REQUIREMENTS FOR LONGITUDINAL HOM DAMPING IN SUPERCONDUCTING RECIRCULATING LINACS*

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Abstract

Transverse beam breakup[1] provides the primary current limitation in the operation of superconducting recirculating linacs and requires the significant damping of transverse-deflecting higher order modes. The need to damp the coexisting longitudinal HOMs in these nominally isochronous machines, however, is not as clear. Isochronicity implies that energy variations induced by excitation of longitudinal modes do not translate directly into position and current modulations. Such modulations, if present, could enhance the initial excitation, effectively closing a potentially unstable feedback loop. Design optimization of cavity structures may suggest that no longitudinal damping be provided. On the other hand, easing of the isochronicity requirement may provide desired flexibility in lattice design. In this note, limits are placed on the requirements for longitudinal HOM damping and on the tolerances for isochronicity which are driven by possible longitudinal multipass phenomena.

Physical Motivation

Consider a series of bunches passing through a single accelerating cavity in a two-pass, recirculating linac, the simplest multipass configuration. If the bunches are equally spaced, they will produce a current at harmonics of the bunching frequency out to the rolloff of the bunch spectrum. If the positions of the bunches, however, are modulated at some small amplitude, side bands will form in the current power spectrum.

Suppose there is is an initial excitation of some longitudinal higher order mode. Let the bunches enter the cavity on the first pass perfectly spaced. On exiting the cavity, the energy of the bunches will be modulated by the mode. If the isochronicity of the recirculation optics on the second pass through the cavity is not perfect, the energy modulation will be translated into a spacing modulation. As discussed previously, this modulation will generate a side-band current whose magnitude is scaled by the magnitude of the perturbation, and whose frequency matches (with sampling aliasing) that of the exciting HOM. Thus, on the second pass, the resulting current can enhance the excitation of the HOM that created it. A feedback loop is formed which is analogous to that which generates multipass transverse beam breakup. The threshold condition for instability is met when an excitation produces, through the induced current generated, a self-enhancement which matches the original cavity excitation. One significant difference, however, is that the induced current can only achieve a value equal to the average beam current, and saturation will occur.

In the following, a model is presented of this instability for a simple one-cavity, two-pass configuration, and limits are placed on the necessary damping for longitudinal higher order modes. Comparisons to bunch-by-bunch simulations are presented, and saturation effects are discussed.

Current Spectrum of a Modulated Bunched Beam

Consider a sequence of bunches of charge q spaced in time at $t_b = \frac{2\pi}{\omega_b}$. At a reference point, the current is of the form (disregarding the finite bunch length)

$$I(t) = q\sum_k \delta(t - kt_b) \qquad (1)$$

(all sums from $-\infty$ to $+\infty$)

On performing a Fourier decomposition we have

$$I(t) = \frac{q}{t_b}\sum_n e^{in\omega_b t} \qquad (2)$$

and the result that a uniform sequence of point bunches produces a signal at all harmonics of the bunching frequency.

Now consider a sequence of bunches whose arrival time has a small amplitude modulation at a frequency $\nu\omega_b$. The current will now be

$$I(t) = q\sum_k \delta\big(t - t_o - kt_b - \Delta\sin(\nu\omega_b kt_b + \phi)\big) \qquad (3)$$

where we have allowed for an arbitrary time delay in average arrival of t_0 and an arbitrary phase ϕ of the perturbation. (The choice of the argument $\nu\omega_b kt_b$ rather than $\nu\omega_b t$ will become apparent in later discussions.) The Fourier transformed current is given by

$$\tilde{I}(\omega) = \frac{q}{\sqrt{2\pi}}\sum_k \int_{-\infty}^{+\infty} dt e^{i\omega t} \times \delta\big(t - t_0 - kt_b - \Delta t\sin(\nu\omega_b kt_b + \phi)\big) \qquad (4)$$

or

$$\tilde{I}(\omega) = \frac{e}{\sqrt{2\pi}}\sum_k e^{i\omega t_0}\ e^{i\omega kt_b}\ e^{i\omega\Delta t\sin(2\pi\nu k+\phi)} \qquad (5)$$

