APPLICATION OF THE MODE MATCHING TECHNIQUE TO THE ANALYSIS OF WAVEGUIDE ARRAYS

A. Jöstingmeier, M. Dohlus and N. Holtkamp, DESY, D-22607 Hamburg, Germany

Abstract

In this contribution the mode matching technique is applied to compute the absorption characteristics of a two-dimensional array of rectangular waveguides. This analysis is motivated by a proposal of a broadband absorber for TESLA. Outside the waveguide array the so-called Rayleigh expansion is used which represents the electromagnetic field in terms of space harmonics whereas the complete modal spectrum of eigenmodes is taken into account inside the waveguides. In the case of normal incidence the validity of the presented method is confirmed by MAFIA computations. The absorption characteristics for various angles of incidence are calculated for a broad frequency range. The accuracy of the results is checked by a study of convergence. It is shown that the Rayleigh expansion has to be modified if the waveguide array is excited by an ultra-relativistic beam instead of an incoming plane wave. Numerical results for the beam parameters are presented for an array of parallel-plate waveguides and compared with those obtained by other methods.

1 INTRODUCTION

A HOM (higher order modes) absorber for TESLA has been proposed in [1] which consists of an array of rectangular waveguides surrounding the beampipe (Fig. 1). This absorber is used to extract the HOM in the THz region from the superconducting accelerating structure and to attenuate the extracted fields by the ohmic wall losses of the rectangular waveguides.

For the sake of simplicity we consider instead of the circular waveguide array a two-dimensional infinite planar grating (Fig. 2). The absorption characteristics of this model are expected to be very close to those of the original absorber because the free-space wavelength is much shorter than the curvature of the structure in the relevant frequency range.

Such a grating can be analyzed by the application of the mode matching technique. Above the grating the electromagnetic field is expanded in terms of an infinite series of spatial harmonics, which is known as the Rayleigh expansion; and inside the waveguides the field is represented by the complete spectrum of TE and TM waveguide modes. Matching the waveguide aperture tangential electromagnetic field, yields an infinite algebraic system of equations the unknown of which are the field expansion coefficients [2].

The ratio describing how much of the power of the incoming wave is coupled into the waveguides is denoted as the grating efficiency. It is used to estimate the power absorption properties of the structure. This quantity is calculated for a grating with typical absorber dimensions over a broad frequency range and for various angles of incidence.

If we use a grating as HOM absorber we have to keep in mind that such a structure itself also contributes to the beam impedance. The second part of this paper is therefore dedicated to the computation of the beam parameters of infinite periodic structures.

The mode matching technique is employed to calculate the electromagnetic field which is excited by an ultra-relativistic bunch of particles in the presence of an infinite array of parallel-plate waveguides (Fig. 3). Subsequently the beam parameters are calculated by making use of the results of the field analysis. In [3] it has been demonstrated that the results of this planar model can also be used for a circular configuration if certain scaling laws are taken into account.

The Rayleigh expansion for the field representation above the grating is known to be complete which means that it can be used to represent any kind of pseudo-periodic field. Nevertheless it has to be modified for an ultra-relativistic beam since the phase advance of the exciting

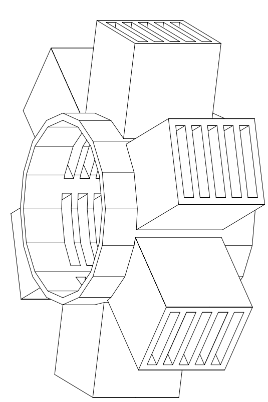

Figure 1: Sketch of the waveguide array absorber.

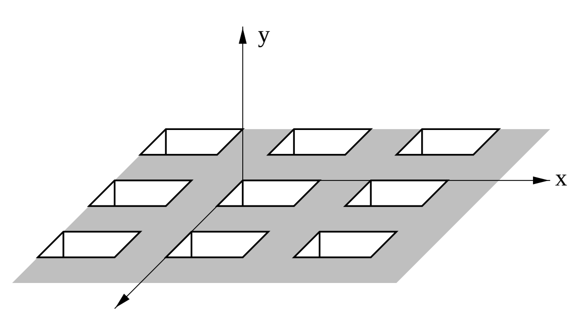

Figure 2: Two-dimensional planar grating.

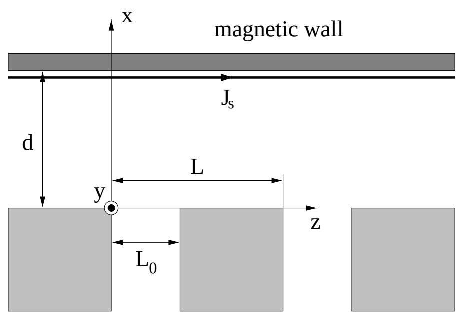

Figure 3: Infinite array of parallel-plate waveguides which is excited by an ultra-relativistic current backed by a magnetic wall.

