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CAVITY-WAVEGUIDE COUPLING THROUGH A LARGE APERTURE*

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Abstract

In designing a high-power, single-feed rf-cavity, one needs to consider the field disturbance caused by the large coupling aperture between the cavity and the waveguide. In this paper, we present an approach for studying this kind of field disturbance for modes in the frequency domain by using the three-dimensional code MAFIA. We demonstrate that by judiciously choosing the boundary conditions at the appropriate location along the waveguide, one can model the cavity-waveguide system by a closed boundary system and reduce the computation time significantly.

Introduction

In this paper we shall present the results of our preliminary investigations on the field distribution of the fundamental mode of an rf cavity coupled to a waveguide through a large hole. Our study is concentrated on the field asymmetry due to the coupling hole. The concern about the field asymmetry results from the fact that the asymmetric field may deflect the accelerated particle beam and lead to emittance growth.

The coupled cavity-waveguide system has been discussed by many authors using various methods.[1] The results presented in this paper are based on calculations using the recently developed MAFIA[2] computer codes. The theoretical background and approach will be discussed in the next section. Then the procedures and the results of case studies will be presented. For simplicity, all boundaries are assumed to be perfect conductors, and only rectangular or square coupling holes are considered.

The Approach - A Coupled-Cavity Model

In using the three-dimensional MAFIA codes for frequency-domain eigenmode analysis of rf systems, one must take into account the fact that the algorithmic formulation of the problem assumes that the electric and magnetic fields are everywhere in time quadrature, so that neither losses nor energy transport can be treated in the numerical analysis. Thus, in using these codes to analyze the cavity-waveguide coupling problem, we have chosen to analyze a coupled-cavity system that allows for the superposition of normal modes that will give energy flow from the waveguide-like cavity into the accelerator cavity. This flow will essentially empty the waveguide portion of energy if the cavity parameters are properly chosen (see below) and thus represents a realistic cavity-waveguide field configuration. The field asymmetry study will be based on the field pattern at the time when the stored energy is almost all in the cavity of the cavity-waveguide system. We believe this can be a good approximation for a high-Q cavity driven through a waveguide at the frequency of the fundamental mode. To understand the behavior of the energy stored in the coupled-cavity system, we used the following formalism based on the coupled-equivalent circuits.

Consider two series resonant RLC circuits, the first with resistor, inductor, and capacitor values R_1, L_1, and C_1, respectively, and the second with values R_2, L_2, and C_2. We assume the magnetic flux of the two inductors is linked so that a mutual inductance $M = k\sqrt{L_1 L_2}$ exists, where k is the coupling constant, ≤ 1. We also assume the second circuit is connected to a voltage source $v_g(t)$, and the currents in the first and the second circuits are $i_1(t)$ and $i_2(t)$, respectively. Using the complex notations $i_1(t) = \Re\left\{I_1 e^{j\omega t}\right\}$, $i_2(t) = \Re\left\{I_2 e^{j\omega t}\right\}$, and $v_g(t) = \Re\left\{V_g e^{j\omega t}\right\}$, where I_1, I_2, and V_g are possibly complex numbers, and $\Re$ means real part, we can write the loop equations for the two circuits as

$$\left(R_1 + j\omega L_1 + \frac{1}{j\omega C_1}\right) I_1 - j\omega M I_2 = 0 \quad , \tag{1}$$

and

$$-j\omega M I_1 + \left(R_2 + j\omega L_2 + \frac{1}{j\omega C_2}\right) I_2 = V_g \quad . \tag{2}$$

If the frequency ω is the only variable and if we impose the conditions that the power delivered to the first circuit and the ratio I_1/I_2 be maximal, we find that

$$\omega = \omega_0 = \omega_1 = \omega_2 \quad , \tag{3}$$

where $\omega_1 = \sqrt{1/L_1 C_1}$ and $\omega_2 = \sqrt{1/L_2 C_2}$ are the resonant frequencies of the uncoupled circuits. Thus we have seen that if the cavity and the waveguide are modeled as resonant circuits, then the conditions in Eq. (3) ensure that the power delivered to the cavity will be maximal and the dissipation in the waveguide will be minimal. We assume these relations are still true when R_1, R_2, and V_g approach zero.

