HALO FORMATION BY MISMATCH FOR HIGH INTENSITY BUNCHED BEAMS

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Abstract

In high intensity proton linacs even small particle loss causes activation of the accelerator components [1]. Minimizing the losses is therefore a major task of the beam dynamical design. Losses are strongly correlated to the formation of a halo around the beam core. Monte Carlo simulations are presented for bunched beam transfer lines with varying tune depressions and equipartitioning ratios showing halo formation by mismatch due to parametric resonance excitation. For large tune depressions and/or quite unequal transverse and longitudinal temperatures substantial halo formation even for a matched beam occurs due to temperature exchange and/or exciting a higher order mode instability. An envelope instability can be excited in addition. A conservative criteria is presented for identifying parameters for reduced halo formation due to mismatch.

1 INTRODUCTION

In recent years substantial progress has been achieved by identifying the parametric resonance conditions as a major source of halo production in DC and bunched beams [2]. For realistic particle distributions with nonlinear space charge forces particles inside the core have a tune spread. Parametric resonances can occur between single particle tunes and the frequency of the oscillating mismatched beam core.

In this presentation the one dimensional parametric resonance model is generalized to bunched beams. Due to one longitudinal and the two transverse bunch dimensions, 3 eigenmodes exist for the mismatched envelopes.

The three eigenmodes of bunched beams are described in section 2. Correlation between enhanced halo production due to mismatch and parametric resonances are verified by Monte Carlo simulation in section 3. In section 4, halo production even for a matched beam due to the excitation of an envelope or higher order mode instability or as a result of temperature exchange is demonstrated. A conservative criteria is presented for identifying parameters for reduced transverse and longitudinal halo formation due to mismatch.

2 THE THREE ENVELOPE MODES OF MISMATCHED BUNCHED BEAMS

For the analytical approximation of the eigenfrequencies (modes) of the mismatched envelopes it is assumed that the beam is of ellipsoidal shape with uniform charge density giving linear space charge forces. In the rest frame of the bunch, the bunch radii are denoted by a_x, a_y and a_z. The bunch length b in the laboratory system is given by $b = a_z/\gamma$, where γ is the relativistic mass factor. The external forces for are assumed to be linear and periodic in the longitudinal direction s with period length L. The envelope equations are given by

$$a_x'' + k_{xo}^2 a_x - \frac{IK_x}{a_y b} - \frac{\epsilon_t^2}{a_x^3} = 0,$$
$$a_y'' + k_{yo}^2 a_y - \frac{IK_y}{a_x b} - \frac{\epsilon_t^2}{a_y^3} = 0,$$
$$b'' + k_{zo}^2 b - \frac{IK_z}{a_x a_y} - \frac{\epsilon_z^2}{b^3} = 0.$$

Here k_{xo}, k_{yo} and k_{zo} are the external periodic force constants. K_x, K_y and K_z are proportional to the elliptical formfactors and depend on the bunch dimensions too. I is the bunch current and ϵ_t and ϵ_z are the transverse and longitudinal emittances. This system of nonlinear coupled differential equations exhibits oscillating stable or unstable solutions. 'Matched' solutions have the same periodicity as the external focusing system. The oscillation of small mismatched solutions can be characterized by three eigenfrequencies, a pure transverse **quadrupolar mode**

$$\sigma_{env,Q} = 2\sigma_t$$

and a **high** and **low mode** which couple the transverse and longitudinal directions

$$\sigma_{env,H}^2 = A + B,\ \sigma_{env,L}^2 = A - B$$

with

$$A = \sigma_{to}^2 + \sigma_t^2 + \frac{1}{2}\sigma_{lo}^2 + \frac{3}{2}\sigma_l^2$$

and

$$B = \sqrt{\left(\sigma_{to}^2 + \sigma_t^2 - \frac{1}{2}\sigma_{lo}^2 - \frac{3}{2}\sigma_l^2\right)^2 + (\sigma_{to}^2 - \sigma_t^2)(\sigma_{lo}^2 - \sigma_l^2)}.$$

The mismatch mode tunes are expressed by the full and zero current transverse and longitudinal tunes σ_t, σ_{to}, σ_l and σ_{lo}.

The high mode represents a 'breathing' of the ellipsoidal bunch [3]. For the low mode, the bunch breathes in the transverse direction, but the oscillation in the longitudinal direction is of opposite phase.

In Fig. 1, the optical elements and the three matched beam radii are shown for a bunched beam transportline corresponding to the first period at 70 MeV of the coupled cavity (CCL) of the proposed European Spallation Source (ESS). By varying the beam current, the transverse and longitudinal emittance and the quadrupole gradient, a range of full and zero current tunes can be obtained. The radii shown in Fig. 1 correspond to the 214 mA ESS bunch current and 60^o transverse full current tune.

