MULTIPLE COUPLING AND BEAM LOADING OF A RF CAVITY

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Abstract
Future high power accelerators are aiming at increasing the power transferred to the beam through radiofrequency (RF) cavities. Consequently, multiple main coupler drives may be required to withstand the high RF power needed per cavity. An analysis including multiple couplers and beam loading is described here, featuring the interaction between all couplers, the beam and the cavity. In this description, the beam is shown to act as if it were merely an additional drive for the cavity. Special focus is given to the case of superconducting cavities in a continuous (CW) operation mode. Some important conclusions derived from this analysis are briefly discussed as, for example, power coupler fault conditions or a way to handle the commissioning of an accelerator in a CW mode.

1 INTRODUCTION

The superconducting radiofrequency APT (Accelerator Production of Tritium) cavities will be driven simultaneously by two power couplers. Consequently, an analysis involving multiple couplers will be needed to understand the behavior of the interaction between the beam, the cavity and all the couplers, extending the beam loading introduction given in reference [**1**].

2 CAVITY, BEAM AND COUPLERS

Standard notations will be used to define the cavity parameters. If W is the stored energy, P_c the dissipated power and ω_0 the resonance pulsation ($\omega_0 = 2\pi F_0$), then the shunt impedance R and the quality factor Q_0 are defined by $P_c = |V_c|^2 / 2R$ and $Q_0 = \omega_0 \tau_0 = \omega_0 W / P_c$, $\tau_0 = Q_0/\omega_0$ is the filling time of the cavity. A superconducting cavity will have a very high R ($10^{12}\ \Omega$) compared to a normal copper cavity ($5.10^6\ \Omega$). This reduction of losses by more than 10^5 will increase the cavity filling time and the available beam power by the same amount. The cavity voltage V_c is arbitrarily defined through the effective length l of the cavity and the effective accelerating field E_{acc} :

$$V_c = E_{acc}\, l \tag{1}$$

The cavity impedance is

$$Z_0 = \frac{R}{1 + jQ_0\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)} \cong \frac{R}{1 + jQ_0 \frac{2\Delta\omega}{\omega_0}} \tag{2}$$

If the injected time of a particle passing through the cavity is shifted by $\delta t = \phi/\omega$ as compared to the maximum energy gain, then the accelerating voltage will be

$$V_{acc} = V_c \cos\phi \tag{3}$$

It can be shown that the induced voltage of a crossing particle with charge q on each mode of the cavity will simply reduce to $V_{ind} = -q\,(R/Q)\,\omega_0$ and the corresponding energy deposited $W = (R/Q)\omega_0 q^2/2$ will decay after the particle had passed with the time constant τ_0. Therefore, in a steady state regime, a current beam I_b will induce a total voltage $V_0 = -R\, I_b$. The induced voltage is always decelerating the particles. The beam can then be merely seen as a drive line having a fixed current source I_b with an equivalent voltage V_0 and will therefore be labeled drive 0 from now on. Let us now consider many RF lines (labeled 1,2,k) with characteristic impedances Z_{ck} coupled to the cavity through a coupling factor β_k. The external Q of each line k is defined as $Q_k = Q_0/\beta_k$ and the loaded quality factor Q_l results from $Q_l^{-1} = (\sum Q_k^{-1})$. Introducing the alpha coefficients $\alpha_k = Q_l/Q_k = \alpha_0 \beta_k$, these will always be between 0 and 1[a], and obey the closure relation $\sum_k \alpha_k = 1$. The cavity detuning angle ψ is, by definition, $\tan\psi = -Q_l\,(2\Delta\omega/\omega_0)$. Each drive k will see the cavity impedance Z_0 in parallel with all the other drive impedances transformed to the cavity (R/β_m) through its own coupling coefficient β_k (Fig. 1). Consequently, drive k will see the overall impedance :

$$Z = \left(\frac{\beta_k Z_{ck}}{R}\right) \frac{1}{\frac{1}{Z_0} + \sum_{m \neq 0,k} \frac{\beta_m}{R}} = \frac{\alpha_k Z_{ck}}{(1-\alpha_k) - j\tan\psi} \tag{4}$$

