EQUIVALENT LUMPED CIRCUIT STUDY FOR THE FIELD STABILIZATION OF A LONG FOUR-VANES RFQ

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Abstract

The possibility to design an RFQ long respect to the RF wavelength is very important for the feasibility of linacs with different applications, from the high power CW linacs for neutron production to the low power high frequency linacs proposed for hadrotherapy. In particular INFN has been funded for the study of the critical parts of a waste transmutation linac. In this framework the field stabilization of a 352 MHz RFQ has been studied, using the LANL resonant coupling technique; an equivalent lumped circuit approach has been used, and compared with MAFIA simulations. Based on the results of these studies an aluminum model (1:1 scale, 0.04 mm tolerances) has been built and we are now ready for the test of the tuning procedure.

1 CIRCUIT MODEL

A four vanes RFQ resonator is based on a transfer line quadrupolar mode, with dispersion relation:

$$\omega^2 = \omega_o{}^2 + k^2 c^2 \qquad (1)$$

where ω_0 is the cut-off frequency, k is the wave number and c is the asymptotic phase velocity. In an RFQ the boundary conditions are such that the eigen modes are given by $k\ell = \pi n$, with ℓ cavity length and n (integer) mode number. The same dispersion relation (1) is valid for the circuit in Fig. 1, governed by the equation:

$$\frac{d^2V}{dz^2} = -Z_L Y_0 V \qquad (2)$$

with V intervane voltage, $Z_L dz = i\omega L_1\, dz$ coupling impedance, L_1 coupling inductance per unit length, and

$$Y_0 dz = \left(\frac{1}{i\omega L} + i\omega C\right)dz = \frac{1}{i\omega L}\left(1 - \frac{\omega^2}{\omega_0^2}\right)dz$$

with L inductance length, C capacitance per unit length. The solution of (2) can be written as:

$$V = V_0 e^{ikz} \qquad k^2 = -\frac{L_1}{L}\left(1 - \frac{\omega^2}{\omega_0^2}\right)$$

All these parameters have for a RFQ a straightforward meaning; $\omega_0 = 1/\sqrt{LC}$ is the frequency of operation, C is mainly the capacitance per unit length of the quadrupolar vane tips, (of the order of 120pF/m), that can be evaluated with the help of SUPERFISH. Moreover $L_1 / L = \omega_0^2 / c^2$, so as to have the asymptotic phase velocity equal to c. This model can be discretized, keeping the same meaning of the parameters involved, as long as we are only interested in the first modes and the upper cut-off in frequency can be neglected. The resulting dispersion relation is:

$$\left(\frac{\omega_n}{\omega_0}\right)^2 = 1 + \frac{L}{L_1 dz^2} 2(1 - \cos\alpha n)$$

with dz (finite) discretization step and $\alpha = \pi\, dz/\ell$. In the limit of dz→0 this relation reduces to (1), but if one is only interested in the first two modes (n=0,1), one can take $dz = \ell/2$, and the two frequencies correspond to the α=0 and α=π modes by imposing $L_1 / L = 32/\lambda^2$, with $\lambda = 2\pi c/\omega_0$; we shall use this result in the following.

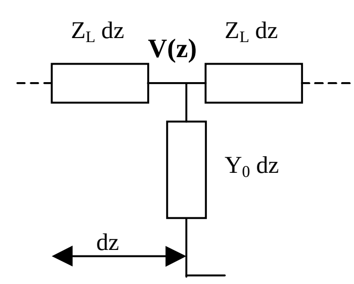

Figure 1: Equivalent circuit for the longitudinal dependence of the quadrupole modes a four vanes RFQ.

In a four vanes RFQ the operating frequency is ω_0, and therefore the field cannot be stabilized as foreseen by the resonant coupling theory, since ω_0 is never between an upper and lower symmetrical frequency[1]. As a consequence some drop of the voltage along the RFQ can be expected in presence of mechanical errors for a long RFQ, since the distance of the operating frequency respect to the first mode frequency is function of λ / ℓ.

