HALO FORMATION IN 3-D BUNCHES WITH DIFFERENT PHASE SPACE DISTRIBUTIONS *

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Abstract

We recently constructed, analytically and numerically, a new class of self-consistent 6-D phase space stationary distributions [1], which allowed us to study the halo development mechanism without being obscured by the effect of beam redistribution. In this paper we consider non-stationary distributions and study how the halo characteristics compare with those obtained using the stationary distribution. In contrast to bunches with a large aspect ratio we find that the effect of coupling between the r and z planes is especially important as the bunch shape becomes more spherical.

1 INTRODUCTION

A realistic treatment of halo formation must take into account 3-D beam bunches and 6-D phase space distributions. Barnard and Lund [2] performed numerical studies with a 3-D beam bunch using the particle-core model, drawing attention to the existence and importance of a longitudinal halo for a spheroidal bunch. However, all studies based on the particle-core model do not address the question of whether halo formation is influenced by the density redistribution which follows for a non-stationary beam, even if it is rms matched (See for example [3]). In fact, halo formation in 2-D due to the redistribution process in rms matched beams was shown, for example, by Okamoto [4] and Jameson [5]. We therefore continued our effort to study the halo development mechanism in 3-D beam bunches in the absence of the redistribution process [1]. Such an approach allowed us to study the fundamental mechanism of halo formation associated with the beam mismatch. To accomplish this we constructed a new class of stationary 6-D phase space distributions for a spheroidal beam bunch [1]. We then explored the formation of longitudinal and transverse halos in 3-D bunches in great detail [1].

Now that we have established the parameters which lead to halo formation in 3-D beam bunches for the 6-D self-consistent phase space distribution, we explore distributions which are not self-consistent, to determine the extent to which the relatively rapid redistribution in the 6-D phase space contributes to the formation of halos. This is the focus of the present paper.

* Work supported by the U.S. Department of Energy

2 NUMERICAL SIMULATIONS

In this paper we compare particle simulations performed for the 6-D stationary distribution given in [1] with the non-stationary 6-D Gaussian distribution, and the non-stationary 6-D uniform distribution. We also consider an axisymmetric beam bunch by putting $a = b$ with a, c being the minor and major semiaxes of our spheroidal bunch, respectively. Both the Gaussian and uniform distributions are constructed in the rms matched sense.

2.1 Stability of the Matched Distribution

Both numerical studies of the unstable modes and multi-particle simulations for the 2-D breathing KV beam with zero mismatch confirmed that the beam is unstable for tune depressions below $\eta = 0.4$ [6]. However, no halo was observed in the corresponding 2-D simulations. Similar studies for other 2-D rms matched distributions which are not stationary solutions of the Vlasov equation showed the existence of a halo for severe tune depression and zero mismatch [4]. The existence of a halo for such rms matched distributions was attributed to the unavoidable plasma oscillations generated by the initial density-redistribution process which is clearly shown in [4].

In our recent 3-D simulations [1] with the stationary distribution no such redistribution occurred. However, for the Gaussian distribution one can see the strong redistribution process which occurs very quickly in both the transverse and longitudinal planes. In contrast to the 2-D simulations [4] this redistribution process happens for both modest and severe space charge. In Fig. 1 we plot the maximum x and z among the million particles in our run for severe ($\eta_z = 0.27, \eta_x = 0.38$) and modest ($\eta_z = 0.65, \eta_x = 0.75$) tune depressions, respectively. Figure 2 shows the phase space $z - p_z$ diagram for $\eta_z = 0.27, \eta_x = 0.38$ without and with a low-density cut [1] which enables us to observe the halo structure clearly. Similarly, one can see the redistribution process for the uniform distribution. One again finds halo formation for both modest and severe space charge [7]. Thus, we have found that an rms matched 3-D beam can produce transverse and/or longitudinal halos (of relatively small extent) for a wide range of space charge intensity even when it is initially perfectly matched. Of course, from a practical point of view such halos are not important because the halo extent is very small for the mismatch factor $\mu = 1.0$. The important consequence is that the redistribution process by itself (zero initial rms mismatch) does not lead to significant emittance growth.

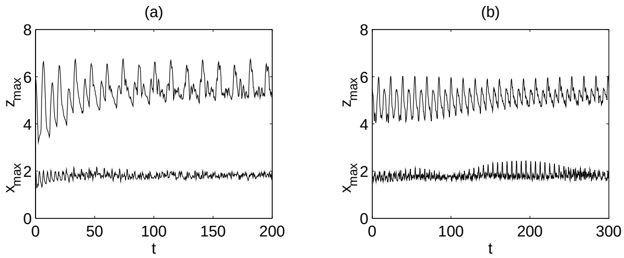

Figure 1: Maximum x and z as a function of time for initially matched beam $\mu_x = \mu_y = \mu_z = 1.0$ with 6-D Gaussian distribution ($c/a = 3$) a) $\eta_x = 0.38$, $\eta_z = 0.27$ b) $\eta_x = 0.75$, $\eta_z = 0.65$.

