2.5D CAVITY CODE WITH HIGH ACCURACY

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Abstract

PISCES II calculates Eigensolutions of any axi-symmetric cavity with 2.5D Finite Element Method and can handle periodic boundary conditions. Dipole and higher multipole solutions are obtained by hybrid finite elements. The accuracy of the frequency in a solution obtained from PISCES II is improved by use of higher order elements. In order to reduce the computation time, the Eigenvalue solver is improved.

1 INTRODUCTION

An RF cavity code with high accuracy is necessary for design with high precision. There are still some needs of an axisymmetric cavity because of its simplicity.

PISCES II can calculate all Eigensolutions in a cavity with axisymmetric boundaries including multipole modes[1,2]. Periodic boundary conditions can also be handled in this code.

2 FORMULATION AND FINITE ELEMENT MODEL

Because either $\vec{E}$ or $\vec{H}$ can be used as the field variable, only the electric field will be shown here. The differential equations to be solved are [3,4,5],

$$\nabla \times \nabla \times \vec{E} + k^2 \vec{E} = \vec{0}, \quad \nabla \cdot \vec{E} = \vec{0} \qquad (in \quad \Omega), \tag{1}$$

where $k^2 = \omega^2 \varepsilon \mu$. In vacuum space $k^2 = \omega^2/c^2$, where c is the speed of light. Boundary conditions are

$$\vec{E} \cdot \vec{n} = 0 \qquad \text{on magnetic boundaries } (\Gamma m) \tag{2}$$
$$\vec{E} \times \vec{n} = \vec{0} \qquad \text{on electric boundaries } (\Gamma e) \text{ and} \tag{3}$$
$$\vec{E}_{right} = e^{i\varphi} \vec{E}_{left} \qquad \text{on periodic ones } (\Gamma p), \tag{4}$$

where $\vec{n}$ denotes the outward normal on the boundary, and φ is the phase advance in the problem .

Because only the problems on axisymmetric domains are considered, we can assume $\sin m\theta$ and $\cos m\theta$ dependencies for E_r, E_z and E_θ components, and then the problem can be reduced to two-dimensional problem:

$$\vec{E} = (E_\theta \sin m\theta, E_r \cos m\theta, E_z \cos m\theta). \tag{5}$$

Then (E_θ, E_r, E_z) are functions of r and z only. The field variables are (rE_θ, E_r, E_z) for $m \geq 1$ and (E_θ, H_θ) for $m=0$.

Using Finite Element Method, Eqs. (1-3) can be reduced to a matrix form of a general Eigenvalue equation:

$$\boldsymbol{M} \cdot \vec{x} = k^2 \boldsymbol{K} \cdot \vec{x}, \tag{6}$$

where $\boldsymbol{M}$ and $\boldsymbol{K}$ are large sparse symmetric matrices, and $\vec{x}$ is an Eigenvector[6]. Because of the hybrid elements, any spurious mode has zero-Eigenvalue and is well separated from the real modes. Usually several Eigensolutions starting from the smallest one but zero are of interest. Unfortunately, this Eigenvalue problem has many zero-Eigenvalue solutions, which correspond to the spurious modes, and thus special care should be taken. For axisymmetric solutions, such as TM0xx or TE0xx, the problem can be expressed by field variable of E_θ or H_θ, which has no zero Eigensolution.

3 PERIODIC BOUNDARY

The periodic boundary condition is implemented using Floquet's theorem[7]. Although $\vec{x}$ has to be a complex vector for this analysis, the final Eigenvalue problem to be solved can be rewritten as real and symmetric by splitting the component into real and imaginary part. Because of this technique, the Eigenvalue solver for the usual boundary conditions is used.

4 GENERAL EIGENVALUE SOLVER

Because Eq. (5) is a general Eigenvalue problem for large sparse symmetric matrix with many zero Eigenvalue solutions, special care had to be taken. The solver is based on the subspace method[8] and uses zero and upper filtering technique[9].

The zero filtering technique requires a rough estimate of the lowest Eigenfrequency(FLO), which can be obtained by the physical dimension of the problem. If FLO is not given by a user and solution with the second order element or multiple solutions are required, PISCES II evaluates FLO from a single mode solution with lowest order element starting with the "guess" value from the physical dimension.

