ABERRATIONS OF THE SPIN OF POLARIZED PROTONS IN A LINAC

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In the majority of physical experiments with polarized proton beams it is important to have, apart from a high degree of polarization, a small value of transverse spin aberrations, especially of the first crossing moments[1].

The behaviour of the polarization vector $\vec{S}$ in a magnetic field is described by the quasi-classical equation[2]

$$\frac{d\vec{S}}{dt} = \frac{e}{m_0\gamma} S \times \left[\frac{g\vec{B}_\parallel}{2} + \left(1 + \frac{g-2}{2}\gamma\right)\vec{B}_\perp\right] \quad (1)$$

where e, m_0 are the charge and mass of particles, γ is the Lorentz factor, g is the gyromagnetic ratio (for protons $g = 5.586$), $\vec{B}_\parallel$ and $\vec{B}_\perp$ are the parallel and perpendicular components of the projection of the field $\vec{B}$ onto the momentum direction (onto the z axis). The change of the polarization vector in the field $\vec{B}$ is determined by the simultaneous solution of eq. (1) and the equations of motion of particles. For a beam with transverse nonzero emittance in its passage through a magnetic field, nonuniform in space, the polarization vector becomes a function of phase coordinates. For example, for a beam with longitudinal polarization the incoherent parts of the transverse components S_x and S_y, which are dependent on the coordinates of particles, will be called spin aberrations and the area occupied by phase points of the beam on the plane: deviation x or y - transverse spin component S_x or S_y will be called "spin" emittance. The "spin" emittance characterizes the beam at the point of view of maximum values of aberrations. The moments of distributions represent a more accurate quantitative characteristic of the quality of a polarized beam. Specifically, in making experiments on the investigation of P-Parity Nonconservation Effects in weak nucleon-nucleon interaction in polarized protons scattering, of interest are the first crossing moments of distributions of S_x and S_y with respect to the x and y axes: $\langle S_x \cdot y\rangle$ and $\langle S_y \cdot x\rangle$, which in the SIN experiment were of the order of 10^{-3} mm[1]. For obtaining polarized beams of such a high quality it is necessary to take special measures both in the source and transfer channel and in the linac itself. However, the optimization of the transfer channel in terms of the minimum values of spin aberrations in largely determined by the linac. Firstly, for a long time the spin undergoes changes precisely in the linac and, secondly, the linac focussing system is practically determined by other requirements: the intensity, emittance, $\Delta p/p$ of the unpolarized beam.

Therefore initially it is important to determine aberrations which arise in the linac channel and then to optimize the transfer channel and source.

In the present report we consider a linac of Moscow Meson Factory type in which it is possible to make experiments with polarized beams [3]. Such a linac represents alternation of the sections where focussing or defocussing of the beam by quadrupole lenses and acceleration occur. Hereafter, for concreteness, we shall assume the beam to be longitudinally polarized

$$\begin{aligned}
\frac{dS_x}{dt} &= -\frac{e}{m_0\gamma} S_z B_y M \\
\frac{dS_y}{dt} &= \frac{e}{m_0\gamma} S_z B_x M \qquad (2)\\
\frac{dS_z}{dt} &= \frac{e}{m_0\gamma}\left(S_x B_y M - S_y B_x M\right)
\end{aligned}$$

It is taken into account here that $S_y B_z \ll S_z B_y$, $S_x B_z \ll S_z B_x$ and $M = 1 + \frac{g-2}{2}\gamma$.

Let us differentiate the third equation and substitute the first two equations into it:

$$\frac{d^2 S_z}{dt^2} + \left(\frac{e M B_\perp}{m_0 \gamma}\right)^2 S_z = 0. \quad (3)$$

The spin precession frequency $\Omega_s = \frac{eMB_\perp}{m_0\gamma}$ depends on the field distribution function and on the fact along what trajectory the particle moves: $B \sim G(t)\cdot r(t)$, where $G(t)$ is the field gradient in the lens and $r(t)$ is the deviation of the particle from the axis. The condition for the absence of a parametric resonance of spin precession for particles in a beam can be written as

$$\frac{e M L_f G r_0}{2 m_0 \gamma \beta c \mu} < \sqrt{\frac{2}{3}} \quad (4)$$

where μ is the phase advance of radial oscillations, L_f is the focussing period and r_0 is the beam envelope. It is fulfilled in a large energy range from 0.3 MeV and over and is extended to the whole linac. Therefore the system of equations (2) can be integrated in the approximation of smallness of the values:

$$\frac{\Delta S_z}{S_z} \ll 1, \quad \tilde{S}_x = \frac{S_x}{S_z} \ll 1, \quad \tilde{S}_y = \frac{S_y}{S_z} \ll 1.$$