We now prove that at the limits of $R_1 = R_2 = 0$ and $V_g = 0$ the conditions in Eq. (3) imply that if the system is initially energized with all the energy in one of the resonant circuits, then sometime later all the energy will be in the other circuit. We will show that under the conditions in Eq. (3) the current ratio I_2/I_1 will have the same magnitude but different signs for the two eigenmodes with frequencies near ω_0.

In the case of zero resistance and driving voltage, Eqs. (1) and (2) reduce to

$$\left(j\omega L_1 + \frac{1}{j\omega C_1}\right) I_1 - j\omega M I_2 = 0 \quad , \tag{4}$$

and

$$-j\omega M I_1 + \left(j\omega L_2 + \frac{1}{j\omega C_2}\right) I_2 = 0 \quad . \tag{5}$$

The above equations can have nontrivial solutions only when the system determinant vanishes, i.e.,

$$\begin{vmatrix} (1-\omega^2/\omega_1^2) & -\omega^2 M C_1 \\ -\omega^2 M C_2 & (1-\omega^2/\omega_2^2) \end{vmatrix} = 0 \quad . \tag{6}$$

The determinant yields a quadratic equation in ω^2 (using the relation $M^2 C_1 C_2 = k^2/\omega_1^2\omega_2^2$):

$$1 - \left(\frac{1}{\omega_1^2} + \frac{1}{\omega_2^2}\right)\omega^2 + \frac{(1-k^2)}{\omega_1^2\omega_2^2}\omega^4 = 0 \quad . \tag{7}$$

This quadratic equation has two roots; let us denote them as ω_+^2 and ω_-^2. By the usual properties of quadratic equations, we see that the sum of these roots $\omega_+^2 + \omega_-^2 = (\omega_1^2 + \omega_2^2)/(1-k^2)$ and the product of the roots $\omega_+^2\omega_-^2 = \omega_1^2\omega_2^2/(1-k^2)$.

Before proceeding to solve for the roots of the quadratic equation [see Eq. (7)], a discussion of the properties of the solutions is in order. The angular frequencies ω_+ and ω_- are the normal frequencies of oscillation of the coupled system. As will subsequently be seen, if the coupled system is oscillating at the frequency ω_+, the currents in the two resonant circuits will be 180° out of phase; when oscillating at the frequency ω_-, the currents will be in phase. Any arbitrary undriven oscillation of the coupled system can be written as a linear superposition of these two normal modes of oscillation. For the application we have in mind, we would like to be able to energize the system so that, initially, all the energy is in one of the resonant circuits and,

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at some later time, all the energy is in the other circuit. This desire places a requirement (to be demonstrated later) on the normal mode solutions, namely, that if the ratio of the currents is

$$I_2/I_1 = -\alpha \quad , \tag{8}$$

when oscillating at frequency ω_+, then we must have the ratio

$$I_2/I_1 = +\alpha \quad , \tag{9}$$

when the system is oscillating at ω_-.

Let us proceed to solve for the current ratio when the frequency is ω_+ by solving Eq. (4):

$$\left.\frac{I_2}{I_1}\right|_{\omega_+} = \frac{1}{\omega_+^2 M C_1}\left(1 - \frac{\omega_+^2}{\omega_1^2}\right) \quad . \tag{10}$$

If we now solve for the ratio I_2/I_1 when the system is oscillating at frequency ω_-, by solving Eq. (5) we obtain

$$\left.\frac{I_2}{I_1}\right|_{\omega_-} = \frac{\omega_-^2 M C_2}{(1 - \omega_-^2/\omega_2^2)} \quad . \tag{11}$$

Setting the right-hand sides of these last two equations equal to the negative of one another yields the equation

$$\left(1 - \frac{\omega_+^2}{\omega_1^2}\right)\left(1 - \frac{\omega_-^2}{\omega_2^2}\right) = -\omega_+^2\omega_-^2 M^2 C_1 C_2 = -\frac{k^2\omega_+^2\omega_-^2}{\omega_1^2\omega_2^2} \quad . \tag{12}$$

By using the relation for the product of the roots of the quadratic determinantal equation, this equation simplifies to

$$1 - \left(\frac{\omega_+^2}{\omega_1^2} + \frac{\omega_-^2}{\omega_2^2}\right) + \frac{(1+k^2)}{(1-k^2)} = 0 \quad . \tag{13}$$

It may readily be seen that if $\omega_1^2 = \omega_2^2$, then, by using the relation for the sum of the roots of the determinantal quadratic equation, this equation is satisfied. Thus our application requires that the two resonant circuits must have the same frequency when uncoupled.

