

Transverse Effects of a Waveguide Coupling Slot\*

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Abstract

High-power operation of rf cavities may require that the waveguide supplying energy to the system be coupled to the cavity with a large coupling slot. It is a concern that this coupling slot may be so large that appreciable asymmetry will exist in the fields of the fundamental mode and transverse effects will be introduced on the beam. Computation of these effects is complicated by the facts that the fundamental mode frequency must propagate in the waveguide and resonant cavity codes cannot handle energy transfer across the boundaries of the problem. We have successfully computed these effects using 3-D computer codes in MAFIA, both in the frequency and time domains. This paper will report the computed results showing good agreement with experimental measurements and indicating that the transverse effects can be compensated. The results include mode distortions and transverse forces. The methods of calculation will also be described.

Introduction

High-power, heavily beam-loaded cavities must be supported with rf power through waveguides and large coupling slots. It is a concern that such a coupling slot will break the cylindrical symmetry of the cavity substantially and introduce an appreciable transverse force on the beam. Calculations of this force have been made with three different methods and explained as follows.

First, investigations of this asymmetry effect have been made with an analytical method. The prescription of Bethe<sup>1</sup> was used for the case of a small coupling hole. In this prescription, the coupling hole is replaced with a distribution of radiating magnetic and electric dipoles, which can be calculated using the undisturbed field distribution of the waveguide mode. The deflecting field can then be calculated using these dipoles as driving terms. This prescription was extended to the case of a large hole.<sup>2</sup> Results for a pillbox cavity were obtained showing that the amplitudes of deflecting effects will increase with the third power of the radius of the hole. Because of the analytic nature of the method, the application to cavity shapes away from pillbox will be difficult.

Second, the problem was investigated using calculations in the frequency domain. In principle, calculation in the frequency-domain is the method of choice for this problem but requires an open boundary condition at the end of the waveguide. The lack of an open boundary condition in the present 3-D code has forced us to obtain results indirectly. Although the method of these calculations is described in a separate paper in these proceedings,<sup>3</sup> a summary of the calculations will be presented here because the results of the mode fields give a good insight into mode distortion caused by the waveguide asymmetry.

Third, an attempt was made recently to solve this problem using time-domain calculations. In a time-domain calculation, the need for an open-boundary condition can be avoided by observing causality. The calculated transverse deflections using this method show excellent agreement with measurements by Boeing Aerospace Corporation (BAC).<sup>4</sup> In this paper, this method of calculation will be given in detail.

Besides the descriptions of the 3-D calculations in the frequency and time domain, the transverse force is shown to be compensated by a trough on the side of the cavity diametrically opposite the waveguide coupling slot. The geometry used in all the calculations is shown in Fig. 1 and is the designed cavity shape and waveguide arrangement as supplied by BAC.

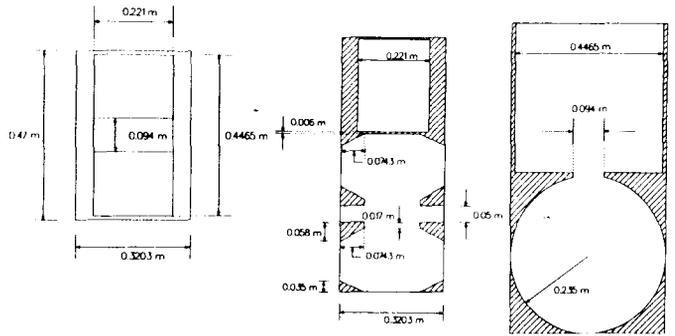


Fig. 1. Schematic diagram of the cavity and waveguide arrangement as supplied by the Boeing Aerospace Corporation.

