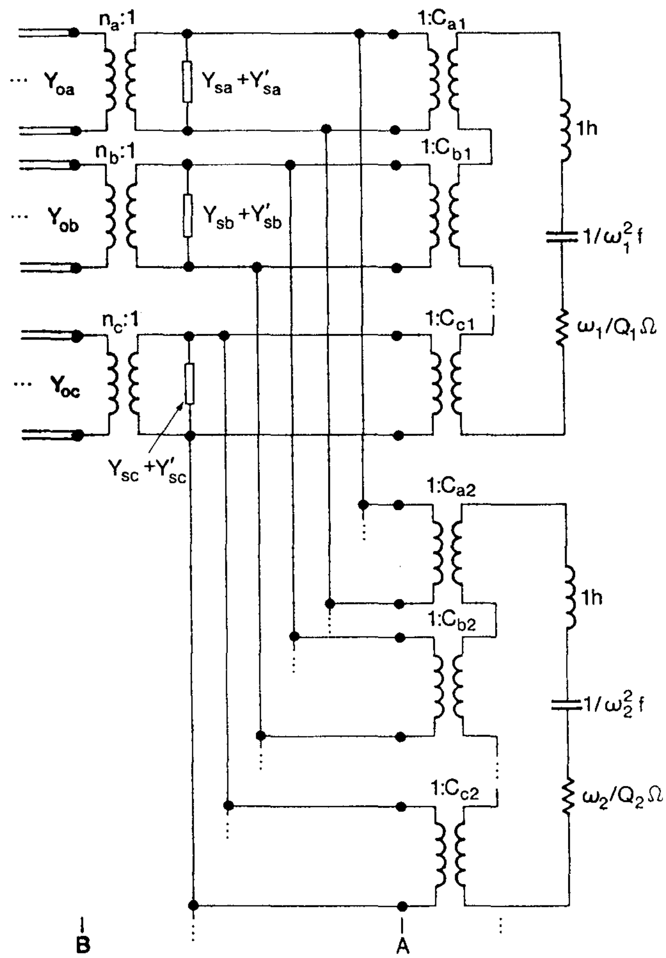

Fig.2

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