SIMULATIONS OF THE NONLINEAR TRANSVERSE RF FIELD EFFECTS ON THE BEAM DYNAMICS IN LOW ENERGY X-BAND SW LINACS*

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Abstract

The detailed beam dynamics simulations in a low energy X-band SW linacs are presented. The codes *CAV* and *TRACK* are developed. They are used to investigate beam dynamics in a model structure including the nonlinear transverse RF fields. Examples are given of the study of transverse effects in different cavity shapes and in different synchronous states. The results are compared and found to agree well with those from analyses.

1 INTRODUCTION

In ref.[1], we discuss the RF fielde nonlinear effects on the emittance dominated beam in low energy linacs analyticly. In low ernergy linacs, the initial phase region of captured particles is very large and usually in the decelerating region. From the simply theory, the nonlinear transverse effects on the beam is associate with the relative radial positon of particles, the nonlinear components of RF fields and the synchronous state between particles and RF fields. The RF nonlinear transvers effects on the beam and the emittance growth mainly occurs in the first cavity, especially over the decelearting phase region. These effects weaken quickly and vanish in the quasi-synchrous section and synchrous. In the symmetric π mode cavities, the nonlinear components of RF fields are least. But the nonlinear effects on the beam in an assymetric first cavity can be less than the beam with the same initial phase in a symmetric one.

A few formulas for longitudinal and transverse dynamics have been derived including bunching, accelerating and higher space harmonics. The results of the theory should be compared with the ones of simulation to test and verify this theory.

We mainly concentrate on the first cavity in simulation.

2 MODEL

A model linac structure is considered to compare the analysis of this structure by the methods in ref.[1] with the numerical simulation by *CAV* and *TRACK*. In order to amplify the effects of the nonlinear RF fields, a larger beam radius and RF power are chosen. This model structure opperates in *9300MHz* and consists of *8* cavities (shown in Fig. 1). The microwave power inputed in the structure is *1.1MW* and the initial *rms* normalized tranceverse emittance of the electon gun is *12.02mm.mrad*. The initial phase region of captured particles is from -150^0 to -30^0.

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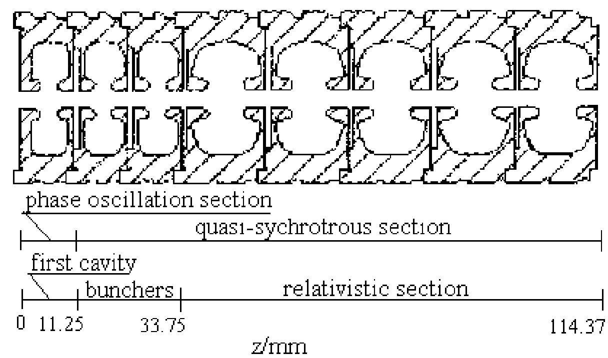

Fig. 1 Schematics of the model structure

3 DESCRIPTION OF THE SIMULATION CODES

The RF fields in the linacs are calculated by CAV. TRACK is coded by multi-particle tracking and calculates the beam dynamics including the nonlinear RF fields. It uses the time as independent variable and calculates the effects of Coulomb interactions among the electrons by variable-radius-disc model[2]. The electrons are emitted randomly with a profile that can be arbitrary distribution in radius. Both forward and backward particles are tracked. We introduce *power distribution factors* to TRACK to changing the RF field distribution conveniently.

4 EMITTANCE GROWTH

4.1 Emittance Growth Occurs in the First Cavity Mainly

The rms normalized emittances at the entrance of the model, at the exit of the first cavity, at the exit of the bunchers and the exit of the model are calculated separately. From simulation results (shown in Fig. 2), we can see the emittance growth due to RF nonlinear fields occurs in the first cavity mainly. The drop of emittance at the exit of the model whose initial phase is *-150^0* results from the particles loss on the wall. So we concentrate on the first cavity when investigating the emittance growth.