Frequency-Domain Calculation

The frequency-domain calculation was performed using the eigenvalue solver in MAFIA,<sup>5</sup> a family of codes that solves Maxwell equations in three-dimensional geometries. Details of the calculation are given in Ref. 3. The calculation shows that, although the code lacks the outgoing-wave boundary condition required at the end of the waveguide away from the cavity, a metal boundary can be employed instead. By clever manipulation of the position of the boundary, the code can model an energy flow from the waveguide toward the cavity.

Results of the calculations clearly show a displacement of the field center (see Fig. 2), which can be viewed as the consequence of mixing the TM<sub>110</sub>-like deflecting mode with the TM<sub>010</sub>-like fundamental mode. The displacements of the fundamental mode were calculated for various cavity shapes. For the MCTD (Modular Components Test Demonstration) cavity, a displacement of 0.7 mm is calculated, which compares well with the 0.5 mm as measured by BAC.<sup>4</sup>

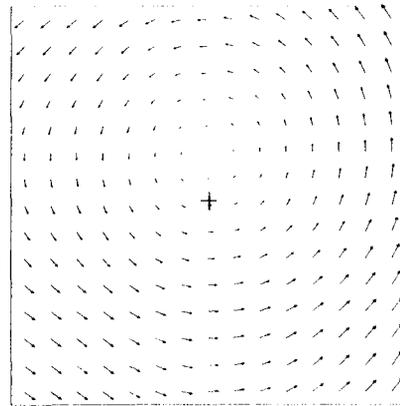


Fig. 2. Magnetic field pattern of the fundamental mode for a cavity coupled to a waveguide. The geometric center of the cavity is marked as +.

Time-Domain Calculations

Method of Calculation

The time-domain calculations were made using the computer code T3 in the MAFIA code family. The code computes electromagnetic fields excited by charged bunches in a 3-D geometry by numerically integrating the Maxwell equations in the

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time domain. The results are presented in terms of wake functions. A wake function is a function of time  $t$  and is the total force that a unit charge will experience if it passes through the cavity at a time  $t$  behind the head of the exciting charge bunch.

To study the transverse force of the fundamental mode, first one needs to establish a fundamental mode field pattern in the cavity. In a time-domain calculation, this field can be induced by passing multiple bunches through the cavity. In this calculation, nine bunches were used. To ensure that the induced field would be predominantly fundamental mode in nature, these bunches were chosen to have a relatively long bunch width ( $\sigma_{rms} = 16$  cm) and a bunch frequency of 433 MHz (the fundamental mode frequency of the cavity). The success in selectively exciting only the fundamental mode is demonstrated using TBCI, a 2-D version of T3. Figure 3a shows the longitudinal wake for axially symmetric modes during and after the passing of nine bunches. During the passing of the bunches, there is a linear buildup of the field indicating that the mode has been resonantly excited. After the bunches have passed, there is an oscillation with constant amplitude. Fourier analysis shows that the oscillation has no appreciable frequency component other than 433 MHz. Figure 3b shows similar wake functions for the dipole-like modes. The resultant wake functions have amplitudes significantly smaller than those from a single bunch. In fact, with the long bunch width and higher mode frequency, the front and tail of the bunches are respectively exciting and de-exciting the mode.

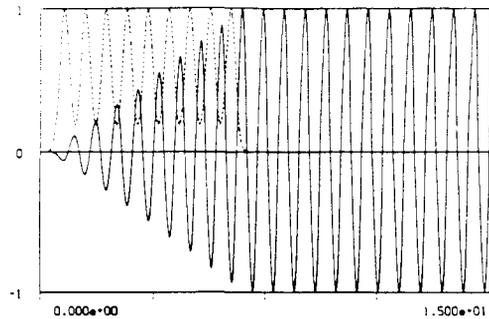


Fig. 3a. Longitudinal wake function (solid curve) for axially symmetric modes during and after the passing of nine bunches through an axially symmetric cavity. The bunch shape is represented by the dashed curve.

