CALCULATION OF CAVITY/WAVE GUIDE COUPLING*

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I. Introduction

The excitation of an electromagnetic mode in a cavity usually involves feeding energy from a power source through a wave guide into the cavity. The analytic solution of Maxwell's equations in the coupling region between the cavity and the wave guide is usually not feasible. Instead, the coupling is usually described by one or two parameters which can be measured. In this paper we describe how cavity codes can be used to determine these parameters for an arbitrary coupling geometry.

II. Wave Guide Impedance

We follow Slater's formalism[1] for the analysis of cavity modes. Our configuration is shown schematically in Fig. 1, where the cavity is joined to a wave guide.

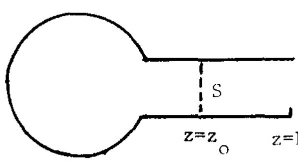

Figure 1

In particular, we will expand the fields into a complete set of modes $\vec{E}_m(\vec{x})$, $\vec{H}_m(\vec{x})$ which are orthonormal in the cavity/wave guide region bounded by the metallic wall and the surface S located at $z = z_o$. We will assume that only one mode, p, can be propagated in the wave guide, and choose the location of S to be far enough into the wave guide so that all other guide modes are negligible at S. The fields $\vec{E}_m$ and $\vec{H}_m$ satisfy

$$\nabla \times \vec{E}_m = k_m \vec{H}_m \quad , \quad \nabla \times \vec{H}_m = k_m \vec{E}_m \quad , \tag{1}$$

$$\vec{n} \times \vec{E}_m = 0 \text{ on walls and S.}$$

The actual fields $\vec{E}, \vec{H}$ (we take the time dependence to be exp $j\omega t$), which satisfy Maxwell's equations:

$$\nabla \times \vec{E} = -jk\, Z_o \vec{H} \quad , \quad \nabla \times Z_o \vec{H} = jk\, \vec{E} \tag{2}$$

are expanded in terms of $\vec{E}_m, \vec{H}_m$ according to

$$\left.\begin{aligned} &\vec{E} = \sum V_m \vec{E}_m \ , \ \vec{H} = \sum I_m \vec{H}_m \ , \\ &V_m = \int dv\, \vec{E}\cdot\vec{E}_m \ , \quad I_m = \int dv\, \vec{H}\cdot\vec{H}_m . \end{aligned}\right\} \tag{3}$$

If we multiply Eq. (2) by $\vec{H}_m, \vec{E}_m$ and integrate over the cavity volume, it is straightforward to show that

$$Z_o k_m I_m = jk\, V_m \quad , \tag{4}$$

$$k_m V_m + \int_S dS\, \vec{n}\cdot\vec{E} \times \vec{H}_m = -jk\, Z_o I_m \quad , \tag{5}$$

$$Z_o I_m = \frac{jk}{k^2 - k_m^2} \int_S dS\, \vec{n}\cdot\vec{E} \times \vec{H}_m \ , \tag{6}$$

where the integral is Eq. (6) is carried out only over the surface S, and where $\vec{n}$ is the outward normal to S.

Our immediate objective is to calculate the impedance of the cavity/wave guide combination at the surface S. To do this we recognize that the fields $\vec{E}_m$ and $\vec{H}_m$ will be proportional to $\vec{e}_p(x,y)$, $\vec{h}_p(x,y)$, the normalized transverse fields for mode p in the wave guide. Specifically we write within the guide:

$$\left.\begin{aligned} &\vec{E}_m^{\perp} = v_{mp}\, \vec{e}_p(x,y) \sin \beta_p (z - z_o) \\ &\vec{H}_m^{\perp} = Z_o \frac{v_{mp}}{Z_p} \vec{h}_p(x,y) \cos \beta_p (z - z_o) \end{aligned}\right\} \tag{7}$$

where v_{mp} is the coefficient to be determined by the solution of Maxwell's equations in the coupling region, and $z = z_o$ is the axial location of the surface S. The propagation constant in the guide is β_p and Z_p is the characteristic impedance of the wave guide mode p at the frequency $k_m c$. On the surface S, we write

$$\vec{E}_S = V\, \vec{e}_p \quad , \quad \vec{H}_S = I\, \vec{h}_p \quad . \tag{8}$$

