THE NONLINEAR TRANSERVERSE RF FIELD EFFECTS ON THE BEAM DYNAMICS*

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Abstract

The nonlinear transverse RF field effects on the low energy and emittance dominated electron beams are investigated theoretically including the bunching and accelerating. Without any external magnetic focusing devices and in pursuit of high shunt impedance, the nonlinear transverse RF field effects should not be neglected. The evolution of the beam phase space distribution in asymmetric cavity is studied by taking into account the nonlinear transverse RF fields. As a result of the nonlinear transverse RF field effects, the emittance growth occurs mainly in the first cavity of bunchers and is associated with the relative position of the particles deviating from the axis, the nonlinearity of the RF fields and the synchronous state of the beam. A few formulas for energy, phase and emittance growth are derived.

1 INTRODUCTION

Low energy electron linacs used for industry, medicine and research need the higher quality beam. The nonlinearity of RF fields, magnetic focusing fields and space charge will cause the phase space tearing and emittance growth. And this distortion of the phase space can not be corrected.

The RF fields in S-band low energy electron linacs are considered as linearity and this approximation are good enough due to a smaller ratio of beam size to beam aperture. In order to get higher breakdown threshold and shunt impedance, higher microwave frequencies should be used (e.g. X-band linacs). Then the nonlinearity of RF fields becomes serious. With the increasing of RF frequencies, $r/(\beta_p\lambda)$ (r: the particle radial position; λ: the RF field wavelength; β_p: the relative phase velocity) increases and a decreases correspondingly (a: the iris radius). The cavities are optimized for higher shunt impedance by decreasing a and r/a is larger. In the meantime the magnetic focusing lens is not used to keep linac size small and r/a can not decrease. The condition of paraxial beam is no long satisfied in our case.

In NLC design, the beam sizes are microns and the iris radius is 4 mm, so the cubic term is down by about $(10^{-3})^2$ from the linearity[1]. But in our case, the beam radius is about 1 mm and the iris radius is no larger than 2 mm.

The nonlinear effects of magnetic focusing fields and space charge have been studied[2][3]. An approximate theory of electron beam dynamics in laser-driver RF guns has been developed[4] and the emittance growth due to the effects of nonlinear RF fields in laser-driver RF guns has been studied by simulations[5].

In low energy and low current SW linacs, the electron beams are emittance dominated. The beams are bunched in the first a few cavities and accelerated along the tube. Contrary to the laser-driver RF guns, the initial phase region of captured particles is very large (e.g. 120^0) and usually in the decelerating region. In the following discussion, the influence of initial phase should be considered. The initial emittance is set to zero and the space charge forces are turned off. There is no magnetic focusing lens and the beam is not paraxial.

2 RF FIELDS IN SW LINACS

In order to represent the fields near the axis of a multi-cell π-mode structure, the RF fields are approximated by a power expansion truncated after r^3, yielding

$$E_z = \sum_{n=1}^{\infty} a_n Sin\left(\frac{nkz}{\beta_p}\right) + \frac{k^2r^2}{4}\sum_{n=2}^{\infty} a_n Sin\left(\frac{nkz}{\beta_p}\right) \quad (1.1)$$

$$E_r = -\frac{kr}{2\beta_p}\sum_{n=1}^{\infty} n a_n Cos\left(\frac{nkz}{\beta_p}\right) - \frac{k^3r^3}{16\beta_p^3}\sum_{n=2}^{\infty} n\left(n^2-1\right)a_n Cos\left(\frac{nkz}{\beta_p}\right) \quad (1.2)$$

$$H_\theta = \frac{jkr}{2\mu_0 c\beta_p}\sum_{n=1}^{\infty} a_n Sin\left(\frac{nkz}{\beta_p}\right) + \frac{jk^3r^3}{16\mu_0 c\beta_p^3}\sum_{n=2}^{\infty}\left(n^2-1\right)a_n Sin\left(\frac{nkz}{\beta_p}\right) \quad (1.3)$$

where, $a_i = \frac{2}{D}\int_0^D E_z(r=0) Sin\left(\frac{kz}{\beta_p}\right)dz$; $D = \frac{\beta_p\lambda}{2}$ is the cell length; $i = 1,2,3,\cdots$.

It is because the coefficient of the first harmonic nonlinear term is zero that the π-mode structure is used. If we use symmetric cavities, the coefficient of the second harmonic nonlinear term is zero.

So called symmetric cavity is the one that has a symmetry plane perpendicular to the axis and asymmetric cavity is the one which has not such plane.