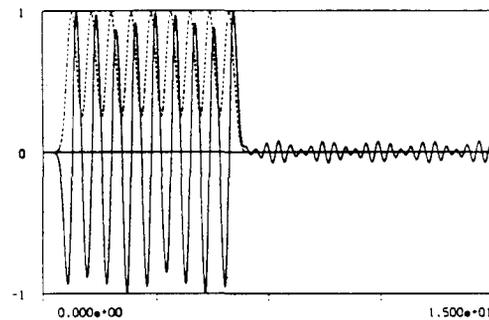


Fig. 3b. Longitudinal wake function (solid curve) for dipole-like modes during and after the passing of nine off-axis bunches through an axially symmetric cavity. The bunch shape is represented by the dashed curve.

The requirement of an outgoing wave boundary condition was circumvented using causality considerations. Although a metal-like boundary condition was put at the end of the waveguide, it will be equivalent to an outgoing wave boundary condition for field study in the cavity until the reflections from this boundary arrive at the cavity. The time when the reflections arrive can be found both analytically and with simulations. Analytic calculations showed that, at 433 MHz, the wave travels in the waveguide at half of the speed of light. The speed of the wave including dispersion effects was found by observing the arrival of wave fronts at various locations along the waveguide. Results

showed that no significant fields are traveling faster than three-fourths of the speed of light. Therefore, using a waveguide 4 m long, one can have a time window of eight cycles at 433 MHz for study after the bunches have passed through and before the arrival of the reflected waves.

### Results of Calculations

The transverse and longitudinal forces experienced by a charge are proportional, respectively, to the transverse and longitudinal wake functions during the time window discussed in the last section. The wake functions integrated on axis for a geometry with waveguide (see Fig. 1) are shown in Fig. 4. They show that a transverse force is introduced by the coupling to the waveguide. This force is in time quadrature with the accelerating force, indicating that it is magnetic in origin. This

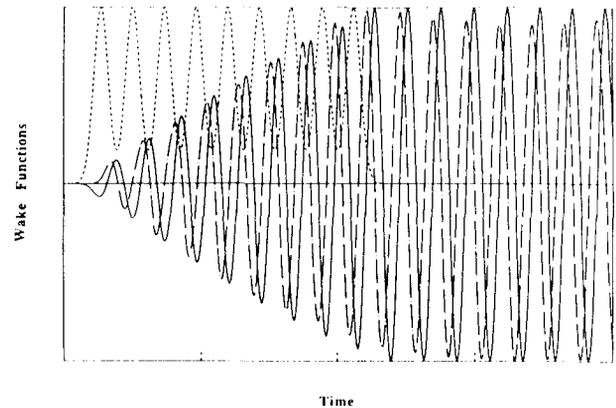


Fig. 4. Longitudinal (solid curve) and transverse (dash-dotted curve) wake function during and after the passing of nine bunches on-axis through the cavity and waveguide arrangement. The bunch shape is represented by the dashed curve.

observation is in agreement with the frequency-domain work, which shows that the deflections are due to a small mixture of  $TM_{110}$ -like mode. With the center of a beam bunch arriving at the center of the cavity for maximum acceleration, the transverse force is zero at the center of the bunch, with the front of the bunch deflected towards the waveguide and the tail away from the waveguide. The differences in deflections for the head and tail of a bunch will be defined as the deflection shear. The deflection shears were calculated as a function of the width of the coupling slot for a bunch length of 25 ps and a beam energy of 15 MeV. Figure 5 shows the results compared with the experimental measurement from BAC. Good agreement is evident.