3 HALO FORMATION BY PARAMETRIC RESONANCE EXCITATION

It is important to do multiparticle calculations of the bunched beam transfer line and compare the results with the model as due to phase space filling the multiparticle simulations have nonlinear space charge forces included. However the rms quantities are mainly determined by the linear part of the space charge forces. Monte Carlo simulations are done with 20 000 particles which interact fully in 3d. A 6d waterbag distribution is used as input.

Due to the nonlinear space charge forces, particles have tunes which are distributed between the full current and zero current tune. Due to oscillation of the mismatched radii single particles can experience parametric resonances. The condition for exciting a parametric resonance either radially or longitudinally is given by

$$\frac{\sigma^p_{t,l}}{\sigma_{env}} = \frac{m}{n} = \frac{1}{2}, \frac{1}{3}, \ldots$$

with

$$\sigma_t \leq \sigma^p_t \leq \sigma_{to},$$
$$\sigma_l \leq \sigma^p_l \leq \sigma_{lo}$$

where σ_{env} is one of the three envelope tunes of the mismatched radii and $\sigma^p_{t,l}$ the single particle tune.

The low order resonances are the most dangerous ones. For the radial direction the 1/2 parametric resonance is always excited by the quadrupolar mode. The high or low mode can excite a parametric resonance either in the transverse or longitudinal direction. The frequency of the high mode should be limited below 180^o in order to avoid an envelope instability.

Figs 2 and 3 are the results of Monte Carlo simulations of the bunched beam transfer line in Fig. 1. Shown is the 99.9% total to rms emittance ratio in x-direction. The transverse and longitudinal rms emittances stay constant for a 20% initial mismatch. In Fig. 2 the matched case is compared to a 20% quadrupolar mode excitation. An increase of the 99.9% emittance is visible due to exciting the 1/2 parametric resonance. Fig. 3 compares the matched case with a 20% radially and 30% longitudinally excited high mode with a numerically determined mode frequency of 168^o. No halo formation due to mismatch is visible here as no single particles have a transverse tune of 84^o. The full current transverse design tune is 60^o.

4 STABILITY CRITERIA FOR HALO FORMATION DUE TO MISMATCH

Halo production due to mismatch gets complicated if the rms emittances are changing. Reasons for emittance change can be an envelope instability, temperature exchange or particle redistribution under high space charge forces. Substantial halo production even for a matched beam will occur in these cases.

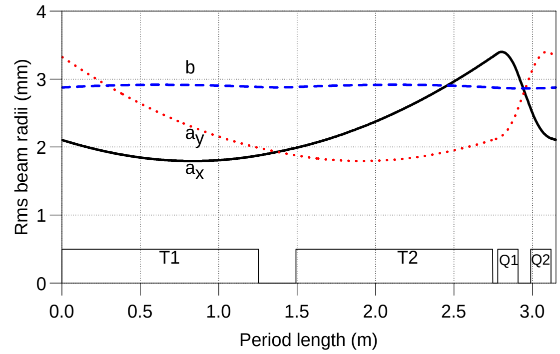

Figure 1: The matched beam radii along one period. T1, T2 bunching cavities, Q1, Q2 quadrupoles

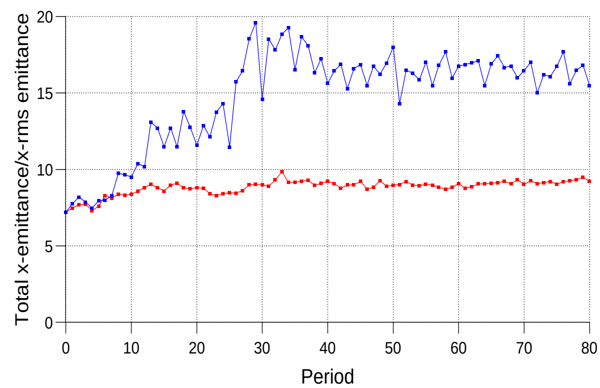

Figure 2: 99.9% total to rms emittance ratio for a matched (bottom) and a quadrupolar mode excited case (top)

