BEAM BREAK-UP AND RESISTIVE WALL INSTABILITY IN A STEADY-STATE FREE ELECTRON LASER IN THE MICROWAVE REGIME

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Abstract

The beam break-up instability and resistive wall instability caused by interaction between beam and induction gap, vacuum chamber wall, in a steady-state free electron laser in the microwave regime are considered. The large energy spread induced by free electron laser performance is theoretically proved not to lead to Landau damping of both instabilities when the synchrotron frequency is of order or larger than the betatron frequency(This is an intrinsic nature of a steady-state free elctron laser).

Introduction

A microwave FEL is regarded as a possible candidate of high power microwave sources in the frequency region of 6~30 GHz for which the high energy accelerator society gets tired of waiting. In fact, the collaboration of LLNL and LBL has dramatically demonstrated the successful single-stage experiment of 35 GHz and 1 GW [1]. However this single-stage experiment is not straightforwardly extrapolated to a multi-stage level which is motivated by its use in a two-beam accelerator[2], because there are still unsolved many basic problems of microwave's extraction without RF breakdown[3], phasing of output RF[4], or the stability of a driving beam itself over a long distance[5]. From an accelerator physics point of view, an essential issue of the two-beam accelerator concept is how far a kiloamp electron beam in a steady-state FEL can be propagated with tolerable loss of beam quality. Major obstacles to long distance transport of kiloamp beams are the so-called BBU and transverse resistive wall instability. The former arises as a result of interaction between beams and induction gap, and the existance of wall charges and currents induced on the waveguide surface of finite conductivity by the displacement of beam centroid gives rise to the latter instability. So far the synchrotron oscillation in a large bucket of steady-state FEL has been supposed to induce a relatively large energy spread associated with a large spread in the betatron wave number k_β, eventually resulting in Landau damping of both instabilities. This hypothesis seems not true in the case of a steady-state FEL where the synchrotron frequency ν is in general the same order of magnitude as the betatron frequency, as described in Appendix. Recently it has been theoretically proved by the present author[5] that Landau damping of BBU due to a large energy spread is not expected in a steady-state FEL; in addition Whittum[6] has demonstrated the same result for the resistive wall instability by computer simulations.

Main purposes of the present paper are to summarize the analysis of BBU in a steady-state FEL, to extend the techniqe used there to the resistive wall instability, and to find the characteristic distance formulas: L_{BBU} and L_R. Based on these formulas, the possible limit of multi-stage for a proposed FEL is given.

BBU and Resistive Wall Instability

BBU

The analysis is based on the BBU model provided in Ref. 7 where the induction gaps are treated as discretely distributed along the structure with spacing of L_g and the BBU cavity mode is characterized by its angular frequency ω_λ, quality factor Q, and transverse shunt impedance $Z_\perp/Q$. The mode is excited by a dipole current source term which is proportional to the product of the beam current I_B and transverse displacement $\langle\xi\rangle$. The transverse position of the beam centroid is determined by the linear focusing, the cavity fields, and the planar wiggler field. Now, introducing the variable $\tau=t-z/c$ which measures the time-delay behind the beam pulse head and averaging the perturbed betatron oscillation over a wiggler period and over one period of induction module, the BBU equations become

$$\left(\frac{\partial^2}{\partial\tau^2}+\frac{\omega_\lambda}{Q}\frac{\partial}{\partial\tau}+\omega_\lambda{}^2\right)\Delta(\tau,z)=\frac{I_B}{I_0}\omega_\lambda{}^3\frac{Z_\perp}{Q}\langle\xi(\tau,z)\rangle \qquad (1a)$$

$$\left[\frac{\partial}{\partial z}(1+\varepsilon\cos\nu\zeta)\frac{\partial}{\partial z}+k_\beta{}^2\right]\xi_{\varepsilon,\varphi}(\tau,z)=\frac{\Delta(\tau,z)}{L_g\gamma_0} \qquad (1b)$$

$$(\zeta=z+\varphi/\nu)$$

where Δ the z-averaged normalized transverse momentum change of the beam centroid, $\xi_{\varepsilon,\varphi}$ the transverse position of each particle, ε and φ the maximum relative energy deviation and the initial phase for each particle, I_0 the Alfven current, ν and k_β the synchrotron and betatron frequency, respectively, γ_0 the synchronous energy assumed to be constant in the following discussion. Here we assume a clude relation among characteristic distance: $L_{BBU}>\frac{2\pi}{k_\beta}, \frac{2\pi}{\nu}>L_g\gg\lambda_W$(wiggler wave-length). Eq. (1a) represents the time-evolution of momentum gain proportional to magnetic wake fields in the induction gap located at z after pulse head arrival and Eq. (1b) represents the orbital-evolution of transverse position of i-th particle in the slice at pulse position τ behind the pulse head.