A possible way to stabilize the RFQ is suggested by our circuit; we introduce a capacitance every two cells. In other words we cut the transmission line into segments of

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length 2dz, for each segment we consider only the n=0 and 1 modes, and we couple the segments with an impedance $Z_{Ca} = 1/i\omega C_a$. The result is a biperiodic system governed by the equations

$$\begin{cases}(1-x-\beta\gamma x+\gamma)u_n+\beta\gamma x u_{n-1}-\gamma u_{n+1}=0\\(1-x-\beta\gamma x+\gamma)u_m-\gamma u_{m-1}+\beta u_m=0\end{cases}$$

with m=2j+1, n=2j, j integer,

$$\omega_c \equiv \frac{1}{\sqrt{L_1 2dzC_a}},\ \beta \equiv \frac{\omega_c^2}{\omega_0^2},\ \gamma \equiv \frac{2C_a}{Cdz},\ x \equiv \frac{\omega_0^{\ 2}}{\omega^2}.$$

It is interesting to observe that the unknown x appears in the non-diagonal terms and this is therefore a so-called extended eigen value problem. Nevertheless, since we have a lossless chain, one can look for a solution of the form: $u_n = e^{i\alpha}u_{n-2}$. This condition leaves two independent voltages, for example u_n and u_{n+1}, and therefore the recursive relations determine a couple of linear equations that have solution different from zero if:

$$x=\frac{\left[\beta\gamma^2(1-\cos\alpha)+1+\beta\gamma+\gamma\right]\pm\sqrt{\left[\beta\gamma^2(1-\cos\alpha)+1+\beta\gamma+\gamma\right]^2-(1+2\beta\gamma)(1+2\gamma)}}{(1+2\beta\gamma)} \quad (3)$$

This is the new dispersion relation. The two bands, corresponding to the two sign determination of the square root, have even dependence on α, and the fundamental frequency $x=1$, corresponding to $\alpha=0$, is not any more the lowest. Moreover we have the coalescence of the two bands at $\alpha=0$ when the square root vanishes, i.e. for $\beta=1$. Therefore the stabilization of the fundamental mode is realized when $\omega_0=\omega_C$, condition that resembles the coalescence condition for the coupled pillbox biperiodic chains.

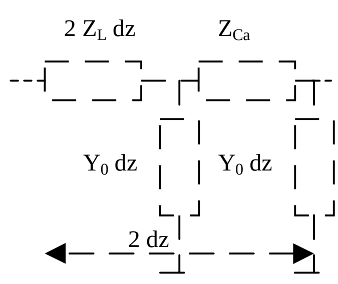

Figure 2: Biperiodic equivalent circuit.

In Fig. 3 we show the numerical solution a chain of 10 resonators (γ=.024, β=1). In the upper part we plot the normalized frequency of the modes, together with the dispersion relation (3) for an infinite chain of resonators; the agreement is good, meaning that in this case the termination conditions do not play an important role. The fundamental mode and the two neighbor modes are plotted in the lower part of Fig 3. These two modes have the same long range dependence, but the oscillation between next cells is in opposite phase. As a consequence, for a small (1%) perturbation in the eigen-frequency of the first resonator, the long range drop of the field is compensated, but the short range oscillation (first mode for each wave-guide segment) is enhanced. In this sense the stabilization is not complete.

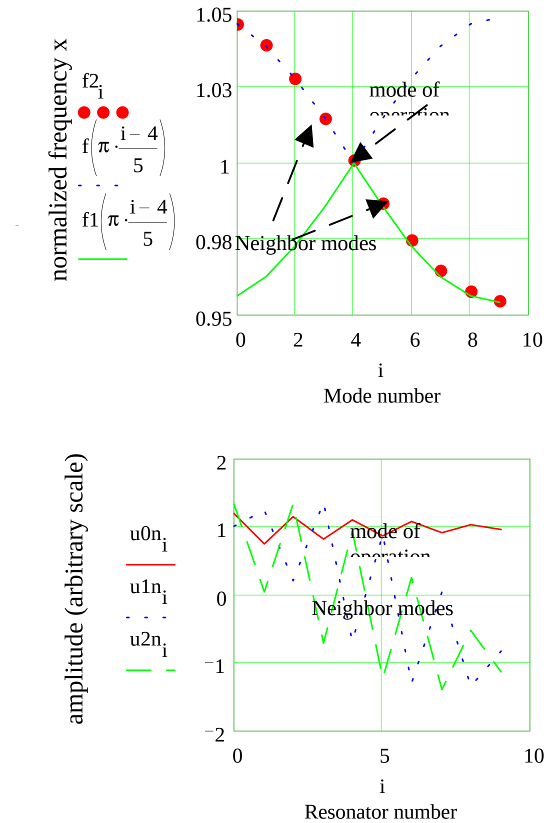

Figure 3: Numerical solution of the equivalent circuit (γ =.024, β =1, 1% perturbation in the frequency of the first resonator) and dispersion relation for an infinite chain of resonators.

2 SCALING FOR A SEGMENTED RFQ

The lumped circuit model of the previous section has been developed so to dimension a 352 MHz RFQ (about 6 m long) using the LANL technique for field stabilization[2]. We just remind that the RFQ is split in segments (of length 2 dz) coupled throw a coupling cell with the geometry shown in the MAFIA plot of fig. 4.

