TEMPERATURE DISTRIBUTION CALCULATIONS ON BERYLLIUM WINDOWS IN RF CAVITIES FOR A MUON COLLIDER *

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Abstract

We report the temperature distribution calculations on beryllium windows in RF cavities for a muon collider. The cavity resembles a closed pill-box cavity with the conventional beam iris aperture covered by thin beryllium (Be) foils to enhance the electric fields on the beam axis. The design resonant frequency for the cavity is 805 MHz. To reduce the RF power losses, the cavity may be operated at liquid nitrogen (LN) temperature. The heating caused by RF power dissipations on the Be windows is a concern for either at the room or at the LN temperature operations. Based on the MAFIA simulations, considering the heat conduction inside the Be windows only, a pill-box and an approximated *linear* model are used to calculate the temperature distributions analytically on a flat and a tapered window, respectively. Preliminary calculations suggest that no special cooling designs be needed for 0.127 mm thickness of Be windows while the cavities operate at 30 MV/m of electric fields on beam axis with 30 μs pulse length and 10 Hz repetition rate.

1 INTRODUCTION

Significant efforts have been devoted to exploring the feasibility of designing and constructing a high luminosity muon collider. Among many technical challenges, experimental demonstration of ionization cooling for high intensity μ^+ and μ^- beams is one of the most critical paths towards the building of a muon collider. Initial simulation studies have indicated that the six-dimension phase space volume of muon beams can be cooled by as much as a factor of 10^5 or 10^6 through an ionization cooling channel [1]. The cooling channel under study is composed of 20 to 30 sections, while each section consists of liquid hydrogen absorber and RF acceleration cavities surrounded by alternating super-conducting solenoids. The muon beam loses its momentum (energy) both transversely and longitudinally while passing the absorber. Nevertheless the longitudinal momentum loss is compensated by coherent acceleration fields in the RF cavities and therefore a net transverse momentum loss is left $\Rightarrow$ **transverse cooling**! A highest possible acceleration gradient RF structure is a must for restoring the longitudinal momentum loss in the limited lifetime of the muon beam. Among few proposed RF structures [2], an interleaved $\pi/2$ standing wave (SW) structure, shown in Figure 1, is favored. The accelerating cavity cell resembles a regular closed cylindrical pill-box cavity, but with the beam iris aperture covered by thin Be foils. The structure produces an on-beam-axis field equal to the maximum surface field (e.g. $E_{\rm acc}/E_{\rm surface} = 1$) while for conventional RF structures with beam iris $E_{\rm acc}/E_{\rm surface} \approx 0.5$. Moreover the muon beams traverse these foils with negligible energy losses. MAFIA simulations indicate the cavity has an effective shunt impedance of 37 MΩ/m at $\beta = 0.87$ [2]. To further reduce RF power loss (or the costs for RF sources) the cavity may be operated at LN temperature. Potential benefits operating at LN temperature will depend on both the electrical and thermal properties of the Be windows. However, these properties at low temperature are not very well known. The thermal property in particular has strong influence on the mechanical and engineering designs for the windows and cavities. Experimental exploration on these issues is currently in progress at BNL. Nevertheless based on the limited data [4], we have estimated the temperature raise the Be windows so that proper temperature control or special cooling designs are employed if necessary. Two models, a pill-box and an approximated *linear* have been used to calculate the temperature distributions. Analytical formulae and numerical results are presented and compared. The linear model is also used for the calculation of tapered windows.