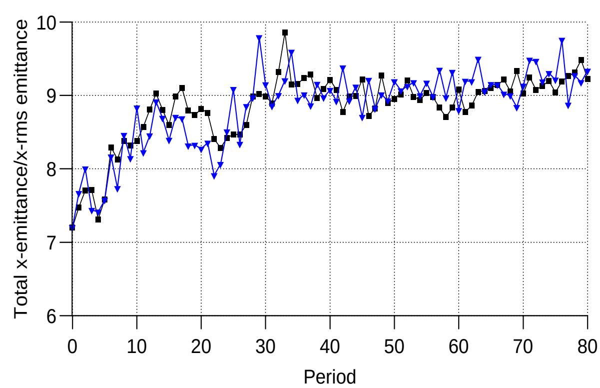

Figure 3: 99.9% total to rms emittance ratio for a matched (squared) and a high mode excited case (triangles)

4.1 High Mode Envelope Instability

The coupled set of bunched beam envelope equations with linear space charge forces can have unstable solutions due to the periodic external focusing. This is expected to happen if one of the envelope tunes is close to or above 180^o. Fig 4 shows a Monte Carlo simulation for an unstable bunched beam transfer line, which has a high mode frequency of almost 180^o. Clearly visible is a dramatic increase of the transverse total emittance.The transverse rms emittances grow by a factor of 4.

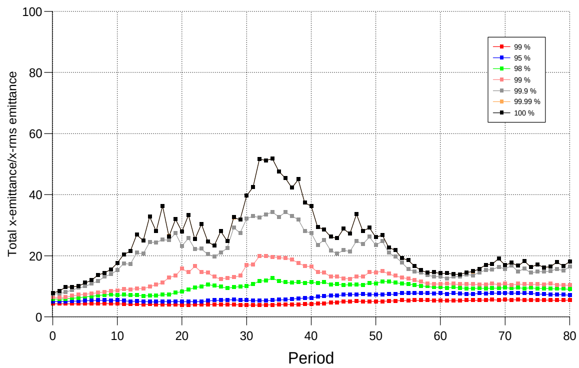

Figure 4: Excitation of the high mode envelope instability.

4.2 *Temperature exchange*

In Fig. 5 the transverse and longitudinal rms emittances are shown for an initially matched beam, where the transverse temperature is chosen to be a factor 2.3 higher than the longitudinal one. Visible is an exchange of emittance at the beginning, followed by damped oscillation afterwards. The ratio of total to rms emittance stays constant in all three planes. This is consistent with a temperature exchange [4,5].

4.3 *Higher Order Mode Instability*

Results of Monte Carlo simulations for an initially matched, equipartitioned beam with transverse and longitudinal tune depressions below 0.4 are shown in Fig. 6. Visible is the excitation of a higher order mode instability [6,7]. The total emittance growth is earlier and much more pronounced than the rms emittance growth which is not shown.

4.4 *Stability Criteria for Halo Formation due to Mismatch*

Based on Monte Carlo simulations for designs of bunched beam transferlines, a conservative stability criteria for halo fromation due to mismatch can be given. If the high mode envelope instability is avoided then a linac design insensitive to mismatch is given if the tune depressions are above 0.8 and the transverse to longitudinal temperature ratios between 1/3 and 2. The asymmetric temperature boundary reflects the situation of a bunched beam. Two hot transverse temperatures feeding one cold longitudinal are more dangerous than vice versa.

It should be pointed out that 'islands of stability' for a matched beam can exist outside this boundary [8] but they may suffer from substantial halo production due to mismatch. For a larger initial mismatch, pronounced halo formation has been observed in DC beams [9] and for equipartitioned, self-consistent bunched beams [10].

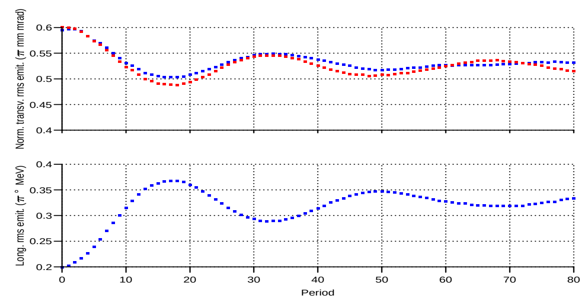

Figure 5: Rms emittances showing temperature exchange from the transverse (top) to the longitudinal (bottom) plane

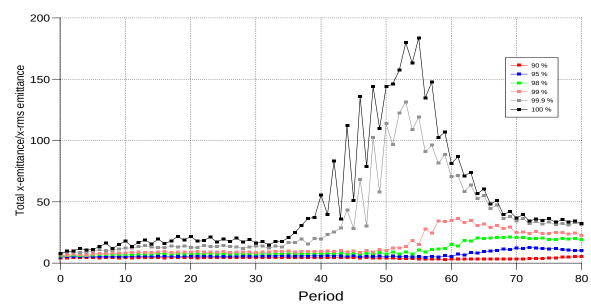

Figure 6: Excitation of a higher order mode instability

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