Using similar technique, upper filter is also applied for acceleration of the convergence. If the upper frequency (FUP) are not given by a user, PISCES II uses the highest Eigenvalue in the subspace after it settled. In this case the FUP value is adjusted adaptively.

Because the method is based on the iterative method, initial vectors should be given at the beginning. For the problem with the lowest order element, the initial vectors are given by random numbers. Before starting with the second order element, the solution vectors are obtained from the lowest order element and "prolonged" to the

second order solutions by linear interpolation. Because fairly good initial solutions are obtained by this way and FUP is also available from the beginning, the solution time is reduced up to half comparing with direct start with the random vectors.

5 INPUTS TO PISCES II

A mesh data example for a sphere as shown in Fig. 1 is listed in Fig. 2. All the internal units are in SI. The first line contains a title with less than 80 characters. The second line is problem constants, where only the unit scale for cm is specified in the list. The options are explained later. The third line has three numbers, which are the number of nodes, the number of elements and the number of boundary points plus one for closure of the line. Node coordinates are specified being enclosed by brackets. Followings are the element data, which specifies the three coordinates of the vertices by the sequential numbers of the coordinate sets. The fourth positions are reserved. Then the boundary points are specified in the same way as above. Each boundary condition for a segment between n and n+1 boundary points is specified by a character constant. The encoding of the boundary
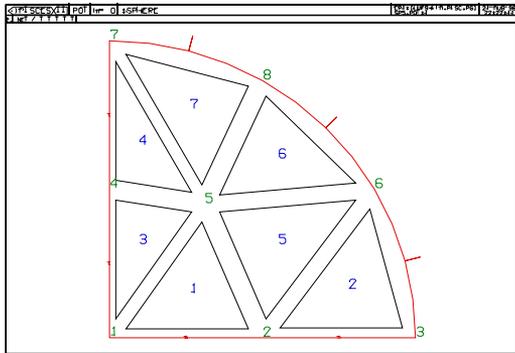

Fig. 1   Mesh example for a sphere.