Equations (3), (6) and (7) allow us to write

$$Z_o I = \sum_m \frac{Z_o}{Z_p} I_m v_{mp} = jk\, V \sum_m \frac{v_{mp}^2}{k^2 - k_m^2} \cdot \frac{Z_o^2}{Z_p^2} \ . \tag{9}$$

From Eq. (9) we obtain for the admittance of the cavity/wave guide combination as a function of frequency

$$Z_o\, Y(k) = jk \sum_m \frac{v_{mp}^2}{k^2 - k_m^2} \cdot \frac{Z_o^2}{Z_p^2} \ , \tag{10}$$

where we need to change the sign if the wish to view the impedance or admittance looking <u>toward</u> the cavity.

III. Definition of Q_{ext}

If the wave guide is matched to its characteristic impedance, Z_p, there will be only an outgoing wave in the wave guide. In this case the "resonant" frequency, k, will correspond to modes which decay due to the power flow out through the wave guide. The corresponding Q_{ext} can be obtained by assuming the dominance of a single term in Eq. (10), with $Y(k) = 1/Z_p$ and is

$$Q_{ext}^{-1} = \frac{\text{Im } k}{k} = \frac{v_{mp}^2 \, Z_o}{k_m \, Z_p} \quad . \qquad (11)$$

We therefore can rewrite Eq. (10) as

$$Z_p \, Y(k) \cong \frac{j}{Q_{ext}} \cdot \frac{k \, k_m}{k^2 - k_m^2 - jkk_m/Q_c} \quad , \qquad (12)$$

where the cavity losses have been included by the addition of the term involving Q_c. In fact the equivalent Q of the cavity/wave guide combination is readily seen from Eq. (12) to be

$$1/Q_{equiv} = 1/Q_c + 1/Q_{ext} \quad . \qquad (13)$$

In writing Eq. (12) we have discarded all non-resonant terms in the sum over m in Eq. (10). This will actually be correct if we choose a particular location, which has not otherwise been specified up to this point, for the surface S.

IV. Numerical Determination of Q_{ext}

If we terminate the wave guide by a metallic wall at z = L, the resonance condition for $Q_c \to \infty$ becomes

$$\cot \beta_p L = \sum_m \frac{k \, k_m}{k^2 - k_m^2} Q_{ext}^{-1} \quad . \qquad (14)$$

Here

$$\beta_p^2 = k^2 - \gamma_p^2 \quad , \qquad (15)$$

where γ_p is a parameter depending only on the geometry of the guide. With the metallic wall one can use existing computer programs like SUPERFISH[2], URMEL[3], and MAFIA[4] to obtain k as a function of L.

If we plot L against $\lambda_p \equiv 2\pi/\beta_p$, we expect a dependence of the form shown in Fig. 2.