3 ANALYSIS OF LONGITUDINAL BEAM DYNAMICS

The low energy linacs consist of a first cavity, bunchers and relativistic cavities. In analysis, the first cavity is the phase oscillation section and bunchers and relativistic cavities are the quasi-synchronous section (shown in Fig. 1). For convenient, we neglect the nonlinear of longitudinal electric fields.

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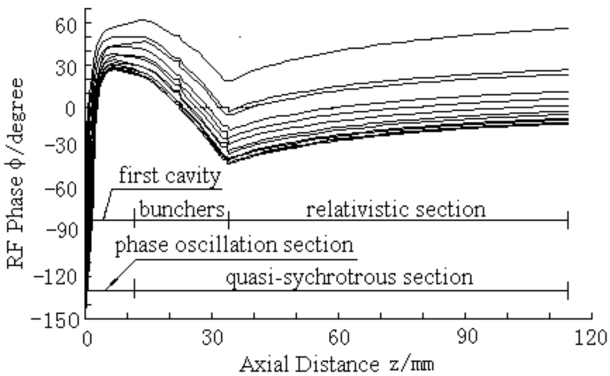

Fig. 1 Schematic of linac distributions

Because the particles are in the decelerating region when they are injected, we assume D_0 is the length of decelerating region (shown in Fig. 2) and $\overline{\beta}_{e0}$ is the average velocity over D_0. ϕ_0 is the initial phase of particles and γ_0 is the initial relativistic factor. γ_i and ϕ_i are the relativistic factor and the phase at the exit of *ith* cavity separately. D_i is the cell length of *ith* cavity. $\overline{\beta}_{ei}$ is the average velocity in the *ith* cavity except $\overline{\beta}_{e1}$ which is the average velocity over $(D_1\text{-}D_0)$ in the first cavity.

We assume $\frac{k}{\overline{\beta}_{e0}} D_0 + \phi_0 = 0 \Rightarrow D_0 = -\frac{\overline{\beta}_{e0}}{k}\phi_0$.

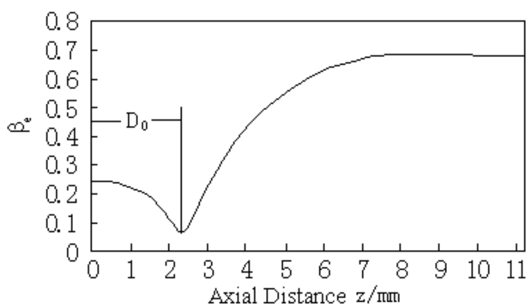

Fig. 2 Schematics of β_e in the first cavity

The average relative velocity is assumed to be constant over its integral region below. At the exit of the first cavity, the relativistic factor particles is

$$\gamma_1 = \gamma_0 + \int_0^{D_0} \frac{e}{m_0 c^2} E_z(z) Sin(\omega t + \phi_0) dt + \int_{D_0}^{D_1} \frac{e}{m_0 c^2} E_z(z) Sin(\omega t + \phi_0) dt$$

$$= \gamma_0 + \frac{e}{m_0 c^2} \frac{\overline{\beta}_{e0}}{k} \left[\sum_{n=1}^{\infty} a_n \frac{Sin\left(\frac{n\overline{\beta}_{e0}}{\beta_{p1}}\phi_0\right) - \frac{n\overline{\beta}_{e0}}{\beta_{p1}} Sin(\phi_0)}{1 - \left(\frac{n\overline{\beta}_{e0}}{\beta_{p1}}\right)^2} \right]$$

$$+ \frac{e}{m_0 c^2} \frac{\overline{\beta}_{e1}}{k} \sum_{n=1}^{\infty} a_n \left[\frac{-Cos\left(\frac{\overline{\beta}_{e0}}{\overline{\beta}_{e1}}\phi_0\right) Sin\left(\frac{n\overline{\beta}_{e0}}{\beta_{p1}}\phi_0\right)}{1 - \left(\frac{n\overline{\beta}_{e1}}{\beta_{p1}}\right)^2} + \frac{n\overline{\beta}_{e1}}{\beta_{p1}} \frac{(-1)^n Sin\left(\frac{\beta_{p1}}{\overline{\beta}_{e1}}\pi\right) + Sin\left(\frac{\overline{\beta}_{e0}}{\overline{\beta}_{e1}}\phi_0\right) Cos\left(\frac{n\overline{\beta}_{e0}}{\beta_{p1}}\phi_0\right)}{1 - \left(\frac{n\overline{\beta}_{e1}}{\beta_{p1}}\right)^2} \right] \quad (2)$$