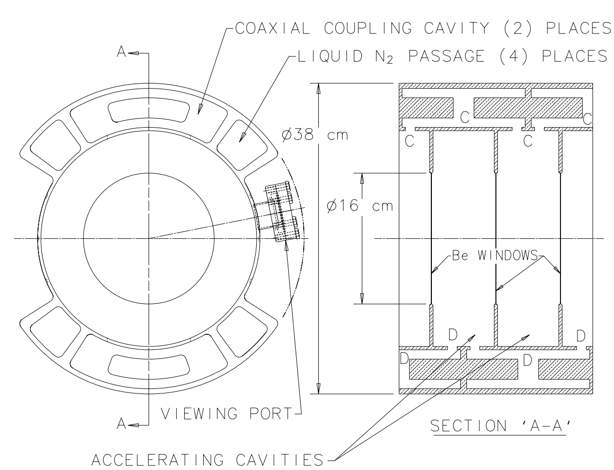

Figure 1: An interleaved $\pi/2$ SW RF structure proposed for a muon cooling experiment at FNAL. Two cavity chains are interleaved with a $\pi/2$ phase shift. Each chain operates at its own $\pi/2$ mode separately. Each cavity cell has a resonance frequency of 805 MHz. Conventional beam irises are covered by thin Be foils with radii of 8 cm. RF coupling between the cells is conducted through open slots on adjacent walls. The accelerating resembles a closed pill-box cavity and the coupling cell is a loaded coaxial type cavity.

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2 RF POWER LOSSES

The electro-magnetic field continuously loses its energy to the cavity wall. In order to maintain the field, the energy loss has to be provided by rf sources. The total RF energy (power) flowing from the field into the wall can be computed by

$$P = \frac{R_s}{2} \oint_S H_t \cdot H_t^* ds, \quad (1)$$

where P is average power loss over one RF period; $R_s = \sqrt{\frac{\mu_0 \omega}{2\sigma}}$ is surface resistance of the wall; H_t is tangential component of magnetic field on the surface; S is the area of the cavity wall being studied. Equation (1) holds only if the conductor thickness and radii of curvature are much greater than δ, the skin depth. This is the case for most of the accelerator applications. H_t usually has to be obtained from numerical simulation codes such as MAFIA and SUPERFISH etc.. In some special cases (simple geometry), analytical solutions are available or can be used as good approximations. The $\pi/2$ interleaved SW structure, for example, the field in the accelerating cell can be well approximated by pill-box solutions. Using the analytical solutions and a *linear* approximation model solution in particular greatly simplify the calculation for temperature distributions. Considering the cylindrical symmetry of the accelerating cells, it is convenient to define a power loss and a density functions, $P(r)$ and $\rho(r)$ by re-writing Equation (1),

$$P(r) = \int_0^r \rho(r')(2\pi r')dr', \quad \rho(r) = \frac{R_s}{2} H_t \cdot H_t^*, \quad (2)$$

where we have assumed that only TM_{010} is the mode of our interest and $P(r)$ and $\rho(r)$ depend on r only. [1]

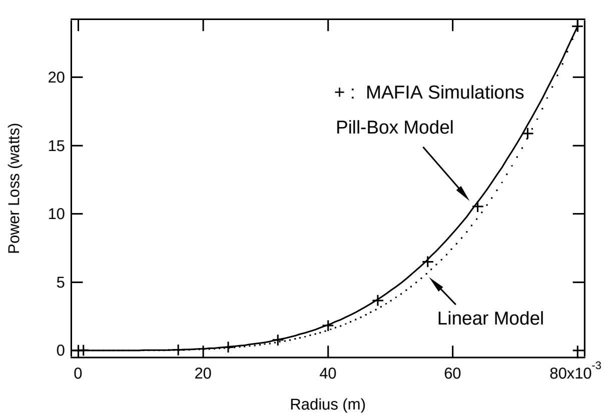

Figure 2: Power loss distribution on a flat Be window (conductivity of copper was used for this calculation, it should be easy to scale to the Be case once the data is available) calculated from the MAFIA simulations (+ sign), analytical formulae for the pill-box cavity (solid line) and the *linear* approximation (dotted line). The resonance frequency of the cavity is 805 MHz.

[1] The coupling slots in the cavities may introduce asymmetric field components, but these components are usually small and have negligible effects on on the power loss distribution.