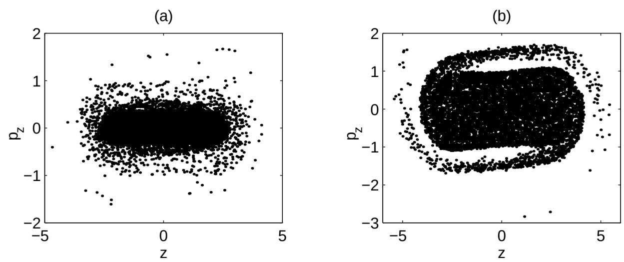

Figure 2: Longitudinal phase space diagram at $t = 50$ of initially matched beam $\mu_x = \mu_y = \mu_z = 1.0$ with 6-D Gaussian distribution ($c/a = 3$, $\eta_x = 0.38$, $\eta_z = 0.27$) a) without low-density cut (with 32,768 particles plotted) b) with low-density cut (with 20,000 particles plotted).

2.2 Initially Mismatched Beam

Numerical 3-D simulations with the initially mismatched non-stationary distributions described above confirm all the characteristics of halos observed for the stationary distribution [1]. The main difference is that for a non-stationary distribution the halo extent is larger (especially for the Gaussian) than the halo extent of the stationary distribution with the same initial mismatch parameters. As an example, in Fig. 3 we show the maximum x, z, emittance growth, $z - p_z$ diagram without the low-density cut and $r - p_r$ diagram (with angular momentum $|L_z| < 0.1$ to make the "peanut" diagram relatively clear) with initial $\mu_x = \mu_y = \mu_z = 1.5$ for the uniform distributions. The 6-D stationary distribution constructed in [1] gives a picture of halo development almost identical to the uniform non-stationary distribution except for a slight difference in the halo extent.

A systematic study for bunches of different shape (c/a) and mismatch factor μ (with simultaneous mismatch in all planes) was presented recently [1]. Below we present some examples of the mismatch in the transverse plane only. To compare our results with those available for a transverse

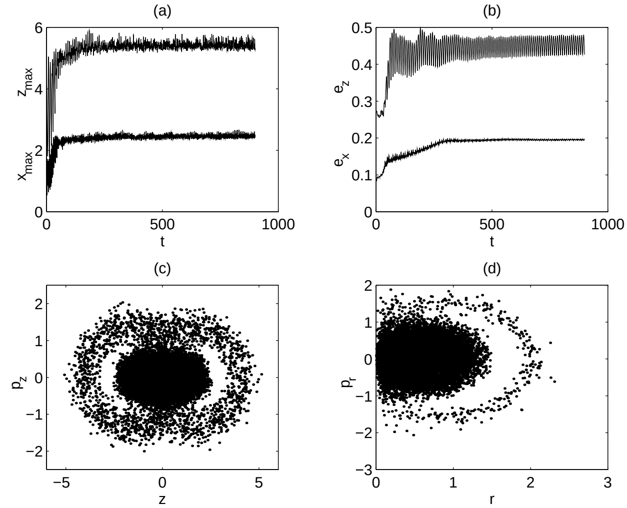

Figure 3: 6-D uniform distribution $\mu_x = \mu_y = \mu_z = 1.5$ ($c/a = 3$, $\eta_x = 0.53$, $\eta_z = 0.39$) a) maximum x and z b) emittance growth c) $z - p_z$ diagram at $t = 900$ (with 32,768 particles plotted) d) $r - p_r$ diagram at $t = 900$ for particles with the angular momentum $|L_z| < 0.1$ (with 25,000 particles plotted).

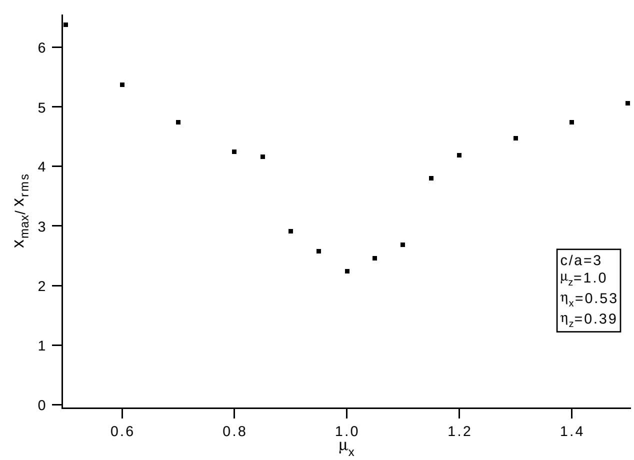

Figure 4: Extent of the transverse halo for the 6-D stationary distribution with zero longitudinal mismatch $\mu_z = 1.0$ ($c/a = 3$, $\eta_x = 0.53$, $\eta_z = 0.39$).

halo [8] we show in Fig. 4 the dependence of the transverse halo extent on the mismatch for fixed space charge, with tune depressions $\eta_z = 0.39$, $\eta_x = 0.53$. The main difference is the behavior near $\mu = 1.0$ which clearly shows the existence of threshold for halo formation in beams with stationary distributions. Similar behavior exists for the longitudinal halo [7] (the extent of the longitudinal halo is smaller than that of the transverse halo).