Since the linac channel possesses quadrupole symmetry, all further reasoning will refer only to the plane x. We write simultaneously the integral equations of motion and spin for one quadrupole lens:

$$\tilde{S}_x = \tilde{S}_{x0} - \frac{e\,M}{m_0\gamma\beta c}\int_0^L B_y\,dz$$
$$\left.\frac{d\,x}{d\,t}\right|_L - \left.\frac{d\,x}{d\,t}\right|_0 = \frac{e}{m_0\gamma}\int B_y\,dz \qquad (5)$$

where L is the lens length and β is the relative particle velocity. The simultaneous solution for a series of lenses can be represented as:

$$\tilde{S}_x = \tilde{S}_{x0} - \frac{M}{c}\Bigg\{\underbrace{\frac{\dot{x}_1}{\beta_1} - \frac{\dot{x}_{10}}{\beta_1}}_{\text{first lens}} + \underbrace{\frac{\dot{x}_2}{\beta_2} - \frac{\dot{x}_{20}}{\beta_2}}_{\text{second lens}} + \dots \qquad (6)$$
$$+ \frac{\dot{x}_N}{\beta_N} - \frac{\dot{x}_{N0}}{\beta_N}\Bigg\}$$

or by regrouping the terms:

$$\tilde{S}_x = \tilde{S}_{x0} - \frac{M}{c}\Bigg\{\frac{\dot{x}_N}{\beta_N} - \frac{\dot{x}_{10}}{\beta_1} - \underbrace{\left[\frac{\dot{x}_{20}}{\beta_2} - \frac{\dot{x}_1}{\beta_1}\right]}_{\text{first accelerating gap}} - \qquad (7)$$
$$\underbrace{\left[\frac{\dot{x}_{30}}{\beta_3} - \frac{\dot{x}_2}{\beta_2}\right]}_{\text{second accelerating gap}} - \dots - \left[\frac{\dot{x}_{N0}}{\beta_N} - \frac{\dot{x}_{N-1}}{\beta_{N-1}}\right]\Bigg\}$$

Noting that the angular divergence of the trajectory $\dot{x}\,/\,\beta$ in the accelerating gap is much less than that in the quadrupole lens, we may assume that the expressions in the square brackets are equal to zero:

$$\tilde{S}_x = \tilde{S}_{x0} + \frac{M}{L_{f0}}\left(\frac{dx_0}{d\tau} - K\frac{dx}{d\tau}\right) \qquad (8)$$

where $K = \frac{\beta_0}{\sqrt{\beta}}$, $\tau L_f = \beta c\, t$.

Using the smooth approximation of the solution of the equations of motion in the accelerating-focussing channel, we obtain:

$$\tilde{S}_x = \tilde{S}_{x0} + \frac{M}{L_{f0}}\left\{x_0 K\mu\sin\mu\tau - \dot{x}_0(K\cos\mu\tau - 1)\right\} \qquad (9)$$

Sinse our objective is to determine the contribution of the linac to spin aberrations, then $\tilde{S}_{x0} = 0$. Knowing the transformation of the phase ellipse of the beam in the plane (xx) and substituting eq. (9) into these transformations, we obtain functions of variation in the parameters of "spin" emittance along the channel length. In particular, for S_{xmax} we have obtained the simple expression:

$$S_{xmax} = \frac{M}{L_{f0}}\left(\frac{2V\lambda\mu}{\gamma}\right)^{1/2}\left(1 + K^2 - 2K\cos\mu\tau\right)^{1/2} \qquad (10)$$

where V is the normalized emittance and λ is the wavelength of the accelerating field.

In the same way we can obtain the first moment:

$$\langle S_x \cdot x\rangle = \frac{V\,M}{4\gamma\beta_0}\sin\mu\tau. \qquad (11)$$

Fig. 1 shows the behaviour of S_{xmax} along the linac channel, which was obtained by the numerical method. The results agree with fair accuracy with the analytical ones.