Therefore, the reflection coefficient ρ_k of drive k will be :

$$\rho_k = 2\alpha_k \cos\psi\, e^{j\psi} - 1 \tag{5}$$

In order to get the different RF powers traveling on each line, we will use the incident and reflected voltages normalized to their relative coupling coefficient, that is $\left(V_{ik}/\sqrt{\beta_k}\right)$ and $\left(V_{rk}/\sqrt{\beta_k}\right)$. Each normalized drive should exhibit the same voltage $\frac{(V_{ik} + V_{rk})}{\sqrt{\alpha_k}} = \text{constant}$. The cavity voltage is $V_c = V_{i\,0} + V_{r\,0}$, equal to the sum of all the induced voltages from each drive :

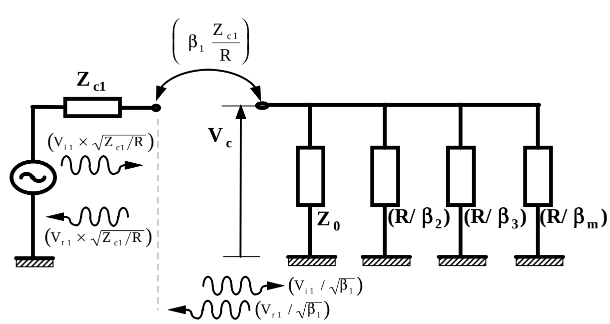

Figure 1. Impedance transform as seen from drive 1.

[a] This is an advantage over the use of the β (those can be infinite) as it significantly reduces computation errors.

$$\frac{V_c}{\sqrt{\alpha_0}} = \sum_m (1+\rho_m)\frac{V_{im}}{\sqrt{\alpha_m}} = \frac{(V_{ik}+V_{rk})}{\sqrt{\alpha_k}} \qquad (6)$$

The above basic equations (6) will give all the return voltages including the cavity one. Once the incident powers (with their phases) and the coupling coefficients α_k are set, all the reflected powers in each line, the beam power and the cavity accelerating field may be deduced.

3 EXAMPLE OF TWO COUPLERS

Let us look more deeply at the case where two couplers are driving a single cavity. If the injected powers are P_1 and P_2 with phases θ_1 and θ_2, then, the corresponding normalized incident voltages are :

$$\begin{cases} V_{i1} = \sqrt{2RP_1}\, e^{j\theta_1} \\ V_{i2} = \sqrt{2RP_2}\, e^{j\theta_2} \end{cases} \qquad (7)$$

And the reflected voltages derived from equations (6) :

$$\begin{cases} \frac{|V_c|e^{j\phi}}{\sqrt{\alpha_0}} = 2\alpha_0\cos\psi e^{j\psi}\frac{|V_{i0}|e^{j\theta_0}}{\sqrt{\alpha_0}} + 2\alpha_1\cos\psi e^{j\psi}\frac{|V_{i1}|e^{j\theta_1}}{\sqrt{\alpha_1}} + 2\alpha_2\cos\psi e^{j\psi}\frac{|V_{i2}|e^{j\theta_2}}{\sqrt{\alpha_2}} \\ \frac{V_{r1}}{\sqrt{\alpha_1}} = 2\alpha_0\cos\psi e^{j\psi}\frac{|V_{i0}|e^{j\theta_0}}{\sqrt{\alpha_0}} + \left(2\alpha_1\cos\psi e^{j\psi}-1\right)\frac{|V_{i1}|e^{j\theta_1}}{\sqrt{\alpha_1}} + 2\alpha_2\cos\psi e^{j\psi}\frac{|V_{i2}|e^{j\theta_2}}{\sqrt{\alpha_2}} \\ \frac{V_{r2}}{\sqrt{\alpha_2}} = 2\alpha_0\cos\psi e^{j\psi}\frac{|V_{i0}|e^{j\theta_0}}{\sqrt{\alpha_0}} + 2\alpha_1\cos\psi e^{j\psi}\frac{|V_{i1}|e^{j\theta_1}}{\sqrt{\alpha_1}} + \left(2\alpha_2\cos\psi e^{j\psi}-1\right)\frac{|V_{i2}|e^{j\theta_2}}{\sqrt{\alpha_2}} \end{cases} \qquad (8)$$