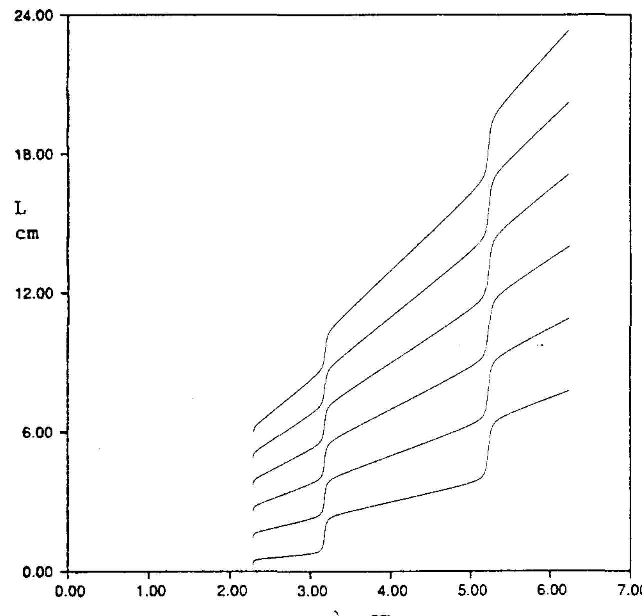

Figure 2

In fact, Fig. 2 has been obtained from an analytic treatment of the cavity/coax wave guide shown in Fig. 3.

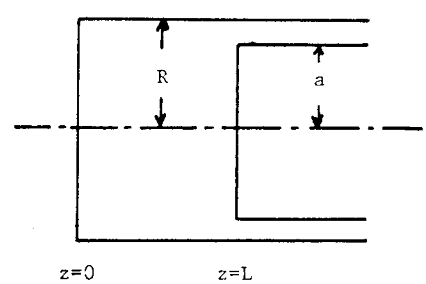

Figure 3

Cavity modes for $0 \le z \le L$ are matched to wave guide modes (j) in the coax region, and the impedance is obtained from a J x J determinantal equation, where J is the number of coax modes used. We have done the analysis for R = 2 cm, a = 1.96 cm, L = 2 cm and find good convergence with J = 2. The straight lines $L = n \lambda_p/2$ represent essentially wave guide excitations and the transitions from one value of n to an adjacent one take place as one goes through the resonances at $k = k_m$, as predicted by Eq. (14).

The values of Q_{ext} for each m are determined by the slope of the curves in Fig. 2 as the resonance curves cross the open circuit lines corresponding to $L = (2n + 1)\ \lambda_p/4$. The resulting value of Q_{ext} for othe TM_{010} like mode in the cavity in Fig. 3 is $Q_{ext} \cong 52$.

In Fig. 4 we show an expanded view of the TM_{010} mode region of Fig. 2 as well as the points obtained by using SUPERFISH as described above. Because SUPERFISH only gives azimuthally symmetric modes we can only obtain Q_{ext} for such modes. The location of the surface S which causes all non-resonant terms in Eq. (14) to add to zero is determined as the value of L obtained by the extrapolation of the wave guide resonance lines to $\lambda_p = 0$. In the present case this yields $z_o \cong -.06$ cm.

V. References

* Work Supported by the Department of Energy.

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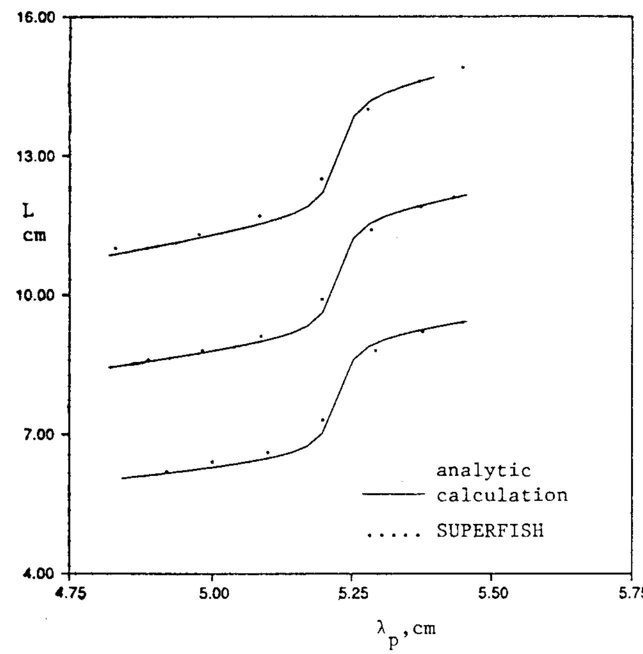

Figure 4