where we have used $\omega_b t_b = 2\pi$. From the identity

$$e^{ix\sin y} = \sum_\mu J_\mu(x)\ e^{i\mu y} \qquad (6)$$

we have

$$\tilde{I}(\omega) = \frac{e}{\sqrt{2\pi}}\sum_k\sum_\mu e^{i\omega t_0}\ e^{i\omega kt_b} J_\mu(\omega\Delta t) e^{2\pi i\mu\nu k + i\mu\phi} \qquad (7)$$

and, on collecting terms,

$$\tilde{I}(\omega) = \frac{e}{\sqrt{2\pi}}\sum_k\sum_\mu e^{i(\omega t_0+\mu\phi)} J_\mu(\omega\Delta t) e^{ikt_b(\omega+\mu\nu\omega_b)} \qquad (8)$$

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Finally, it follows from the identity

$$\sum_n \delta(\omega - n\omega_0) = \frac{1}{\omega_0}\sum_n e^{i\frac{2\pi}{\omega_0}\omega n} \tag{9}$$

that

$$\tilde{I}(\omega) = \frac{e}{\sqrt{2\pi}}\sum_\mu\sum_n e^{i(\omega t_0 + \mu\phi)} J_\mu(\omega\Delta t)\omega_0\delta(\omega + \mu\nu\omega_b - n\omega_b) \tag{10}$$

Longitudinal HOM Impedance

Consider a charge q passing through a cavity at time $t = 0$, the Fourier current spectrum in this case is given by simply

$$\tilde{I}_p(\omega) = \frac{1}{\sqrt{2\pi}}q \tag{11}$$

The voltage $V(t)$ induced by traversal of this charge through a cavity is

$$V(t) = \frac{q\omega_r R}{2Q}\cos\omega_r t\; e^{-\frac{\omega_r}{2Q}t} \qquad (t > 0) \tag{12}$$

where R is the longitudinal shunt impedance and ω_r is the resonant frequency. The Fourier conjugate voltage $\tilde{V}(\omega)$ is

$$\tilde{V}(\omega) = -\frac{1}{\sqrt{2\pi}}q\frac{\omega_r R}{4Q}\left[\frac{1}{i\omega_r + i\omega - \frac{\omega_r}{2Q}} + \frac{1}{-i\omega_r + i\omega - \frac{\omega_r}{2Q}}\right] \tag{13}$$

The longitudinal impedance $Z(\omega)$ is defined by

$$Z(\omega) = \frac{\tilde{V}(\omega)}{\tilde{I}(\omega)} \tag{14}$$

and for this case is given by

$$Z(\omega) = \frac{-\omega_r R}{4Q}\left[\frac{1}{i\omega_r + i\omega - \frac{\omega_r}{2Q}} + \frac{1}{-i\omega_r + i\omega - \frac{\omega_r}{2Q}}\right] \tag{15}$$

Voltage Induced by a Modulated Current

Consider a sequence of equally spaced bunches injected into a two-pass, cw recirculating linac with a single accelerating cavity. Let there be an excitation $V(t)$ of a higher order mode of the linac cavity at frequency $\nu\omega_b$. As the bunches pass through the cavity their energy will be modulated at the HOM frequency. Although the lattice, which carries the bunches back to the cavity for a second pass, is ideally isochronous there may be a residual dependence of the recirculation time on bunch energy. Define the slip factor η by the relation

$$\Delta T = \eta t_0 \frac{\Delta E}{E} \tag{16}$$

where ΔT is the time offset of a bunch of energy offset $\frac{\Delta E}{E}$ for an on-energy recirculation time t_0 and first-pass energy (at the cavity) of E. The energy modulation varying as $\sin(\nu\omega_b t + \phi)$ induced on the first pass will through η cause a modulation of the arrival time of bunch m for the second pass of the form

$$t_m = mt_b + \Delta t\sin(\nu\omega_b m t_b + \phi) + t_0 \tag{17}$$

and will generate the current described in Equation (4), that is,

$$\tilde{I}(\omega) = \frac{e}{\sqrt{2\pi}}\sum_n e^{i\omega t_0}e^{i\omega n t_b}e^{i\omega\Delta t\sin(2\pi\nu n + \phi)} \tag{18}$$