The cavity voltage phase is, like in equation (3), noted ϕ, the reference phase taken opposite to the beam induced voltage, $V_{i0} = -R\,I_b$ with $\theta_0=\pi$. The cavity power is $P_c = |V_c|^2/2\,R$, the beam power $P_b = \mathrm{Re}(V_c I_b^*) = |V_c| I_b \cos\phi$, and $\sum_k (P_{ik}-P_{rk}) = \sum_{k\neq 0}(P_{ik}-P_{rk}) - (P_c+P_b) = 0$ show that power is conserved. The reflected voltages are :

$$V_{rk} = \sqrt{\beta_k}\, V_c - V_{ik} \qquad (9)$$

To minimize the generator power, all $V_{r\,k}$ should be set to zero. That imposes to have for any line k :

$$\begin{cases} \theta_k = \phi \\ V_{ik} = \sqrt{\beta_k}\, V_c \end{cases} \qquad (10)$$

And the external Q of each line have to be chosen so that

$$Q_k = \frac{P_c}{P_{ik}} Q_0 = \frac{|V_c|^2}{2(R/Q)P_{ik}} \qquad (11)$$

A specific relation follows between the detuning angle ψ of the cavity and the beam phase ϕ :

$$\tan\psi = -(1-2\alpha_0)\tan\phi \qquad (12)$$

In particular, if ($\alpha_0 = 1/2$), corresponding to the case of a matched single coupler ($Q_1 = Q_0$), then $\psi = 0$ for any phase ϕ. Whereas if ($\alpha_0 \ll 1$), general case for running a superconducting cavity with beam ($Q_0 \gg Q_1$), then $\psi \approx -\phi$. The cavity will have to be detuned by an angle equal to the opposite of the beam phase. For the APT cavities, where the beam phase is (- 30 degrees), this corresponds to a frequency shift of 947 Hz off resonance.

4 SENSITIVITIES

In order to match the beam, external couplings have to be adjusted using (11), powers and phases using (10), and detuning frequency using (12). Correlatively, the cavity voltage V_c and the phase beam ϕ will be the desired ones. Table I gives the sensitivity of each parameter.

Parameter	**Nominal Values**	**Drive Phase**	**Drive Power**	**External Q**	**Beam Current**
Variation	-	$\theta_1 = \theta_{1ref}$ + - 10°	$P_1=P_{1ref}$ (1+ -10%)	$Q_1=Q_{1ref}$ (1+ -10%)	$I_b=I_{bref}$ (1+ -10%)
P_1 Reflected (W)	0	1500	119	472	2097
P_2 Reflected (W)	0	4872	393	0	2097
Beam Power (kW)	420	414	420 + - 20	420	416
Phase Angle Φ (°)	-30	-30 + - 7,7	-30 - + 1,15	-30 + - 0,02	-30 + - 5,1
Energy Gain (MeV)	4,20	4,14	4,20 + - 0,20	4,20	4,20 - + 0,42

Table I - Sensitivities of reflected powers, beam power, phase angle and energy gain on various parameters.