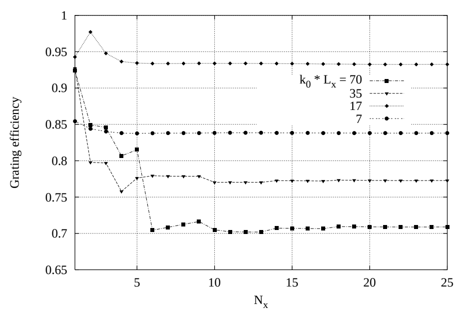

Figure 4: Grating efficiency as a function of the number of field expansion terms for various frequencies.

current is in this case equal to the vacuum wavenumber. Consequently the zeroth order spatial harmonic satisfies the Laplace equation instead of the usual wave equation. Thus we have to use a constant and a linearly increasing function in the x-direction as expansion terms for the y-component of the magnetic field instead of the zeroth order spatial harmonic of the standard Rayleigh expansion.

2 ABSORPTION CHARACTERISTICS OF A RECTANGULAR WAVEGUIDE ARRAY

The mode matching technique leads to an infinite system of equations for the unknown field expansion coefficients which has to be truncated if the method is implemented on a computer. Therefore it is essential to study the convergence of the results with respect to the number of field expansion functions.

Fig. 4 shows the grating efficiency as a function of the maximum order of the spatial harmonics in the x-direction for a grating with typical dimensions of the proposed absorber. The normalized wavenumbers $k_0 L_x = 70$, 35, 17 and 7 correspond to frequencies of approximately 2000, 1000, 500 and 200 GHz, respectively. For a frequency of 200 GHz accurate results are already obtained for $N_x = 5$ whereas $N_x = 20$ is required for a frequency of 2000 GHz. Thus $N_x = 20$ is used for all further calculations which means that approximately a (1500×1500) linear system of equations has to be solved. Assuming this parameter, a typical frequency scan with 1000 points requires about 2 d of cpu-time on a modern workstation.

If the electric field vector of the incoming wave goes along the x- or the z-direction the grating analysis reduces to a waveguide discontinuity problem which can also be solved using the MAFIA computer code [4]. Fig. 5 shows the excitation of the TE_{10}, the TE_{30} and the TE_{50} rectangular waveguide modes as a result of both methods. It is found that in the investigated frequency range from 100 GHz to about 700 GHz the results agree very well.

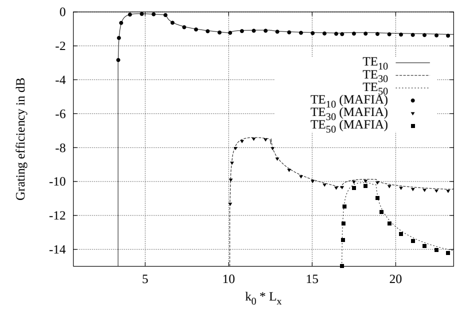

Figure 5: Comparison between the MAFIA computer code and the presented mode matching technique.

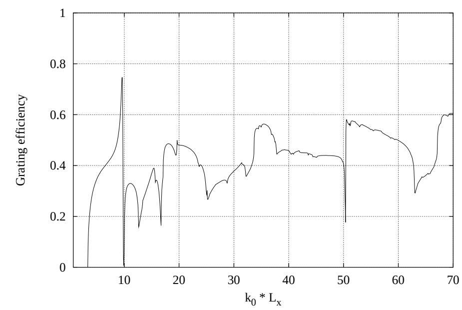

Figure 6: Absorption characteristics for a nearly grazing incident field.

In the case of normal incidence the grating efficiency is greater than 0.6 (except for frequencies which are very close to the cutoff frequency of the fundamental mode of the rectangular waveguide) which means that more than 60% of the power of the incoming wave is extracted by the grating. Nevertheless the grating efficiency decreases for obliquely incident fields. But even if the angle between the direction of propagation of the incoming wave and the grating interface is only $15°$ the average grating efficiency is still about 40% (Fig. 6).

3 BEAM PARAMETERS OF AN INFINITE PERIODIC STRUCTURE

Fig. 7 shows the real part of the beam impedance for a planar grating as a function of frequency. The maximum normalized wavenumber which is $k_0 L = 200$ corresponds to a frequency of about 10 THz for a period length of the grating of 1 mm. The beam impedance is calculated at 32768 frequency points and 200 spatial harmonics are taken into account.

The beam impedance is a smooth function of $k_0 L$ in the frequency range from dc to $k_0 L = \pi$. On the other hand it starts to oscillate rapidly at for higher frequencies where the first higher order spatial harmonic turns from evanescent to propagating with respect to the x-direction. This leads to resonances between the magnetic wall and

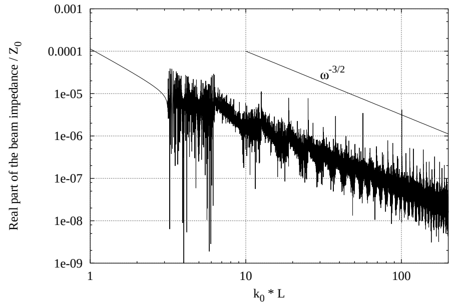

Figure 7: Real part of the beam impedance for a planar grating as a function of frequency. Parameters: $L = 4/15$ mm, $L_0 = 0.5L$ and $d = 35$ mm.