Introducing a new variable $\eta_{\varepsilon,\varphi}=(1+\varepsilon\cos\nu\zeta)^{1/2}\xi_{\varepsilon,\varphi}$ instead of $\xi_{\varepsilon,\varphi}$ and performing Fourier transform of Eqs. (1a), (1b) in the variable τ to ω, we have

$$(\omega_\lambda{}^2-\omega^2+i\frac{\omega\omega_\lambda}{Q})\tilde{\mathcal{A}} = \omega_\lambda{}^3\frac{I_B Z_\perp}{I_0 Q}\langle\tilde{\xi}\rangle \quad (2a)$$

$$\frac{\partial^2\tilde{\eta}_{\varepsilon,\varphi}}{\partial z^2}+\left[k_\beta{}^2+\varepsilon(\frac{\nu^2}{2}-k_\beta{}^2)\cos\nu\zeta\right]\tilde{\eta}_{\varepsilon,\varphi} = \frac{\tilde{\mathcal{A}}}{L_g\gamma_0} \quad (2b)$$

where transformed quantities are denoted by tildes. Nonlinearization of the Mathieu coefficient in a Mathieu-like equation is known to admit exact solutions[5]; the homogeneous solutions of Eq. (2b) are approximately written by

$$x_\pm(Z)=(1+G\cos 2Z)^{1/2}\exp[\pm i\frac{\sqrt{1+\lambda}}{2} \times\sin^{-1}\left[(1-G^2)^{1/2}\frac{\sin 2Z}{1+G\cos 2Z}\right]]$$

where $Z=\nu\zeta/2$, $G=\varepsilon\frac{\nu^2-2k_\beta{}^2}{\nu^2-4k_\beta{}^2}=\varepsilon g$, $\lambda=-\frac{1-4(k_\beta{}^2/\nu^2)}{1-g^2\varepsilon^2}$

Since $G<1$ for $k_\beta\sim\nu$, $\sin\phi=(1-G^2)^{1/2}\sin\nu\zeta/(1+G\cos\nu\zeta)\simeq\sin\nu\zeta/(1+G\cos\nu\zeta)$, then $\phi=\nu\zeta-G\sin\nu\zeta$. From this result and $\sqrt{\lambda+1}/2\simeq k_\beta/\nu$, we have

$$\eta^\pm_{\varepsilon,\varphi}(z)=(1+\frac{G}{2}\cos\nu\zeta)\sum_{n=-\infty}^{\infty}J_n(\frac{k_\beta}{\nu}G)e^{\pm i(k_\beta-n\nu)\zeta} \quad (3)$$

, employing terms of the Bessel function. Using the Green function $G(z,z')$ evaluated from Eq. (3), the solution to Eq. (2b) is given by

$$\tilde{\eta}_{\varepsilon,\varphi}(\omega,z)=\tilde{\mathcal{F}}(\omega,z)+\frac{1}{L_g\gamma_0}\int_{-\infty}^{z}G(z,z')\tilde{\mathcal{A}}(\omega,z')dz' \quad (4)$$

where $\tilde{\mathcal{F}}$ is the initial value term. Multiplying both sides by $(1+\varepsilon\cos\nu\zeta)^{1/2}$, substituting the expression $\tilde{\mathcal{A}}=h(\omega)\langle\tilde{\xi}\rangle$ where

$$h(\omega) = \frac{\omega_\lambda{}^3}{\omega_\lambda{}^2-\omega^2+i\omega\omega_\lambda/Q}\frac{I_B}{I_0}\left(\frac{Z_\perp}{Q}\right)$$

derived from (2b), into Eq. (4), and averaging its both sides over the distribution of energy spread and initial phase, we have a Volterra equation of the 2-nd kind for $X(z) = \langle\tilde{\xi}(\omega,z)\rangle$,

$$X(z)=Q(z)+\frac{h(\omega)}{L_g\gamma_0 k_\beta}\sum_{m=-\infty}^{\infty}a_{mm}\int_{-\infty}^{z}dz'X(z') \times\sin[(k_\beta-\nu m)(z-z')]$$