Setting $\omega_1 = \omega_2 = \omega_0$ in the determinantal equation, one has

$$1 - 2\frac{\omega^2}{\omega_0^2} + (1 - k^2)\frac{\omega^4}{\omega_0^4} = 0 \quad , \tag{14}$$

which has the solutions

$$\omega_+^2 = \omega_0^2\frac{1}{1-k} \quad , \tag{15}$$

and

$$\omega_-^2 = \omega_0^2\frac{1}{1+k} \quad . \tag{16}$$

Substituting Eq. (15) into Eq. (10) gives, after some manipulation,

$$\left.\frac{I_2}{I_1}\right|_{\omega_+} = -\sqrt{\frac{L_1}{L_2}} \quad , \tag{17}$$

whereas for ω_- this ratio has the opposite sign, as desired.

Procedures and Results

Based on the theory formulated in the last section, the following procedures have been pursued:

1. The initial MAFIA run is made on the geometry of a circular cavity coupled to a rectangular cavity through a coupling hole. The rectangular cavity is formed by terminating the waveguide at one-half or at one full wavelength of the waveguide mode (the TE_{110} mode) which has the same frequency, ω_0, as the fundamental mode of the circular cavity (the TM_{010} mode). In the results, there are two modes having frequencies near ω_0, the ω_+ and the ω_- modes, as expected. Attention is focused on these two modes.
2. In the subsequent runs, we vary the length of the rectangular cavity until the ratios of the maximal electric fields in the two cavities are almost equal for the ω_+ and the ω_- modes.
3. The vector sum of the fields of these two modes is taken in such a way as to cancel the fields in the rectangular cavity. This sum is then taken as the desired field configuration for asymmetry study. We investigated the fields on the planes perpendicular to the symmetry axis of the cavity. By interpolating the magnetic field over the mesh, one can obtain the distance between the field center, where the magnetic field is zero, and the symmetry axis of the cavity. A particular one of these distances, namely, that in the plane passing transversely through the center of the cavity, is referred to as the *field-center offset* in the following and is used to characterize the field asymmetry. For practical interest, the ratio between the magnetic field at the cavity center and the maximal magnetic field in the cavity, called the *magnetic field ratio* in the following, is also presented.

Following the procedures outlined above, we have studied three types of cavities. The results are summarized below.

Pillbox Cavity

Figure 1 shows one of the simplest cases of the coupled cavity-waveguide system, in which a pillbox cavity is coupled with a rectangular waveguide through a rectangular aperture. For simplicity, we assume there is no beam hole in the circular cavity.

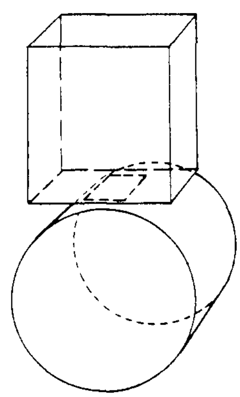

Fig. 1. Diagram of a pillbox cavity coupled to a waveguide.

For this kind of geometry, we concentrate on a circular cavity with a radius of 25 cm and an axial length of 32 cm. The waveguide has a cross section of 40 cm by 20 cm, with the short side parallel to the symmetry axis of the circular cavity. The frequency of the fundamental mode of the closed circular cavity is 456 MHz.

Figures 2 and 3 show the magnetic fields of the two eigenmodes with frequencies near the frequency of the fundamental mode of the circular cavity. In this case, we have chosen to terminate the waveguide near one full wavelength of the waveguide mode. In order to show the field asymmetry, a quite large square coupling hole of 20 cm by 20 cm is considered. The result of combining these two modes is shown in Fig. 4. Figure 5 shows

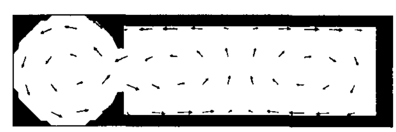

Fig. 2. The magnetic field of the ω_- mode.

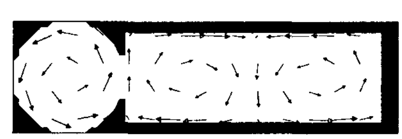

Fig. 3. The magnetic field of the ω_+ mode.