This current, as described in the previous section, will induce a voltage given by

$$V(t) = -\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} d\omega e^{-i\omega t}\left[\frac{\omega_r R}{4Q}\right] \times \left[\frac{1}{i\omega_r + i\omega - \frac{\omega_r}{2Q}} + \frac{1}{-i\omega_r + i\omega - \frac{\omega_r}{2Q}}\right] \times \frac{q\omega_0}{\sqrt{2\pi}}\sum_\mu\sum_n e^{i(\omega t_0 + \mu\phi)}J_\mu(\omega\Delta t)\delta(\omega + \mu\nu\omega_b - n\omega_b) \tag{19}$$

After integration we have

$$V(t) = -\frac{I_0\omega_r R}{4Q}\sum_\mu\sum_n \times e^{i\left((n-\mu\nu)\omega_b(t_0 - t) + \mu\phi\right)} J_\mu\left((n-\mu\nu)\omega_b\Delta t\right) \times \left[\frac{1}{i\omega_r + i\omega_b(n-\mu\nu) - \frac{\omega_r}{2Q}} + \frac{1}{-i\omega_r + i\omega_b(n-\mu\nu) - \frac{\omega_r}{2Q}}\right] \tag{20}$$

Analysis of Longitudinal Multipass BBU

In the limit of a small coherent modulation of the bunching frequency, the J_0 and the J_1 terms of the expansion will dominate. The J_0 term to lowest order is independent of the amplitude of modulation and describes simple energy loss to the higher order mode. Since it does not provide feedback with respect to the modulation amplitude it will not at this level of approximation contribute to a possible instability. The $J_{\pm 1}$ terms, on the other hand, do provide such a feedback mechanism.

Define the tuning angle ψ_n of the HOM by the relation

$$\tan\psi_n = \frac{\left(\omega_r - \omega_b(n-\nu)\right)}{(\omega_r/2Q)} \tag{21}$$

Then the resonant denominators may be reexpressed by, for example,

$$-\frac{1}{i\left(\omega_r - \omega_b(n-\nu)\right) + \frac{\omega_r}{2Q}} = \frac{1}{1 + i\tan\psi_n}\frac{2Q}{\omega_r} = e^{-i\psi_n}\cos\psi_n\frac{2Q}{\omega_r} \tag{22}$$

With this definition $V(t)$ at bunch-crossing times mt_b is given by

$$V(mt_b) = \sum_n I_0 R J_1\left((n-\nu)\omega_b\Delta t\right)\cos\psi_n \times \left[\cos\left((n-\nu)\omega_b t_0 + (\phi - \psi_n) - (n-\nu)\omega_b m t_b\right)\right] \tag{23}$$

where the $\pm$ symmetries of the μ and n sums have been invoked.

For a narrow resonance, one particular term such that $|n - \nu|\omega_b \approx \omega_r$ will dominate. Using the approximation $J_1(x) = x/2$ for small x, and Equation (16), and keeping the dominant term, we have that the modulation ΔT induced by the excited voltage is

$$\Delta T_{\text{induced}} = \cos\psi_n \frac{\eta t_0}{E_0}\frac{I_0 R}{2}(n-\nu)\omega_b \Delta t \times \cos\big((n-\nu)\omega_b t_0 + (\phi - \psi_n) + (n-\nu)\omega_b m t_0\big) \tag{24}$$

However, the initial perturbation is given by

$$\Delta T_{\text{perturb}} = \Delta t \sin(\nu\omega_b m t_b + \phi) \tag{25}$$

The condition for a self-generating modulation is

$$\Delta T_{\text{induced}} = \Delta T_{\text{perturb}} \tag{26}$$

This relation yields the following conditions for the the threshold of coherent motion:

$$\cos\ \psi_n \frac{\eta t_0 I_0 R}{2E_0}(n-\nu)\omega_b = 1 \tag{27}$$

and

$$\frac{\pi}{2} = (n-\nu)\omega_b t_0 - \psi_n \tag{28}$$

A worst-case estimate of the threshold current is obtained under the assumption that $|cos\psi_n| = 1$; that is, when $|n-\nu|$ $\omega_b = \omega_r$. Consequently, the minimum threshold current I_{th} for longitudinal multipass beam breakup is given by

$$I_{th} = \frac{2E_0}{\eta R \omega_r t_0} \tag{29}$$

Comparison with Computer Simulations

To verify the analytic threshold conditions derived, computer simulations have been undertaken which model the longitudinal dynamics of linac bunches in the presence of higher order cavity modes. In the time domain, point bunches are allowed to excite a single HOM on both the first and second passes through the linac. Energy modulation induced on the first pass is translated into a return-time modulation for the second pass.