Corresponding, the phase at the exit of the first cavity

$$\phi_1 = k\int_0^{D_1} \frac{dz}{\overline{\beta}_{e1}} + \phi_0 = k\int_0^{D_0} \frac{dz}{\overline{\beta}_{e0}} + k\int_{D_0}^{D_1} \frac{dz}{\overline{\beta}_{e1}} + \phi_0 = \frac{k}{\overline{\beta}_{e1}}(D_1 - D_0) \quad (3)$$

In the quasi-synchronous section,

$$\gamma_i = \gamma_{i-1} + \int_0^{D_i} \frac{e}{m_0 c^2} E_z(z) Sin(\omega t + \phi_{i-1}) dt$$

$$= \gamma_{i-1} - \frac{e}{m_0 c^2} \frac{\overline{\beta}_{ei}}{k} \frac{\overline{\beta}_{ei}}{\beta_{pi}} \sum_{n=1}^{\infty} n \frac{Sin\left(\frac{\pi}{\left(\overline{\beta}_{ei}/\beta_{pi}\right)} + \phi_{i-1}\right) + Sin\phi_{i-1}}{1 - \left(\frac{n\overline{\beta}_{ei}}{\beta_{pi}}\right)^2} \quad (4)$$

$$\phi_i = k\int_0^{D_i} \frac{dz}{\overline{\beta}_{ei}} + \phi_{i-1} = \frac{k}{\overline{\beta}_{ei}}(D_i - D_{i-1}) + \phi_{i-1} \quad (5)$$

4 ANALYSIS OF TRANSVERSE BEAM DYNAMICS

In the following, the radius r and the average relative velocity are assumed to be constant over their integral region. After the first cavity,

$$\Delta P_{r1}\big|_{non} = \int_0^{D_0} \frac{F_r\big|_{non}}{m_0 c \overline{\beta}_{e0} c} dz + \int_{D_0}^{D_1} \frac{F_r\big|_{non}}{m_0 c \overline{\beta}_{e1} c} dz$$

$$= \frac{\pi^2 D_1 e}{2 m_0 c^2} \left(\frac{r}{\beta_{p1}\lambda}\right)^3 \left(\frac{1}{\beta_{p1}} - M_0^2\right) \sum_{n=2}^{\infty} \frac{n(n^2-1)a_n}{1-(nM_0)^2} \left[Cos(nM_0\phi_0) - Cos(\phi_0)\right]$$

$$- \frac{\pi^2 D_1 e}{2 m_0 c^2} \left(\frac{r}{\beta_{p1}\lambda}\right)^3 \sum_{n=2}^{\infty} \frac{(n^2-1)a_n}{1-(nM_1)^2} \left\{ M_1 \left(\frac{n^2}{\beta_{p1}} - 1\right) Sin\left(\frac{M_0}{M_1}\phi_0\right) Sin(nM_0\phi_0) + n\left(\frac{1}{\beta_{p1}} - M_1^2\right)\left[-(-1)^n Cos\left(\frac{\pi}{M_1}\right) + Cos\left(\frac{M_0}{M_1}\phi_0\right) Cos(nM_0\phi_0)\right] \right\} \quad (6)$$

where $M_0 = \overline{\beta}_{e0}/\beta_{p1}$, $M_1 = \overline{\beta}_{e1}/\beta_{p1}$. $\Delta P_{r1}\big|_{non}$ is the transverse momentum changing due to the nonlinear components of RF fields through the first cavity.

From eq.(6), the nonlinear transverse effects come from three facts.

(1) $(r/\beta_{p1}\lambda)^3$ expresses the relative radial position of particles.

(2) a_n indicates the nonlinear components of RF fields. They are determined by the shapes of the cavities. $a_{2n}=0$ in the symmetric cavities. But the RF fields in the asymmetric cavity lift slowly along the axis and the decelerating force exerted on the particles in the asymmetric cavity is smaller than the force exerted on the particles with the same initial phase in the symmetric cavity over the decelerating region. The synchronous state of particles in the asymmetric cavity is better than in the asymmetric cavity over the decelerating region.

(3) M_0, M_1 and ϕ_0 show the synchronous state between particles and RF fields. We can change the synchronous state by changing injecting voltage and RF field amplitude in the first cavity.