where the abbreviations: $X(z)=\langle\tilde{\xi}\rangle$, $Q(z)=\langle(1+\varepsilon\cos\nu\zeta)^{1/2}\tilde{\mathcal{F}}\rangle$, and $a_{mm}=\frac{1}{\sigma}\int_0^\sigma J_m^2(\frac{k_\beta}{\nu}g\varepsilon)d\varepsilon$ are used. Here a flat distribution for ε (σ: maximum deviation) is assumed. Utilizing a Faltung theorem, the equation is solved by the Laplace transformation in the variable z to p. The inverse Laplace transformation of $X(p)$ gives

$$\langle\tilde{\xi}(\omega,z)\rangle=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{Q(p)e^{pz}}{1-\frac{h(\omega)}{L_g\gamma_0 k_\beta}\sum_{m=-\infty}^{\infty}\frac{a_{mm}(k_\beta-\nu m)}{p^2+(k_\beta-\nu m)^2}}dp \quad (5)$$

From the theory of residue, the integral is evaluated in the form $X(z)=\sum_j A_j(\omega)e^{p_0^j(\omega)z}$ where $p_0^j(\omega)$ is the zero-point of the denominator in the integrand. The Fourier inverse transformation of X(z) gives

$$\langle\xi(z)\rangle = \sum_j\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}A_j(\omega)e^{p_0^j(\omega)z}e^{+i\omega\tau}d\omega \quad (6)$$

An assymptotic form of the integral may be evaluated by the method of steepest descents to become $\langle\xi(z)\rangle\propto A_j(\omega_s)e^{p_0^j(\omega_s)z+i\omega_s\tau}$ where the saddle point ω_s satisfies

$$\frac{dp_0(\omega_s)}{d\omega}+i\tau=0.$$

The problem finally reduces to a mathematical problem of solving the dispersion relation

$$1 - \frac{h(\omega)}{L_g\gamma_0 k_\beta}\sum_{m=-\infty}^{\infty}\frac{a_{mm}(k_\beta-\nu m)}{p^2+(k_\beta-\nu m)^2} = 0 \quad (7)$$

where $k_\beta\sim\nu$, the summation is dominated by three terms of m=0, ±1 : $a_{00}=1-\delta^2/6$, $a_{11}=a_{-1-1}=\delta^2/12$ ($\delta\equiv k_\beta g\sigma/\nu$), then an assumption of strong focusing ($h/L_g\gamma_0 k_\beta^2$, $h/L_g\gamma_0\nu^2\ll1$) leads to

$$p_0(\omega) = -ik_\beta + \frac{ih(\omega)}{2k_\beta L_g\gamma_0}(1-\frac{\delta^2}{6})$$

After tedious mathematical calculation for seeking the saddle point[6], the asymptotic form

$$\langle\xi(\tau,z)\rangle \propto e^{-ik_\beta z+i\omega_\lambda\tau}\ e^{-\alpha\tau+\sqrt{2|B|\omega_\lambda\tau(1-\delta^2/6)z}} \quad (8)$$

where $\alpha=\omega_\lambda/2Q$ and $|B|=\frac{\omega_\lambda}{2L_g k_\beta\gamma_0}\frac{I_B Z_\perp}{I_0 Q}$

is obtained. The real part of index, $\Psi(\tau)$, takes its maximum value

$$\Psi(\tau_{max}) = \frac{|B|\omega_\lambda}{2\alpha}(1-\frac{\delta^2}{6})z$$

for $$\tau_{max} = \frac{1}{2\alpha^2}\cdot\left[|B|\omega_\lambda(1-\frac{\delta^2}{6})z\right]$$

Eventually, we can arrive at the BBU growth distance as follows,

$$L_{BBU} = z/\Psi(\tau_{max}) = \frac{2L_g k_\beta\gamma_0}{\omega_\lambda Z_\perp}\left(\frac{I_0}{I_B}\right)\frac{1}{1-\delta^2/6} \quad (9)$$

For the present case, the BBU growth distance falls in the below range,

$$L_{BBU}(\sigma=0) < L_{BBU}(\sigma) \leq \frac{6}{5}L_{BBU}(\sigma=0)$$

because of $\left(\frac{k_\beta}{\nu}g\right)^2\sigma^2\sim1$ at most.