As the beam current is varied, the HOM excitation (stored energy) exhibits clear threshold behavior (Figure 1). For this

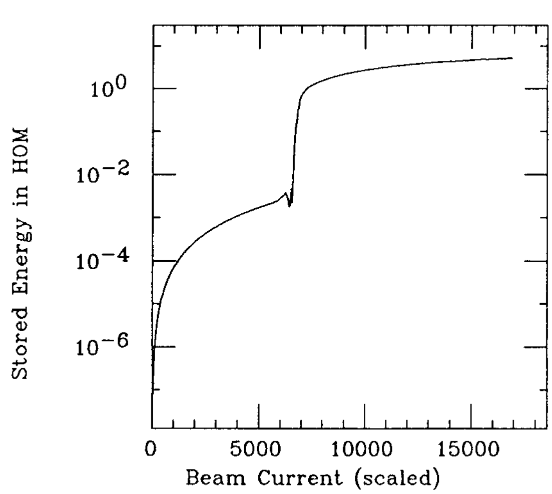

Figure 1 Threshold Behavior for Longitudinal Multipass Beam Breakup.

example, the resonant frequency ω_r is chosen to satisfy the worst-case condition, which through Equation (28) implies that

$$\omega_r = (n + \frac{1}{4})\,\omega_0 \tag{30}$$

where $\omega_0 = 2\pi/t_0$ and n is any integer. Below threshold, the level of excitation is consistent with that driven only by the current generated by the unperturbed sequence of bunches. Above threshold, the level of excitation is enhanced to a value which is bounded by that generated by a sequence of bunches spaced harmonically with respect to the HOM frequency. It is this saturation behavior which distinguishes longitudinal beam breakup from its transverse counterpart.

Damping Requirements for HOM

From a single-particle dynamics point of view, the isochronicity condition (independence of recirculation time with energy error) in a recirculating linac needs to be satisfied only to a fraction of the bunch length, and, in fact, a tighter constraint may conflict with other beam optics goals. In any case, because of the small velocity variation during acceleration, perfect isochronicity cannot be achieved in practice. For example, consider the CEBAF recirculating linac[2] with 45 MeV injection energy and two 400 MeV linac segments connected by recirculation arcs. The passage through the first linac segment yields a slip factor η of the order of 10^{-6} from the point of injection to return to the same point on the second pass. If the recirculation arc is set to cancel this error, the arcs will overcorrect the slip from the endpoint of the first linac segment to the same endpoint on the second pass since for this trajectory—from 445 MeV to 1245 MeV—the motion is more fully relativistic. Later passes, at higher energy, will suffer considerably less from this effect. It will be the case that the limiting Qs estimated will be sufficiently high that cavity-to-cavity differences in the frequencies of higher order modes will make each cavity act independently. Thus the simple model of a single cavity discussed in this note provides a reasonable first approximation. For the CEBAF configuration, with the assumption of isochronous arcs (i.e., slip from the transport along linac segments only) one finds a Q limit of 4×10^{10} after two passes. For more than one recirculation, the dominant slippage will remain from the first pass to second pass. Therefore, the exciting current will scale approximately linearly with recirculation number. For four recirculations (five passes), the threshold quality factor would diminish to $Q \cong 10^{10}$. Residual lattice errors may further reduce this value by an order of magnitude. Although typical undamped Qs are at the 10^9 level, the strongest are damped to the 10^4 level and the effect appears to be negligible. However, the limit is close enough to undamped levels (especially for improved materials) that care must be taken before longitudinal mode damping can be disregarded in the design of superconducting cavities for application to recirculating linacs, especially at lower energies.

References

1. J. Bisognano and R. Gluckstern , 1987 Particle Accelerator Conference, IEEE 87CH2387-9, p. 1078.
2. H. Grunder, CEBAF Status Report, (this conference).