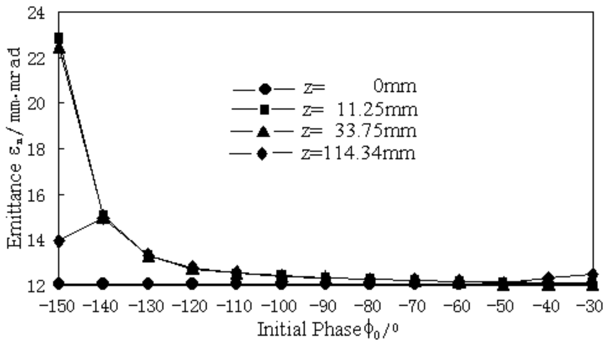

Fig. 2 Comparison of ε_n at different cross section

4.2 The Influence of the Cavity Shape

The nonlinear transverse effects on the beam is associate with the nonlinear components of RF fields which are deceded by the cavity shape. The RF fields in the assymetric cavity lift slowly along the axis and the decelearting for the particles in the asymmetric cavity is smaller than the particles with the same initial phase in the symmetric cavity (shown in Fig. 3). The emittance growth in the asymmetric cavity is smaller because the synchronous state of particles in the asymmetric cavity is better than in the symmetric cavity, even though the nonlinear components of RF fields in the asymmetric cavity is larger than in the symmetric cavity.

ε_{n1}*=22.81mm.mrad* at the exit of the symmetry first cavity as compared to ε_{n1}*=12.51mm.mrad* at the exit of in the assymytry first cavity at the initial phase $\phi_0 = -150^0$.

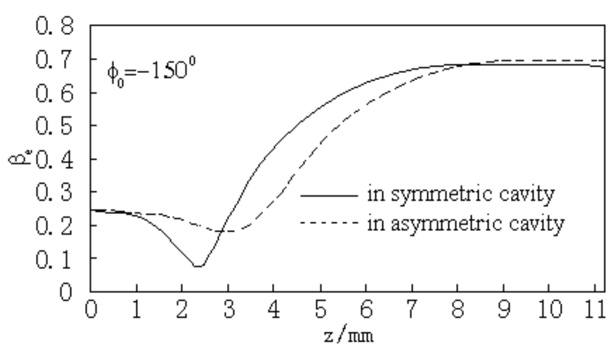

Fig. 3 Schematics of β_e in the different shape first cavity

4.3 The Influence of the Synchronous State

The synchronous state is associated with the initial phase, injecting voltage and RF field amplitude. The following ε_{n1} is the rms normalized emittance at the exit of the first cavity.

If the initial phase is near the sychrotrous phase (0^0), the sychrotrous state is better (shown in Fig. 4). ε_{n1}*=22.82mm.mrad* at the initial phase $\phi_0=-150^0$ as compared to ε_{n1}*=12.72mm.mrad* at the initial phase $\phi_0=-120^0$ in V_{inj} *=16.0kV* and P_0 *=1.1MW*.

If V_{inj} is higher, the sychrotrous state is better (shown in Fig. 5). ε_{n1}*=22.82mm.mrad* in V_{inj} *=16.0kV* as compared to ε_{n1}*=26.80mm.mrad* in V_{inj} *=14.5kV* at the initial phase $\phi_0=-150^0$ and P_0 *=1.1MW*.

If P_0 is higher, a_n is higher and the decelerating is larger, then the sychronous state is worse (shown in Fig. 6). ε_{n1}*=22.81mm.mrad* in P_0 *=1.1MW* as compared to ε_{n1}*=13.72mm.mrad* in P_0*=0.9MW* at the initial phase $\phi_0=-150^0$ and V_{inj} *=16.0kV*.