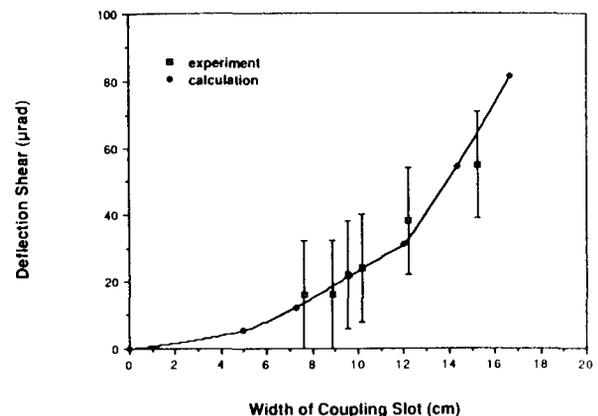


Fig. 5. Deflection shear introduced by the waveguide coupler as a function of the width of the coupler.

Intuitively, the transverse deflection introduced by the waveguide coupling can be compensated by attaching a similar geometry at the side of the cavity that is diametrically opposite the waveguide coupling slot (see Fig. 6). Figure 7 summarizes the results of calculations to determine the correct geometry for compensation. There are two curves corresponding to two different widths of the coupling slot. The first curve (width = 9.4 cm) was produced using a compensation that is an exact replica of the waveguide coupling slot assembly. The deflection shear rapidly approaches a small value, when the length of the compensating waveguide is varied. With the length of the waveguide at 10 cm, the deflection has a value of  $0.9 \mu\text{rad}/\text{cavity}$ . Although this curve does not cross zero, meaning that the transverse force is never exactly compensated, it has the advantage that if a sufficiently long waveguide is used, small deflection is guaranteed. The deflection has very little sensitivity on the length of the waveguide. The second curve (width = 15 cm) was produced and is similar to the first curve, except that the width of the coupling slot increased from 9.4 cm to 15 cm. The deflection shear crosses zero when the length of the compensating waveguide is 1 cm, which indicates that an exact compensation of the asymmetry can be made. If the length of the compensating waveguide is further increased, the deflection shear will be overcompensated. The sensitivity of deflection shear to the length of the waveguide is  $1 \mu\text{rad}/\text{cavity}/\text{mm}$ . In other words, one needs to determine the length of the waveguide to 1 mm if one needs to reduce the deflection shear to  $1 \mu\text{rad}/\text{cavity}$ , which may be difficult to do either with computation or with experiment.

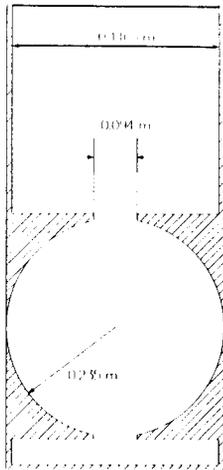


Fig. 6. Schematic with the compensating waveguide added to the cavity and waveguide arrangement.

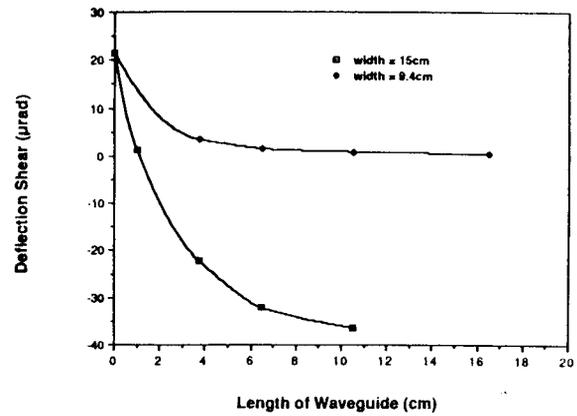


Fig. 7. Deflection shear with the compensating waveguide with two different widths of coupling slot.

### Conclusions

There is a transverse deflection of  $22 \mu\text{rad}/\text{cavity}$  introduced by the waveguide coupling for the present cavity and waveguide design of the Laser Sub-System. This shear is caused by the magnetic field of the small mixture of the  $\text{TM}_{110}$ -like mode in the fundamental mode. This result is in very good agreement with the experimental results obtained by the team at Boeing Aerospace Corporation. We have further shown that this transverse deflection can be compensated with a similar waveguide arrangement diametrically opposite the existing waveguide.

### Acknowledgment

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