The impedance of this coupling-cell has in general an inductive (L_A) and a capacitive component (C_A), determined by the cross talk of the magnetic fields of the neighbor RFQ throw the opening, and by the capacitance of the facing electrode terminations respectively. Nevertheless for practical range of parameters the capacitive term is the dominating one:

$$Z_a = \frac{1}{i\omega C_A}\left(1-\omega^2 L_A C_A\right) \approx \frac{1}{i\omega C_A}\left(1-\omega_0^{\ 2} L_A C_A\right) = \frac{1}{i\omega C_a}$$

With these assumptions we can get very practical scaling relations. From the resonant coupling condition β=1 we calculate the desired coupling capacitance:

$$C_a = \frac{C}{dz}\frac{\lambda^2}{64} \quad (4)$$

that determines the gap between electrode terminations. Moreover the coupling strength is:

$$\gamma = \frac{1}{64}\frac{\lambda^2}{dz^2} \qquad (5)$$

The relative strength of the geometrical errors that can be corrected (for example the relative error of the local frequency) scales like γ; therefore short RFQ segments give a better correction but a higher coupling capacitance and a shorter gap. A short gap can be a problem because of the construction tolerances even if in the lumped circuit model we saw that the tolerances in β are rather loose, and for example for the case in Fig. 3 β=1.2 still gives significant field stabilization.

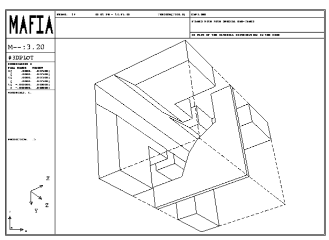

Figure 4: MAFIA simulation geometry in the coupling cell.

3 SIMULATIONS WITH MAFIA

One quarter of the structure has been simulated with MAFIA. Moreover, to reduce the number of mesh points we employed a simplified geometry, six times shorter, that resonates at 5.9 times higher frequency, but preserving the 352 MHz electrode geometry as much as possible (Fig. 4). In this way the “ratio limit”, that is substantially the ratio between the smallest mesh size required (in the gap) and the total structure length, is kept reasonable, and the CPU time required for simulations is affordable.

In Fig. 5 and Fig 6 we show some typical. In Fig 5 the frequencies of the fundamental and of the two neighbor modes are plotted as a function of the gap length. The fundamental mode frequency is of course untouched by the change of capacitance. The equidistance of the spurious modes is reached for a gap of 3.3 mm. With the rough approximation of a planar condenser we get C_a=0.27 pF.

From equation (4), taking into account that for our simplified transverse C=80 pF/m, the capacitance for the coalescence of modes is C_a=0.32 pF. If we consider that the magnetic effects in the coupling cell and the edge electric field due to the finite dimensions of the electrode cross section have been neglected, the 16% agreement achieved is fair.

To test the compensation we have compared the longitudinal behavior of the voltage between the segmented and the uncoupled RFQ in thee presence of a perturbing piston tuner (fig 6).

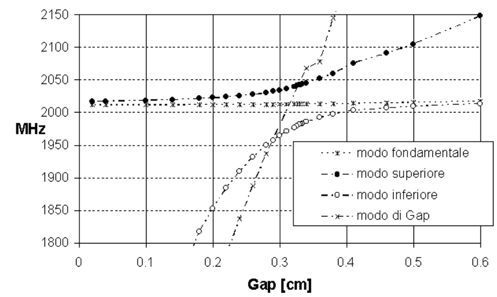

Figure 5: Frequencies of the most relevant modes as a function of gap length.

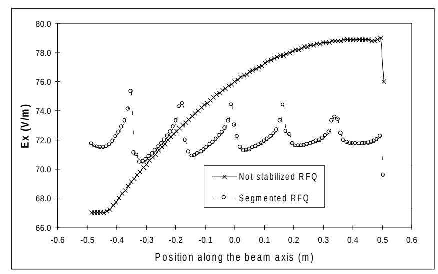

Figure 6: Effect of a perturbation on the RFQ with and without coupling cells. .

4 CONCLUSIONS

We have introduced an equivalent circuit to model an RFQ stabilized with the LANL technique. We think that this rough model catches the essence of the problem, giving the coalescence condition and the pass band amplitude (related to the error correction capability) as a function of few RFQ parameters, namely the operating frequency, the length, the quadrupole capacitance per unit length and the capacitance of the coupling cell. The comparison with MAFIA confirms our scaling laws; however we are not able to test the entire tuning procedure (including the influence of dipole modes) with MAFIA. For these reasons we have built an aluminum model, 3 m long, 352 MHz. The RF measurements are in progress.

5 ACKNOWLEDGMENTS

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