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:SPHERE
 &PROBLEM   UNIT=0.01 &END
     8    7    8
(0,0)  (5,0)  (10,0)  (0,5)  (3.1,4.5)  (8.66025,5)  (0,10)
(5,8.66025)
1,2,5,,2,3,6,,4,1,5,,4,5,7,,5,2,6,,5,6,8,,7,5,8,,
   1,   2,   3,   6,   8,   7,   4,   1,
'A' 'A' 'E' 'E' 'E' 'E' 'E' 'A'
0 0 10 10 10 0 0  0
```

Fig. 2   Mesh data for a sphere with radius of 10 cm.

Table 1: The encoding of the boundary conditions

Code	Boundary condition
'A'	on axis (r=0)
'E''	on electric boundary (metal surface)
'M'	on magnetic boundary (symmetry plane)
'P'	left side boundary for periodic boundary
'Q'	right side boundary for periodic boundary

conditions are listed in Table 1. The last group is the curvature data which specifies the radii of the segments in the same sequence as the boundary conditions. Zero in radius should read as straight line. Each group is read by free format read statement. The mid-line points will be generated in the code if the second order elements are needed. For a periodic boundary problem, the radial coordinates of both the left and right side nodes should coincide.

MESHNET program[10] can convert a TAPE35 data that is generated by LATTICE[11] to an input file for PISCES II. Because the curvature information is not included in TAPE35 data, the curvature data has to be added by hand or by NETREF (NET REFine) program.

Table 2 explains the major problem constants and their default values. Table 3 shows the encoding of the element type. For example, LMTYP=22, the second order nodal elements are used for axisymmetric modes (m=0) while the mixed linear edge/nodal element are used for multipole modes (m>0). For the default setting of LTYP=1, both TE0xx and TM0xx are obtained simultaneously at m=0 so that any mode is not missed in the solution. Although such Eigenvalue problem can be separated into two independent problems, it is solved altogether.

Table 2: The problem constants

name	Default	description
OPTNF	T	The problem constants will be read again
EM	1.0	m in Eq. (5)
MSTEP	1	increment of m
MNUM	1	iterations
PHASE	0.0	phase advance
PHSTEP	0.0	increment of phase
NUMPH	1	iterations
BCLR	''	override boundary condition at left/right sides(2chars)
UNIT	1.	unit scale
CVF	T	enable curved boundary
FIELD	'E'	field variable E or H
LMTYP	1	element mode
NMODE	5	modes to be solved
FUP	0.0	highest frequency
FLO	0.0	lowest frequency
EPS	1E-5	accuracy
ITMX	200	iterations for subspace
MGF	T	enable two step solve
MCG	2	method of CGM
INFILE	''	file for initial value
OTFILE	''	output file

Table 3: The element mode encoding

LMTYP	0	1	2	10	11	12	20	21	22
LTYP (EM=0)	0	1	2	3	1	2	4	1	2
LTYP (EM>0)	I	0	0	0	3	3	3	4	4

LTYP=0 E/H with mixed constant edge/nodal element
LTYP=1 E&H with linear nodal element
LTYP=2 E/H with 2nd order nodal
LTYP=3 E/H with mixed linear edge/nodal element 12 params
LTYP=4 E/H with mixed linear edge/nodal element 14 params

6 EXAMPLE

Figs. 5 and 6 show the relative frequency errors and CPU time as functions of the number of unknowns for the hemisphere problem shown in Fig.1 (r=10cm). The CPU time depends on only the number of unknowns, although the number of nonzero elements in the matrices are about twice. The second order solutions give better accuracy with doubled memory requirement.
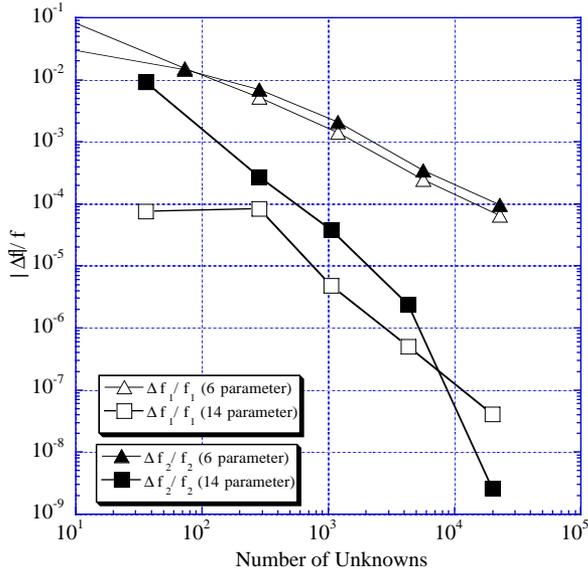

Fig. 5   Relative frequency errors of the second and the third lowest modes in a spherical cavity as a function of the number of unknowns.
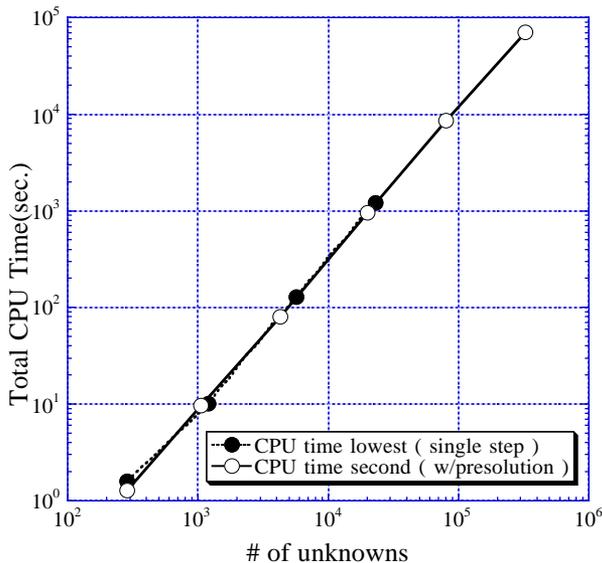

Fig. 6   CPU times as functions of the number of unknowns.

7 CONCLUDING REMARKS

After the improvement of initial vectors and parameters, the dipole problem with 324388 unknowns—23572 nodes, 46519 elements, 624 boundary points, 93662 point (including mid-line points generated internally)—takes about 20 hours for seven Eigensolutions. If FLO and FUP are given, the CPU time is reduced to about 70%. The most time consuming portion is the linear matrix solver. PCGM (Preconditioned Conjugate Gradient Method) with SOR as a preconditioner is currently used for the linear matrix solver. More efficient preconditioner will reduce the CPU time.

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