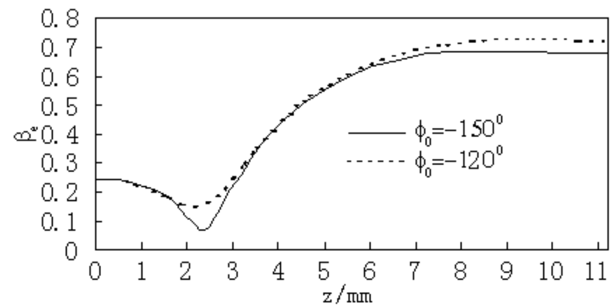

Fig. 4 Schematics of β_e in the first cavity for different ϕ_0

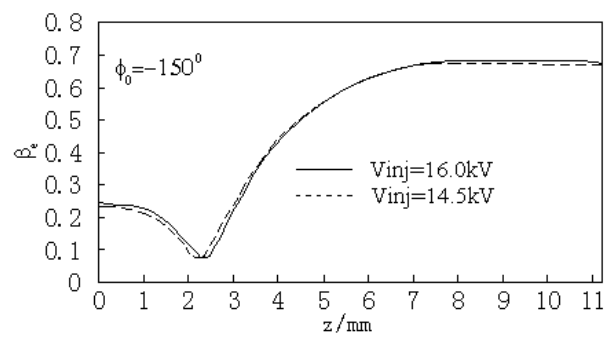

Fig. 5 Schematics of β_e in the first cavity for different V_{inj}

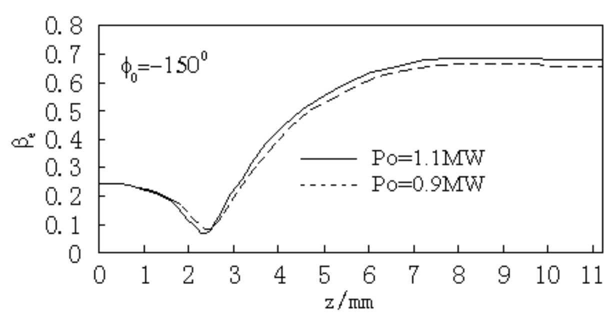

Fig. 6 Schematics of β_e in the first cavity for different P_0

5 COMPARISON BETWEEN ANALYSIS AND SIMULATION

The relativistic factor at the exit of the first cavity,

$$\gamma_1 = \gamma_0 + \frac{e}{m_0 c^2}\frac{\overline{\beta}_{e0}}{k}\left[\sum_{n=1}^{\infty} a_n \frac{Sin\left(\frac{n\overline{\beta}_{e0}}{\beta_{p1}}\phi_0\right) - \frac{n\overline{\beta}_{e0}}{\beta_{p1}} Sin(\phi_0)}{1-\left(\frac{n\overline{\beta}_{e0}}{\beta_{p1}}\right)^2}\right]$$

$$+\frac{e}{m_0c^2}\frac{\bar{\beta}_{e1}}{k}\sum_{n=1}^{\infty}a_n\left[\frac{-Cos\left(\frac{\bar{\beta}_{e0}}{\bar{\beta}_{e1}}\phi_0\right)Sin\left(\frac{n\bar{\beta}_{e0}}{\beta_{p1}}\phi_0\right)}{1-\left(\frac{n\bar{\beta}_{e1}}{\beta_{p1}}\right)^2}+\frac{n\bar{\beta}_{e1}}{\beta_{p1}}\frac{(-1)^n Sin\left(\frac{\beta_{p1}}{\bar{\beta}_{e1}}\pi\right)+Sin\left(\frac{\bar{\beta}_{e0}}{\bar{\beta}_{e1}}\phi_0\right)Cos\left(\frac{n\bar{\beta}_{e0}}{\beta_{p1}}\phi_0\right)}{1-\left(\frac{n\bar{\beta}_{e1}}{\beta_{p1}}\right)^2}\right] \quad (1)$$

Corresponding, the phase at the exit of the first cavity

$$\phi_1 = \frac{k}{\bar{\beta}_{e1}}(D_1 - D_0) \quad (2)$$

where $D_0 = -\frac{\bar{\beta}_{e0}}{k}\phi_0$ defined in ref.[1].

After one cavity under the nonlinear RF fields,

$$\Delta\varepsilon_{xn} = 4\pi\left(\frac{1}{\beta_{p1}\lambda}\right)^3\left|N(\phi_0)\right|\sqrt{\left\langle r^4x^2\right\rangle\left\langle x^2\right\rangle-\left\langle r^2x^2\right\rangle^2} \quad (3)$$

If the distribution of electrons is uniform distribution at the plane which is perpendicular to the axis,

$$\Delta\varepsilon_{xn} = 4\pi\left(\frac{R}{\beta_{p1}\lambda}\right)^3\left|N(\phi_0)\right|\times 0.06R \quad (4)$$

where R is the envelope radius of beam.