Eq. (3) indicates that the frequency modulated betatron oscillation involves the infinite number of eigenmodes with the frequency $|k_\beta \pm n\nu|$ and the relative strength of these modes is determined by the Bessel function term which is a function of the betatron and synchrotron frequencies, and the energy spread. This discrete spectrum of oscillation mode tends to localize at $k_\beta(1-\varepsilon/2)$ in the limit of $k_\beta/\nu\to\infty$, yielding an effective spread in the betatron frequency of the beam. The spread leads to Landau damping of BBU. When $k_\beta/\nu\sim 1$, on the other hand, there are only three dominant modes of k_β, $|k_\beta\pm\nu|$ as above derived. It is easily supposed that interference among different spectra consisting of three lines is quite weak.

Resistive wall instability

The analysis is based on the equation formulated by Caporaso et al. [8] which describes the resistive wall instability. Their equation is modified in the orbital form

$$\left[\frac{\partial}{\partial z}(1+\varepsilon\cos\nu\zeta)\frac{\partial}{\partial z}+k_\beta^2\right]\xi_{\varepsilon,\phi}(\tau,z)=\frac{\beta}{\sqrt{\pi}}\int_0^\tau\frac{\langle\xi(\tau',z)\rangle}{\sqrt{\tau-\tau'}}d\tau'$$

$$\beta=\left(\frac{e}{mc}\right)\frac{I_B}{\pi r_0 b^3}\sqrt{\frac{\mu_0}{\sigma}} \qquad (10)$$

where $1/\sqrt{\tau-\tau'}$ of the right-hand side indicates the resistive wall wake function, b is the vacuum chamber radius, σ is the conductivity of chamber wall material, and μ_0 is the magnetic permeability in vacuum. Solutions of Eq. (10) are written by the term of Green function in the similar way to the previous discussion.

$$\xi_{\varepsilon,\phi}(\tau,z)=f_{\varepsilon,\phi}+(1+\frac{\varepsilon}{2}\cos\nu\zeta)^{1/2}\int_{-\infty}^{z}dz'G(z,z')$$
$$\times\int_0^\tau d\tau'K(\tau-\tau')\langle\xi(\tau',z')\rangle$$

where $f_{\varepsilon,\phi}$ is the initial value term and the abbreviation $K(\tau-\tau')=\beta/\sqrt{\pi(\tau-\tau')}$ is used. Averaging both sides over the distribution of energy spread and initial phase, we have a Volterra equation again,

$$\langle\xi\rangle=\langle f\rangle+\frac{1}{k_\beta}\sum_{m=-\infty}^{\infty}a_{mm}\int_{-\infty}^{z}dz'\sin[(k_\beta-\nu m)(z-z')]$$
$$\times\int_0^\tau d\tau'K(\tau-\tau')\langle\xi\rangle \qquad (11)$$

Utilizing a Faltung theorem, Eq. (11) is solved by two-stages of Laplace transformation; in the variable τ to q,

$$\langle\xi(q,z)\rangle=F(q,z)+\frac{1}{k_\beta}\sum_{m=-\infty}^{\infty}a_{mm}\int_{-\infty}^{z}dz'\sin[(k_\beta-\nu m)$$
$$\times(z-z')]K(q)\langle\xi(q,z')\rangle, \qquad (K(q)=\beta/\sqrt{q})$$

and in the variable z to p,

$$\langle\xi(q,p)\rangle=H(q,p)+\frac{\beta}{k_\beta}\sum_{m=-\infty}^{\infty}\frac{a_{mm}(k_\beta-\nu m)}{p^2+(k_\beta-\nu m)^2}\frac{\langle\xi(q,p)\rangle}{\sqrt{q}} \qquad (12)$$

After algebraic calculation, the double inverse Fourier transformation leads to

$$\langle\xi(\tau,z)\rangle=\frac{1}{4\pi^2}\int_{c'-i\infty}^{c'+i\infty}dq\int_{c-i\infty}^{c+i\infty}dp\frac{H(q,p)e^{zp+\tau q}}{1-\frac{\beta}{k_\beta\sqrt{q}}\sum_{m=-\infty}^{\infty}\frac{a_{mm}(k_\beta-\nu m)}{p^2+(k_\beta-\nu m)^2}} \qquad (13)$$

From the assumption of strong focusing, the pole of the integrand is evaluated as follows,

$$p_0(q)=-ik_\beta+\frac{i\beta}{2k_\beta\sqrt{q}}(1-\frac{\delta^2}{6}) \qquad (14)$$