One can notice the importance of the drive phases. If the maximum allowable reflected power is for example 1‰ (200 W), they should be set equal to the phase angle ϕ to better than 2 degrees. Whereas the drive power, the external Q or the beam current may vary by as much as 6.3% for the same amount of reflection. Note the negligible effect of an external Q variation on ϕ. As a matter of fact, one can even get by without any adjustable coupling. Coupling can be pre-adjusted for each cavity, taking in account the variation of (R/Q) along the linac. Fixed coupling has the advantage of simplicity, reliability, and can avoid unwanted failures. On the other hand, adjustable coupling may reduce the total AC power consumption of the accelerator when operated at reduced current (at half current of 50 mA, a fixed coupling would require 12%[b] more RF power than an adjustable coupler).

5 FAULT SCENARIOS

5.1 Beam Failure

If the beam trips, then the couplers will have to withstand almost full reflection. The accelerating energy gain will exactly double in the cavity. The energy increase will rise with the time constant $\tau = \alpha_0\tau_0$ (46 µs for the 0.82 cavity). The amplitude control loop of the cavity should correct and bring back the voltage to its nominal value. This RF control feedback has generally a bandwidth exceeding 10 kHz and should easily compensate for the field variation. Therefore, the couplers will see the full 210 kW reflection only for about tens of microseconds, while hundred of microseconds after beam failure, the RF drive should be set back to 105 kW per coupler (again with full reflection). By that time, the RF drive can be shut down. Consequently, beam failure has no severe impact, provided RF control ensure that the cavity field would not exceed a given level.

5.2 Cavity Failure

The cavity may fail in many ways. It may experience a sudden change in its resonance frequency (due to mechanical stress or pressure), it may leak, it may exhibit field emission and finally it may quench. In the case of a quench, we are back to the above discussion during beam trip and that should be handled by a quench interlock. A

[b] This reduces to 9% for the whole accelerator taking in account the warm section, the cryoplant load and klystron efficiency.

shift in the resonance frequency would mainly impact the phase and should be corrected by the phase loop. If field emission occurs (inducing excessive Q losses and/or high X-ray levels), then the impact should be minimal. No effect will be seen on the couplers. The Q losses will induce higher cryogenic losses and the X-ray level can be monitored. In either case, the cavity field can be reduced accordingly to an acceptable level without having to react promptly by shutting down the RF or the beam (Here, one may allow a slow reaction time). Finally, in the case of a leak, a vacuum interlock should be set.

5.3 RF Power Failure

The RF system failure may concern power supply, klystron, circulator, divider or waveguide. In any case, the klystron is turned off. Cavity voltage will then be exactly opposite to the one without beam, the beam inducing exactly as much power as what it has been designed to receive. The required action is to detune the cavity far enough from resonance, reducing the beam induced voltage. A detuning frequency of 10 bandwidths (34 kHz) will reduce the beam power loss to less than 0.65% (1.37 kW).

5.4 Coupler Failure

The coupler might fail either because of a window problem (ceramic cracking, multipacting, breakdown, ...) or because of the coaxial part itself (breakdown, leaks, cooling, high dissipation, multipacting, ...). A coupler failure might lead to a serious failure (case of a leak, for example) where the beam and RF should be shut down and the cavity isolated.

6 ACCELERATOR COMMISSIONING

During commissioning, one might use either a pulsed beam and increase the duty cycle, or start at a very low CW current slowly increasing to reach its nominal value. Let us examine this latter case. In order to get the same accelerating energy and the same phase angle ϕ in the cavity, equations (8) give

$$|V_c|e^{-j\psi} = -2\alpha_0 \cos\psi R I_b e^{-j\phi} + 2\sqrt{\alpha_0\alpha_1}\cos\psi |V_{i1}| + 2\sqrt{\alpha_0\alpha_2}\cos\psi |V_{i2}| \quad (13)$$

If one recalls that the nominal value of cavity voltage for the nominal beam current I_{bnom} is

$$V_c = 2\alpha_0 \cos\phi \, (R I_{bnom}) \quad (14)$$

then, taking the imaginary part of (13) leads to the relation $\tan\psi = -(I_b/I_{bnom})\tan\phi$ which is equivalent to :

$$\left(\frac{\Delta F}{F}\right) = \frac{\tan\phi}{2Q_l}\left(\frac{I_b}{I_{bnom}}\right) \quad (15)$$

Therefore, the cavity frequency shift should be kept proportional to the current beam.