2.1 The Pill-Box Model

For the TM_{010} mode in a pill-box cavity, its electro-magnetic fields can be expressed as,

$$E_z = E_0 J_0(kr), \quad H_\phi = H_t = \sqrt{\frac{\epsilon_0}{\mu_0}} E_0 J_1(kr). \quad (3)$$

The power loss and density functions are then given by,

$$\begin{aligned} P_{\rm pb}(r) &= \tfrac{\pi}{2}\eta R_s \left(\tfrac{\epsilon_0}{\mu_0}\right) E_0^2 \left\{ r^2 \left[J_0^2(kr) + J_1^2(kr)\right] \right. \\ &\quad \left. - \tfrac{2r}{k} J_0(kr) J_1(kr) \right\} \\ \rho_{\rm pb}(r) &= \tfrac{R_s}{2}\eta \left(\tfrac{\epsilon_0}{\mu_0}\right) E_0^2 J_1^2(kr) \end{aligned} \quad (4)$$

where $k = (\omega/c) = (u_{01}/a)$, with u_{01} as the first root of zero order Bessel function and a the outer radius of the accelerating cell; η is the duty factor of the RF source.

2.2 Linear Approximation Model

As indicated in Equation (3), $H_t \propto J_1(kr) \approx \frac{k}{2} r$ when $r << a$. We are interested only in the power loss on the Be windows with 8 cm of radii ($0 < r < R =$ 8 cm and $R/a < 0.6$), we therefore may assume H_t to be linearly dependent on r, e.g. $H_t = \sqrt{\alpha} r$ and name it as the linear model. The power loss density in this case is given by,

$$\rho_{\rm ln}(r) = \alpha r^2, \quad (5)$$

which is **quadratically** proportional to r. The linear model diverges more from the pill-box solutions while r is approaching to R. To compensate this deviation, we calculate α by requiring,

$$P_{\rm ln}(r) = P_{\rm Be} \left(\frac{r}{R}\right)^4, \quad \alpha = \frac{2P_{\rm Be}}{\pi R^4}. \quad (6)$$

Where $P_{\rm Be}$ is the total average RF power loss on the Be window (one side). $P_{\rm Be}$ may come from the MAFIA simulations, analytical formulae from the pill-box model or even experimental measurement. For either cases we assume,

$$P_{\rm pb}(R) = P_{\rm ln}(R) = P_{\rm Be}. \quad (7)$$

With this assumption the power loss function by the linear model agrees well (within 6 %) with both the pill-box model and the MAFIA simulations. Nevertheless the linear model is much simpler mathematically, and therefore makes it possible for calculating the temperature distribution analytically tapered windows. As a comparison, Figure 2 shows the power loss calculations versus r on a flat Be window by different models. For all the calculations, we have assumed $E_0 = 30$ MV/m; $\sigma = 5.8 \times 10^7$ /(Ωm) (copper); $d = 0.127$ mm; $\kappa = 200$ W/mk and $\eta = 3 \times 10^{-4}$ (10 Hz repetition rate and 30 μs pulse length).

3 TEMPERATURE DISTRIBUTIONS

The RF power loss on Be windows is dissipated in the form of heat. The heat may be transfered by conduction, convection and radiation. However, the dominant form of the transformation in metals is conduction. For the temperature distribution calculations we consider the heat conduction within Be windows only.

3.1 Heat Equation

It is well known that the conduction of heat obeys [3],

$$\frac{dQ}{dt} = -\kappa A \frac{dT}{dr} \text{ [watts]}, \tag{8}$$

where Q is the heat flow across area A, κ is a constant and known as the coefficient of thermal conductivity for the material of interest, and $\frac{dT}{dr}$ is the temperature gradient at r. Considering the cylindrical symmetry, and neglecting ionization energy loss of muons by passing through the windows, we assume the temperature distribution $T(\vec{r})$ depends on r only, e.g. $T(\vec{r}) = T(r)$. Equation (8) then can be re-written as,

$$\frac{dT}{dr} = -\frac{1}{\kappa d}\left[\frac{1}{r}\int_0^r \rho(r')r'dr'\right], \tag{9}$$

where d is the thickness of the Be window, for tapered windows d is dependent on r.

3.2 The Pill-box Solution

Substituting the power loss density function in Eq. (4) into Eq. (9), for a flat window we obtain,

$$T_{\rm pb}(r) = T_{\rm pb}(0) - \frac{R_s}{4}\eta\left(\frac{\epsilon_0}{\mu_0}\right)\frac{E_0^2}{\kappa d}\left\{r^2\left[J_0^2(kr) + J_1^2(kr)\right] - \frac{r}{k}J_0(kr)J_1(kr) + \frac{1}{k^2}\left[J_0^2(kr) - 1\right]\right\}. \tag{10}$$

Where $T_{\rm pb}(0)$ is the temperature at the center of the window. Note the hottest spot is always at the center of the window (see Figure 3). We have assumed the cooling will keep the out-most edge $(r = R)$ at a constant temperature $T(R)$. The heat at the center takes the longest path to reach the cooling point even though there is no direct heating source in the center.