For the first cavity,

$$N(\phi_0) = \frac{D_1e}{m_0c^2}\frac{\pi^2}{2}\left(\frac{1}{\beta_{p1}}-M_0^2\right)\sum_{n=2}^{\infty}\frac{n(n^2-1)}{1-(nM_0)^2}\left[Cos(nM_0\phi_0)-Cos(\phi_0)\right]$$
$$-\frac{D_1e}{m_0c^2}\frac{\pi^2}{2}\sum_{n=2}^{\infty}\frac{(n^2-1)a_n}{1-(nM_1)^2}\left\{M_1\left(\frac{n^2}{\beta_{p1}}-1\right)Sin\left(\frac{M_0}{M_1}\phi_0\right)Sin(nM_0\phi_0)\right.$$
$$\left.+n\left(\frac{1}{\beta_{p1}}-M_1^2\right)\left[-(-1)^nCos\left(\frac{\pi}{M_1}\right)+Cos\left(\frac{M_0}{M_1}\phi_0\right)Cos(nM_0\phi_0)\right]\right\} \quad (5)$$

where $M_0 = \frac{\bar{\beta}_{e0}}{\beta_{p1}}$, $M_1 = \frac{\bar{\beta}_{e1}}{\beta_{p1}}$.

RF acceleration and RF effects on longitudinal phase space distribution γ_1 and ϕ_1 of analysis is in rough agreement with γ_1 and ϕ_1 of simulation (shown in Fig. 7 and 8). So is the RF field effects on transverse phase space distribution ε_{n1} (shown in Fig. 9).

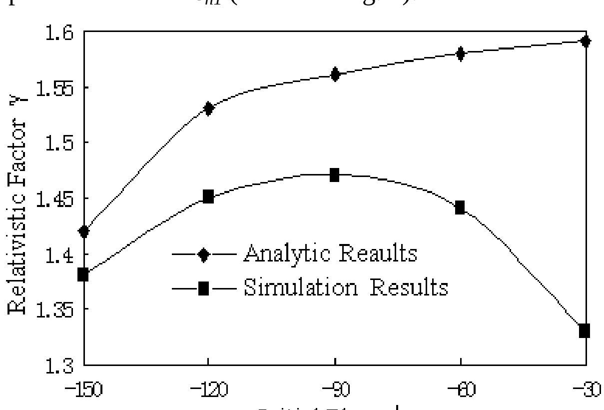

Fig. 7 Comparison of γ at the exit of the first cavity

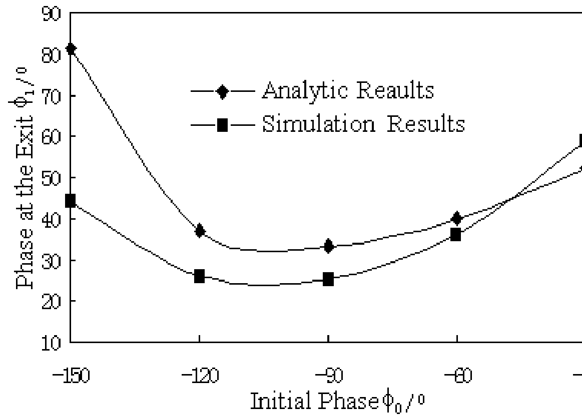

Fig. 8 Comparison of ϕ at the exit of the first cavity

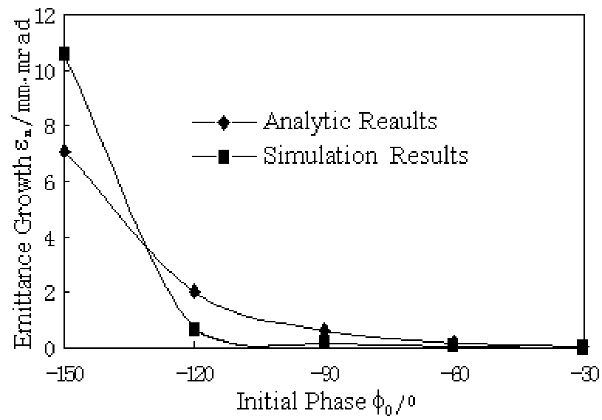

Fig. 9 Comparison of ε_n at the exit of the first cavity

6 CONCLUSIONS

A simulation to investigate the beam dynamics in the presence of nonlinear RF fields in a low energy SW X-band linac structure has been performed. The results have been compared and found to agree well with those from analyses. These programs *CAV* and *TRACK* should be useful in designing the low energy, high beam quality and high shunt resistance SW linacs opperating in higher frequencies.

A *6 MeV* X-band SW linac structure has been designed including the nonlinear RF fields and is presently under construction.

REFERENCE

[1] Sun Xiang and Lin Yuzheng. The Nonlinear Transverse RF Field Effect on the Beam Dynamics. These proceedings.

[2] Yao Chongguo. Electron Linear Acclerators. China Science Press. (1986)