Taking the real part of (13) and using (14) with the nominal input voltages given by (10), one gets

$$2\left(\alpha_1\left|\frac{V_{i1}}{V_{i1nom}}\right| + \alpha_2\left|\frac{V_{i2}}{V_{i2nom}}\right|\right) = \left(1+\frac{I_b}{I_{bnom}}\right) \quad (16)$$

which, considering that the two ratios are identical, gives the input powers on each coupler

$$\left(\frac{P_{ik}}{P_{ik\,nom}}\right) = \frac{1}{4(1-\alpha_0)^2}\left(1+\frac{I_b}{I_{bnom}}\right)^2 \quad (17)$$

In the case of the superconducting cavities, α_0 is very small and can be neglected. The input power required to achieve the same accelerating field at zero current beam is one fourth of the full power at nominal beam. Of course, almost all this power is then fully reflected. If the cavity frequency is changed according to (15) (linearly with beam current) and the input power according to (17) (quadratic with current), everything should remain stable while increasing the beam current. This should provide an easy way for commissioning the accelerator.

7 TRANSIENT ANALYSIS

In a transient mode, the use of the Laplace transform will make all the above relations, established for a steady state regime, straightforwardly extended. The Laplace transform of the cavity impedance is $Z_0 = R\dfrac{p}{p^2 + \dfrac{p}{\tau_0} + \omega_0^2}$

which gives, once transformed through each drive k :

$$Z = \alpha_k Z_{ck} \frac{p/\tau}{p^2 + (1-\alpha_k)\dfrac{p}{\tau} + \omega_0^2} \quad (18)$$

with the time constant $\tau = \alpha_0\tau_0$.

The reflection coefficient is like in equation (5)

$$(\rho_k + 1) = \frac{2\alpha_k (p/\tau)}{p^2 + \dfrac{p}{\tau} + \omega_0^2} \quad (19)$$

The basic equations (6) are still valid in the Laplace transform. In particular, using (6) and (19), one may write the cavity voltage as

$$V_c(p) = \frac{(p/\tau)}{\left(p^2 + \dfrac{p}{\tau} + \omega_0^2\right)}\left(\sum_k 2\sqrt{\alpha_0\alpha_k}\, V_{ik}\right) \quad (20)$$

This function has two poles, solution of $\left(p^2 + \dfrac{p}{\tau} + \omega_0^2\right) = 0$, which are $p_\pm = -\dfrac{1}{2\tau} \pm j\omega_0\sqrt{\left(1-\dfrac{1}{4Q_l^2}\right)} \approx -\dfrac{1}{2\tau} \pm j\omega_0$. The cavity voltage as a function of time is then deduced by inverting the Laplace transform $V_c(t) = \int e^{pt} V_c(p)\,dp$:

$$V_c(t) = \left(A p_+ e^{p_+ t} + B p_- e^{p_- t}\right) + \left(\sum_k 2\sqrt{\alpha_0\alpha_k}\, V_{ik}\right)\cos\psi\, e^{j\psi} e^{j\omega t} \quad (21)$$

The first bracket is the transient one, with the time constant (2τ). The second term is the stationary solution. Finally, using the simplification Q_l » 1, the solution is :

$$V_c(t) = V_c(0) e^{-\frac{t}{2\tau}} e^{j\omega_0 t} + \left(\sum_k 2\sqrt{\alpha_0\alpha_k}\, V_{ik}\right)\cos\psi\, e^{j\psi}\left(e^{j\omega t} - e^{-\frac{t}{2\tau}} e^{j\omega_0 t}\right) \quad (22)$$

References

[1] P. B. Wilson, “High Energy Electron Linacs: Applications to Storage Ring RF Systems and Linear Colliders”, SLAC-PUB-2884, February 1982.