3.3 The Linear Model Solutions

Similarly by substituting the linear model solution in Eq. (5) into Eq. (9), we attain the temperature distribution for a flat window,

$$T_{\rm ln}(r) = T_{\rm ln}(0) - \frac{P_{\rm Be}}{8\pi\kappa d}\left(\frac{r}{R}\right)^4 \tag{11}$$

with $T_{\rm ln}(0) = T(R) + P_{\rm Be}/(8\pi\kappa d)$.

Based on the linear model solution, temperature distribution on a tapered Be window (see Figure 3) can also be calculated. We give the analytical formula below and with some numerical calculations shown in Figure 3.

$$T_{\rm ln}^{\rm tp}(r) = \begin{cases} T_{\rm ln}^{\rm tp}(0) - \frac{P_{\rm Be}}{8\pi\kappa d}\left(\frac{r}{R}\right)^4 & \text{if } r < r_0 \\ T_{\rm ln}^{\rm tp}(0) - \frac{P_{\rm Be}}{2\pi\kappa d R^4}\left\{\frac{r_0^4}{4} + \Phi(r)\right\} & \text{if } r > r_0 \end{cases} \tag{12}$$

$$\Phi(r) = \xi^3\left(1 - \frac{r_0}{\xi}\right)^2(r - r_0) + 2\xi^2\left(1 - \frac{r_0}{\xi}\right)(r^2 - r_0^2) + \frac{\xi}{3}(r^3 - r_0^3) - \xi^4\left(1 - \frac{r_0}{\xi}\right)^3\ln\left[\frac{\xi + r}{\xi + r_0}\right], \tag{13}$$

where $\xi = \frac{R - r_0}{d^* - d}d$. For the tapered window simulations in Figure 3, in addition to the same parameters used in Figure 2 we have taken $d^* = 2d$ and $r_0 = 0.04$ m.

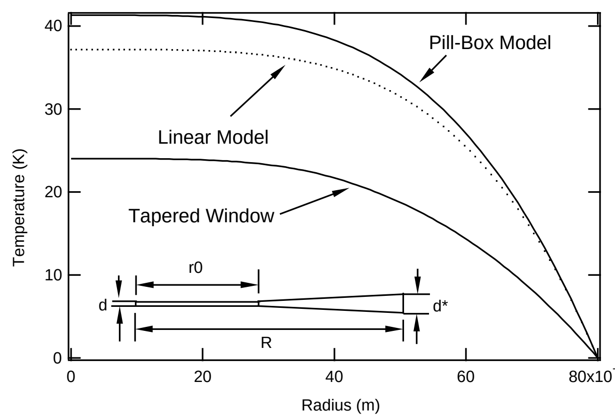

Figure 3: Temperature distribution on a flat Be window with thickness of 0.127 mm and on a tapered window with $d^* = 0.254$ mm. Note the temperature difference of $T(0) - T(R)$ is plotted here. $T(R) \approx 77$ K and $T(R) \approx 300$ K for the LN and room temperature, respectively. The temperature raise of the tapered window is about 15 K lower than the flat one.

4 CONCLUSION

Analytical formulae have been given for the temperature calculations. Numerical results presented here are for copper windows. It can be easily scaled to Be windows accordingly by $T_{\rm Be}(r) = \sqrt{\frac{\sigma_{\rm Cu}}{\sigma_{\rm Be}}}T_{\rm Cu}(r)$ once the Be data becomes available. Note both $\sigma_{\rm Be}$ and $k_{\rm Be}$ may vary with the temperature, but we did not take this into account in our models. However within small temperature range the models still give good approximations. Preliminary results show that the tapered window is better for the heat conduction and even a 0.127 mm thick flat window does not need special cooling designs and yet gives acceptable results from the viewpoint of beam dynamics for the muon cooling.

5 REFERENCES

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