Thus,

$$\langle\xi(\tau,z)\rangle=\frac{1}{4\pi^2}\int_{c'-i\infty}^{c'+i\infty}dqH(q,p_0)e^{zp_0(q)+\tau q}$$

The saddle point of the above integrand, q_s, is calculated from $zdp_0(q_s)/dq+\tau=0$, then

$$q_s=(\pm i)^{2/3}\left[\frac{4k_\beta\tau}{\beta z(1-\delta^2/6)}\right]^{-2/3}$$

Thus, the asymptotic form of the integral becomes

$$\langle\xi(\tau,z)\rangle\propto e^{\frac{3}{2}\left[\frac{\beta z(1-\delta^2/6)}{4k_\beta}\right]^{2/3}\tau^{1/3}} \qquad (15)$$

Finally, we have the growth distance formula,

$$L_R=2\pi\left(\frac{2}{3}\right)^{3/2}\sqrt{\sigma\mu_0}\left(\frac{I_0}{I_B}\right)\frac{r_0 b^3}{\lambda_\beta}/\sqrt{\tau}(1-\frac{\delta^2}{6}) \qquad (16)$$

where $I_0=4\pi(mc/e)/\mu_0$ is the Alfven current.

Summary

The integral equations for the BBU and resistive wall instability have been evaluated in compact forms, introducing a novel technique of nonlinearization of the Mathieu coefficient and the dispersion relations have been derived from these integral equations. In the region of $k_\beta\sim\nu$ of particular interest, we have calculated the poles from the dispersion relations and finally arrived at the formulas for the BBU growth and resistive wall instability growth distances, L_{BBU} and L_R which are functions of the synchrotron and betatron frequencies , and the energy spread. From the expressions of L_{BBU} and L_R, we realize that enlargement in L_{BBU} and L_R due to the energy spread is quite small. Accordingly we conclude that a large energy spread particular for an FEL in the microwave regime doesn't contribute to Landau damping of BBU and the resistive wall instability.

The growth formulas give L_{BBU}=71m and L_R=1.47km with typical parameters [10], $I_B=2kA$, $k_\beta=2\pi/3m^{-1}$, $\sigma(sus)=3.64\times10^7(\Omega\cdot m)^{-1}$, $L_g=2m$, $\gamma_0=40$, $\omega_\lambda Z_\perp=0.4cm^{-1}$, b(chamber radius)=5cm, τ(*pulse length*)=50$nsec$. The value of L_{BBU} is crucial for a steady-state FEL employed in a two-beam accelerator. One would have hoped beam transport over a greater distance for higher conversion efficiency from beam power to microwave power. This requirement may be satisfied in two possible ways. One of those is to use induction gaps with the same accelerating voltage but slightly different deflecting mode frequencies; Landau damping of BBU can be expected because of dephasing by the frequency spread. The other is to introduce a sufficient spread in the betatron number caused by nonlinearity as seen in the ion focusing regime[9]. The latter has been proposed in Ref. 10 where a possibility of ion channel guiding is theoretically anticipated.

The present theory is general for the beam break-up instability in a frequency modulated system. For instance, the present conclusion can be applied to the case of a relativistic klystron[11] (RK) which also is motivated by its use in a two-beam accelerator, if it is driven with a low energy. Unlike a steady-state free electron laser, however, k_β/ν in a relativistic klystron is proportional to $\gamma^{1/2}$; therefore, Landau damping will be expected when an RK is operated with a sufficient large γ. $L_{BBU}(\sigma)$ and $L_R(\sigma)$ in such a case must be analytically derived by solving the original dispersion relation or obtained by computer simulations. However, both are out of the present scope.

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Appendix

The ratio of the betatron frequency to the synchrotron is described by

$$\frac{k_\beta}{\nu}=\frac{1}{2}\sqrt{\frac{cB_w}{E_s}}\left(1+\frac{\lambda_w\lambda_s}{16b^2}\right)$$

where c is the velocity of light, B_w and E_s are the magnetic and signal field amplitudes, respectively, λ_w and λ_s are the wiggler period and the signal wave length in vacuum, respectively, and b the vertical dimension of waveguide. The second term on the right-hand side is normally much smaller than unity. The power density requirement (~ GW/m) in the proposed scheme[12] where permanent wiggler magnets with the nominal surface field of ~1 Tesla are employed yield $0<k_\beta/\nu<1/2$.