However, the existence of a threshold for halo formation observed in 2-D simulations and then confirmed by our 3-D particle simulations turns out to be a feature only observed for self-consistent stationary distributions. For example, Fig. 5 for the 6-D uniform non-stationary distribution has

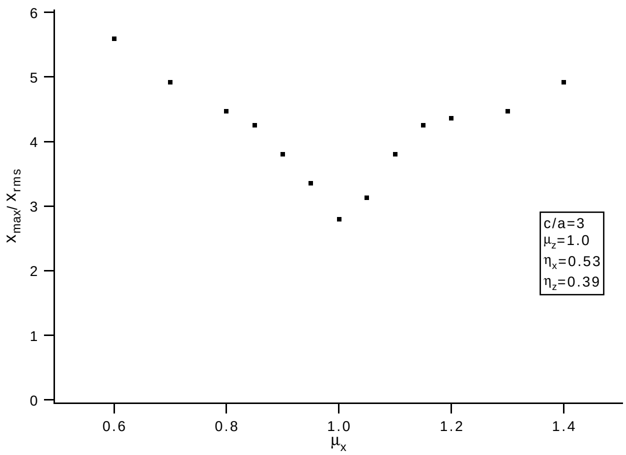

Figure 5: Extent of the transverse halo for the 6-D uniform distribution with zero longitudinal mismatch $\mu_z = 1.0$ ($c/a = 3, \eta_x = 0.53, \eta_z = 0.39$).

no obvious threshold behavior. In fact, we showed above that, in a non-stationary beam, a halo can form even for a zero initial mismatch. Therefore, in a "real" beam which goes through a redistribution and relaxation process one should not expect a threshold for halo formation due to a mismatch.

2.3 Coupling Effects

In performing 3-D simulations we encounter halo formation in a beam bunch, where we clearly see coupling between the longitudinal and transverse motion. It was already noted [1] that due to the coupling between r and z, a transverse or longitudinal halo is observed even for a very small mismatch (less than 10%) as long as there is a significant mismatch in the other plane. Further numerical investigation of this question showed that the effect of coupling becomes extremely important for nearly spherical bunches ($c/a \leq 2$) which is typical of the parameter range of interest for the APT design [9]. For example, for the short bunch with $c/a = 2$, with only a longitudinal initial mismatch ($\mu_z = 1.5$, $\mu_x = \mu_y = 1.0$), one finds particles at large amplitude in both the longitudinal and transverse directions, as can be seen in Fig. 6 for the 6-D stationary distribution. Of course, the intensity of particles in the transverse halo is much smaller than it is when there is in addition a transverse initial mismatch. (In our example in Fig. 6, we have 0.05 percent of the particles in the transverse halo with zero transverse mismatch compared with several percent in the longitudinal halo.) A similar effect due to coupling was seen for the non-stationary distributions.

3 SUMMARY

Recently we constructed, analytically and numerically, a new class of 6-D phase space stationary distributions for an azimuthally symmetric beam bunch which allowed us to study the halo development mechanism in 3-D beam bunches where no phase space redistribution occurs. After we established the parameters which lead to halo formation in 3-D beam bunches for the self-consistent 6-D phase space stationary distribution [1], in this paper we explored rms matched distributions which are *not* self-consistent, to determine the extent to which the relatively rapid redistribution of the 6-D phase space contributes to the formation of halos. We also found that the effect of coupling between the r and z planes is very important in the halo development mechanism and can lead to serious consequencies, especially for a very short beam bunch.

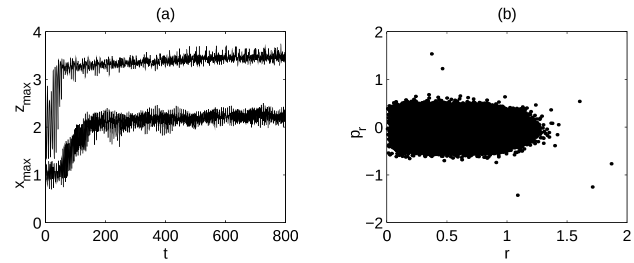

Figure 6: Coupling effect for the 6-D stationary distribution with zero transverse mismatch $\mu_x = \mu_y = 1.0$, $\mu_z = 1.5$ ($c/a = 2$, $\eta_x = 0.55$, $\eta_z = 0.45$) a) maximum x and z b) $r - p_r$ diagram at $t = 800$ for particles with the angular momentum $|L_z| < 0.1$ (with 25,000 particles plotted).

4 ACKNOWLEDGMENT

We thank Tom Wangler for helpful conversations. In addition RLG and AVF wish to thank Andy Jason and the LANSCE1 group for its hospitality during part of these studies.

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