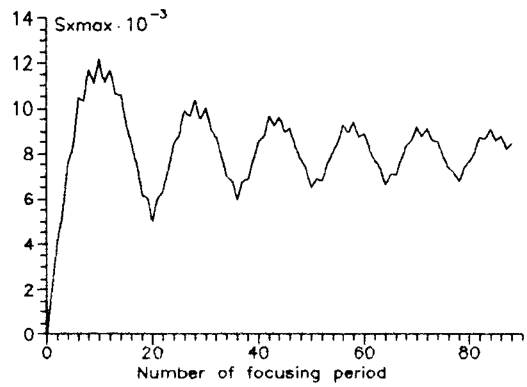

Fig.1. Behaviour of S_{xmax} along the linac channel length.

Owing to this we can draw the following conclusions. Firstly, the final value of S_{xmax} is determined by the parameters of the linac at its entrance and by the normalized beam emittance. Secondly, with increasing energy the average value of S_{xmax} changes slightly

$$\overline{S}_{xmax} \sim \gamma^{-1/2}\left[1 + \left(\frac{\beta_0}{\sqrt{\beta}}\right)^2\right]^{1/2}.$$

Fig. 2 (curve 1) shows a change in the first moment $\langle S_x \cdot x\rangle$ along the channel length, which was obtained numerically.

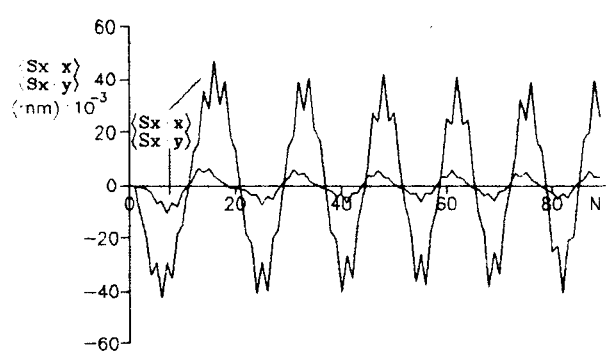

Fig.2. Behaviour of the first moments along the linac channel length.

It is also well described by the function (11), which yields the important conclusion: the maximum value of the moment $\langle S_x \cdot x\rangle$ is independent of the channel parameters but depends only on the beam parameters V, γ, β_0. This can be interpreted as follows: the increase in the phase advance of the channel μ leads, on the one hand, to a decrease in the average beam radius and, on the other, to an increase in spin aberrations (10). Therefore the only possible way of decreasing the maximum value of the first moment $\langle S_x \cdot x \rangle$ is the filtration of emittance at the entrance into the linac.

The crossing moment $\langle S_x \cdot y \rangle$ in the linac arise in the magnetic fringe field $B_z \sim xy$ in quadrupole lenses. Obtaining analytical expressions for them involves great diffculties, but it is obvious that the character of the behaviour of the crossing moments is similar to the behaviour of the direct momets and their value should be decreased by $\sim B_z/B_x$. Fig. 2 (curve 2) illustrates the behaviour of the crossing moment $\langle S_x \cdot y \rangle$ along the linac channel length. The maximum value of the crossing moment $\langle S_x \cdot y \rangle$ is $2 \cdot 10^{-3}$ mm for the accelerating-focussing channel of the linac of Moscow Meson Factory and for the beam emittance $V = 0.15\,\pi$ cm mrad.

In accelerating a longitudinally polarized proton beam in the linac there occur transverse spin aberrations, the maximum values of the first moments of which are: $\langle S_x \cdot x \rangle$ and $\langle S_y \cdot y \rangle$ about $4 \cdot 10^{-2}$, $\langle S_x \cdot y \rangle$ and $\langle S_y \cdot x \rangle$ about $2 \cdot 10^{-3}$ mm for the normalized emittance of $0.15\,\pi$ cm mrad. The character of the behaviour of these parameters is quite analogous to the character of the behaviour of the first moments of the usual phase emittance along the linac channel. The maximum value of the moments is independent of the linac parameters and is only determined by the beam emittance. The change of polarization from longitudinal to transverse does not give any special advantage. The corresponding crossing moments are decreased by a factor of two.

References

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