Fig. 4. The vector sum of the magnetic fields in Figs. 2 and 3.

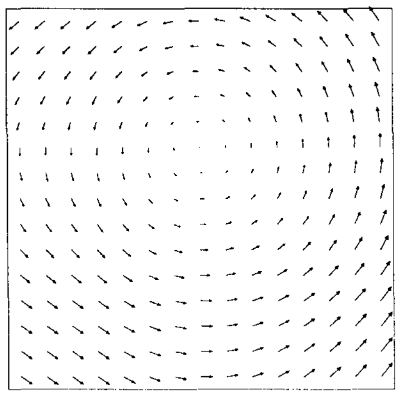

Fig. 5. A close-up of the magnetic field in Fig. 4 near the cavity center shows the field asymmetry. The cavity center is at the center of the picture.

the magnetic field around the center of the cavity, where the cavity center is set at the center of the picture so that the field off-centering can be seen easily. The field-center offset is about 6.7 mm and the field ratio is about 5% in this case.

Boeing Cavity

The Boeing design of the cavity-waveguide coupling is shown in Fig. 6. The rectangular coupling slot is 9.4 cm by 22.1 cm. The MAFIA results show a field-center offset around 0.7 mm, and the field ratio is about 0.5%. The experimentally measured value is about 0.5 mm, in good agreement with our present calculation.[3,4]

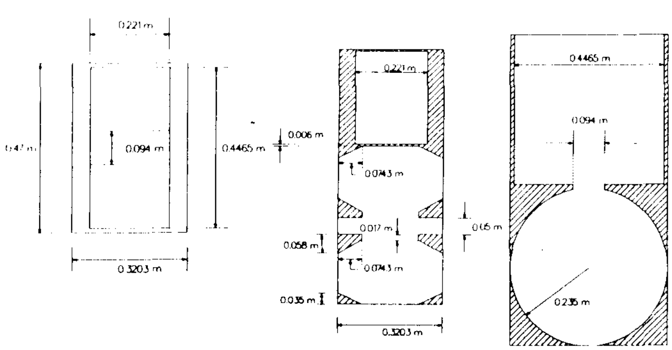

Fig. 6. Diagram of the Boeing cavity and waveguide.

Elliptical Cavity

K. C. D. Chan[5] has suggested that for FEL applications an elliptical cavity (see Fig. 7) may be more favorable for high-quality beam production. Assuming the same coupling hole as in the Boeing cavity, we also have the results of 0.7 mm for the field-center offset and 0.5% for the field ratio; these results are comparable to those of the Boeing cavity.

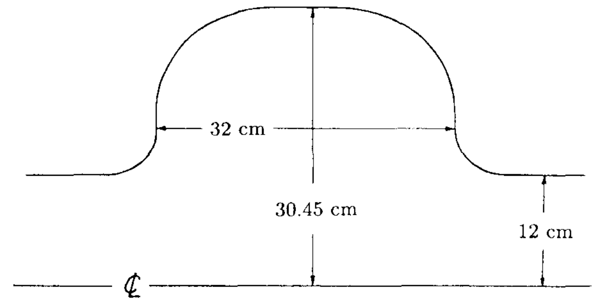

Fig. 7. The profile of the elliptical cavity.

Conclusions and Discussions

The field asymmetry caused by a large coupling hole between a cavity and a waveguide has been examined for the fundamental modes of a pillbox cavity, the Boeing cavity, and an elliptical cavity. In the case of the Boeing cavity, our calculation agrees very well with the experimental measurements.

The assumption that the stored energy in the two coupled cavities can oscillate completely from one cavity to the other requires a location of the short of the waveguide such that the field ratios between the two cavities are equal for the two modes near the fundamental mode of the cavity. In general, one finds that the terminating plane is near the integer multiples of the half wavelength of the waveguide mode, which has the same frequency as that of the closed cavity. We have made several runs with different locations of terminating planes to achieve the equal field ratios. After examining the results from different runs, we found that the equal field ratio does not seem to be a necessary condition. We found that the result obtained under the equal field ratio condition can also be obtained by terminating the waveguide at any location near the integer multiples of the half wavelength of the waveguide mode and normalizing the maximal electric fields in the rectangular cavity to the same value before the vector sum of the fields from the ω_+ and ω_- modes is taken.

Acknowledgment

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