After one cavity in quasi-synchronous section,

$$\Delta P_r\Big|_{non} = \int_0^D \frac{F_r\big|_{non}}{m_0 c\beta_e c} dz = -D\frac{2e}{m_0 c^2\pi}\frac{\pi^3}{2}\left[\frac{1}{\beta_p} - \left(\frac{\beta_e}{\beta_p}\right)^2\right]\left(\frac{r}{\beta_p\lambda}\right)^3 \bullet$$

$$\bullet\left\{\frac{2\cdot 3a_2}{1-\left(\frac{2\beta_e}{\beta_p}\right)^2} Sin\left[\frac{\pi}{2\left(\frac{\beta_e}{\beta_p}\right)} + \phi\right] Sin\left[\frac{\pi}{2\left(\frac{\beta_e}{\beta_p}\right)}\right]\right.$$

$$\left. + \frac{3\cdot 8a_3}{1-\left(\frac{3\beta_e}{\beta_p}\right)^2} Cos\left[\frac{\pi}{2\left(\frac{\beta_e}{\beta_p}\right)} + \phi\right] Cos\left[\frac{\pi}{2\left(\frac{\beta_e}{\beta_p}\right)}\right] + \ldots\right\} \quad (7)$$

where $(\beta_e/\beta_p) \rightarrow 1$, $\beta_p \rightarrow 1$, $\phi \rightarrow 1$,. So $\Delta P_r|_{non} \rightarrow 1$ in quasi-synchronous section.

From eq.(6) and (7), the nonlinear transverse effects occur in the first cavity and the nonlinear transverse effects in the bunchers are more serious than in relative section.

5 EMITTANCE GROWTH

The rms normalized transverse emittance is

$$\varepsilon_{xn} = 4\pi\sqrt{\langle P_{xn}^2\rangle\langle x^2\rangle - \langle P_{xn}x\rangle^2}$$

where <> denotes an average over N particles at z.

For convenient, $p_{xn0} = 0$ is assumed at the entrance of one cavity, or $\varepsilon_{xn0} = 0$.

After one cavity which length is $\beta_p\lambda/2$, we have

$$P_r = \Delta P_r = L(\phi_0)\left(\frac{r}{\beta_p\lambda}\right) + N(\phi_0)\left(\frac{r}{\beta_p\lambda}\right)^3 \quad (8)$$

from eq.(6) and (7), where $L(\phi_0)$, $N(\phi_0)$ is the function of ϕ_0 alone. Then,

$$P_x = \Delta P_x = L(\phi_0)\left(\frac{1}{\beta_p\lambda}\right)x + N(\phi_0)\left(\frac{r}{\beta_p\lambda}\right)^2\left(\frac{1}{\beta_p\lambda}\right)x \quad (9)$$

$$\Delta\varepsilon_{xn} = \varepsilon_{xn} - \varepsilon_{xn0} = 4\pi\left(\frac{1}{\beta_p\lambda}\right)^3 |N(\phi_0)|\sqrt{\langle r^4x^2\rangle\langle x^2\rangle - \langle r^2x^2\rangle^2} \quad (10)$$

If the distribution of electrons is uniform distribution at the plane which is perpendicular to the axis,

$$\Delta\varepsilon_{xn} = 4\pi\left(\frac{R}{\beta_p\lambda}\right)^3 |N(\phi_0)| \times 0.06R \quad (11)$$

where R is the envelope radius of the beam.

From eq.(11), $\Delta\varepsilon_{xn}$ and ΔP_r are the same form. So the analysis and conclusion about ΔP_r can apply to $\Delta\varepsilon_{xn}$ too.

6 CONCLUSION

A simple theory to investigate the nonlinear transverse RF field effects on the emittance dominated beam in low energy SW electron linacs has been performed. The nonlinear transverse effect of RF fields on the beam is associate with the relative radial position of particles, the nonlinear components of RF fields and the synchronous state between particles and RF fields. The nonlinear transverse effects of RF fields on the beam and the emittance growth mainly occurs in the first cavity, especially over the decelerating phase region. These effects weaken quickly and vanish in the quasi-synchronous and synchronous section. In the symmetric π mode cavities, the nonlinear components of RF fields are least. But the nonlinear effects on the beam in an asymmetric first cavity can be less than on the beam with the same initial phase in a symmetric one. It is because the radial RF fields lift slowly in an asymmetric first cavity than in a symmetric one that the synchronous state of particles over the decelerating region in an asymmetric first cavity is better than in a symmetric one, even though the nonlinear components of RF fields in asymmetric cavity are larger.

A few formulas for longitudinal and transverse dynamics have been derived including bunching, accelerating and higher space harmonics. They should be useful in design the low energy SW linacs operating in higher frequencies. The comparison of this theory with the simulation is reported in ref.[6] and they are agreement on the whole.

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