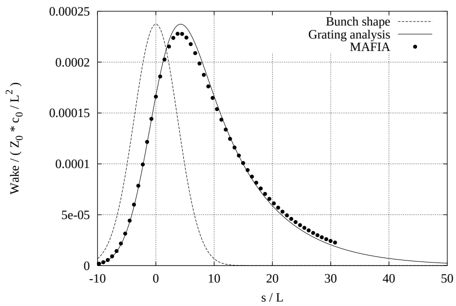

Figure 8: Comparison of the wakefields between the presented mode matching analysis and the MAFIA computer code. Parameters: $L = 4/15$ mm, $L_0 = 0.5L$, $a = 5$ mm and $\sigma = 1$ mm.

the grating interface with a high spectral density because $d = 35\,\mathrm{mm} \gg L = 4/15$ mm.

In Fig. 7 a logarithmic scale is used for both axis. Thus a curve which is proportional to $\omega^{-3/2}$ corresponds to a straight line with a slope of $-3/2$ which is also given in this Fig. From the two curves it can be concluded that the averaged beam impedance also drops as $\omega^{-3/2}$ which has already been shown in [5].

In Fig. 8 the wakefields corresponding to the presented mode matching analysis and the MAFIA computer code [4] are compared. For the MAFIA calculations a corrugated circular beampipe with a length of 200 periods is assumed. The wakefield corresponding to the mode matching technique is obtained by scaling the result from the equivalent planar model. The agreement of the two wakefields is quite good which confirms the validity of the presented method.

The dependence of the wakefield on the bunch length is illustrated in Fig. 9. The results which are presented in this Fig. are valid for a circular configuration with a radius a. The curves converge to an asymptotic wakefield corresponding to an infinitely small bunch length. For the given parameters the wakefield gets very close to the asymptotic curve ($\sigma/L = 0.1$) for a bunch length in the order of one grating period. Such an asymptotic wakefield has also been used in [6] where it is approximated by a special fit.

4 CONCLUSIONS

The mode matching technique has been applied for the analysis of a two-dimensional array of rectangular waveguides which serves as a model for a HOM absorber. A

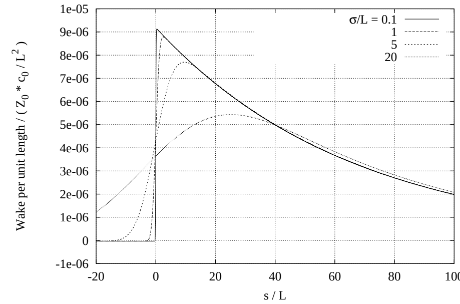

Figure 9: Dependence of the wakefield on the bunch length. Parameters: $L_0 = 0.5L$, $a = 262.5L$.

detailed study of convergence has been carried out in order to demonstrate the accuracy of the presented method. Furthermore the validity of the results has been checked by comparing the excitation of the rectangular waveguide modes with corresponding numbers from MAFIA computations for the special case of normal incidence. The analysis of a grating with typical absorber dimensions has shown that the average grating efficiency is quite high. Although this quantity decreases as we approach the case of grazing incidence the overall absorption properties of such a grating seem to be acceptable. In the second part of this contribution the mode matching technique has been applied to compute the electromagnetic field excited by a bunch of ultra-relativisitic particles traversing a planar grating. It has been shown that the standard Rayleigh expansion which is usually used to represent the field above the grating has to be modified in this case. The beam parameters have been calculated for various structures; and the validity of the presented method has been checked.

5 REFERENCES

[1] A. Jöstingmeier, M. Dohlus, N. Holtkamp and M. Shahabadi, "Computation of the absorption characteristics of a two-dimensional rectangular waveguide array using the mode matching technique", will be published as TESLA report, 1998.

[2] A. Wexler, "Solution of waveguide discontinuities by modal analysis", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-15, pp. 508–517, 1967.

[3] A. Jöstingmeier, M. Dohlus, N. Holtkamp and M. Shahabadi, "Application of the mode matching technique for the computation of the beam parameters of an infinite periodic structure", will be published as TESLA report, 1998.

[4] T. Weiland, "On the numerical solution of Maxwell's equations and applications in the field of accelerator physics", *Particle Accelerators*, vol. 15, pp. 245–292, 1984.

[5] S. A. Heifets and S. A. Keifets, "High-frequency limit of the longitudinal impedance of an array of cavities," *Phys. Rev. D*, vol. 39, pp. 961–970, 1989.

[6] R. Brinkmann *et al.* (ed.), *Conceptual design of a 500 GeV e^+e^- linear collider with integrated X-ray laser facility*, DESY 1997-048, 1997.