

# SIMULATION OF THE TRANSITIONAL PROCESS IN ACCELERATING SECTIONS BY EQUIVALENT CIRCUIT METHOD

S. V. Matsievskiy\*, V. I. Kaminskiy, Ya. V. Shashkov  
National Research Nuclear University MEPhI, Moscow, Russia

## Abstract

Nowadays linac accelerating RF systems design is usually done by the finite difference method. It provides high accuracy of calculations and freedom in topology choosing, but may draw considerable amounts of computer resources with long calculation times. Alternative to this method, equivalent circuit method exists. The basic idea of this method is to build a lumped element circuit, which with certain approximation acts as an original accelerating cell. It drastically reduces the number of equations to solve. This method is long known but usually only used for the particular accelerating structures when speed of calculation is a key-factor. Present paper describes a way to numerically simulate transition processes in arbitrary coupled accelerating cells using the equivalent circuit method. This approach allows simulating transitional processes in accelerating structures significantly faster and allows doing so for structures with high quality factor and many cells – a hard task for conventional transient solvers based on the finite difference method.

## Equations

Each circuit contour is described by the Kirchhoff Equation (1).

$$L \frac{di}{dt} + \frac{1}{C} \int i dt + (R + R_G + R_L)i + \sum_{n=1}^N \frac{1}{C_n^{coup}} \int (i - i_n) dt + \sum_{n=1}^N M_n \frac{di_n}{dt} = e_n \quad (1)$$

It has been shown that for serial connected cells analytical solution of the system of Equations (1) may be presented in a recurrent form [3]. However, for the cases with more complex coupling topologies, in which the matrix of the Laplacian transformations is not tridiagonal, obtaining analytical solution may become problematic.

Solution of the system of Equations (1) may be obtained using classical or operator methods. For the operator method, numerical solution of the algebraic system of Laplacian transformations is straightforward. Several algorithms are available for the numeric calculation of the inverse Laplace transformation. In this paper the classical method of direct solution of the system of DAE equations will be described.

After substituting lumped element parameters with electrodynamic parameters and getting rid of second derivatives, equation takes the form 2.

$$\begin{cases} \frac{dx}{dt} = \hat{x} \\ \frac{d\hat{x}}{dt} + \sum_{n=1}^N -\frac{K_n^H}{2} \frac{f}{f_n} \frac{d\hat{x}_n}{dt} = \sum_{n=1}^N 2\pi^2 f^2 K_n^E x_n - \\ -(1 + \chi_G + \chi_L) \frac{2\pi f}{Q} \hat{x} + 4\pi f^2 x + \\ + \sqrt{\frac{4\chi_G P_g}{\pi f_G Q}} \cos(2\pi f_G t + \varphi) \end{cases} \quad (2)$$

Which, written in matrix form, looks like this:

$$\begin{cases} |\dot{\hat{X}}| = |\hat{X}| \\ |K^H ||\dot{\hat{X}}| = |K^E ||g(t, \hat{x}, x)| \end{cases} \quad (3)$$

Here,  $|\dot{\hat{X}}|$ ,  $|\hat{X}|$  and  $|x|$  are the vectors of normalised amplitudes of electric field and its derivatives.  $|g(t, \hat{x}, x)|$  is the vector of functions, independent of the adjacent cells parameters.  $|K^H|$  and  $|K^E|$  are the matrices of magnetic and electrical connection between cells respectively.

Finally, for numerical solution of the system, only derivative vectors should remain on the left part of equations.

$$\begin{cases} |\dot{\hat{X}}| = |\hat{X}| \\ |\dot{\hat{X}}| = |K^H|^{-1} |K^E ||g(t, \hat{x}, x)| \end{cases} \quad (4)$$

## CALCULATION MODEL

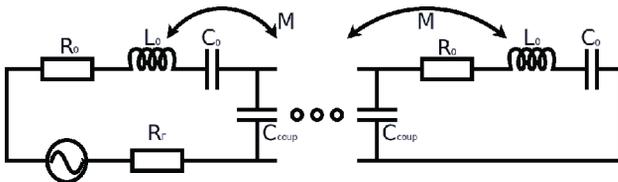


Figure 1: Equivalent circuit of the accelerating structure.

## Equivalent Circuit

In Fig. 1 the equivalent circuit of the accelerating structure is presented. In it distributed parameters of the accelerating structure are substituted with the lumped elements. This circuit may be used to simulate accelerating RF structures with electric and magnetic coupling between cells. It appropriately represents structures operating on  $TM_{01}$  and  $TM_{11}$  wave modes, which covers most types of accelerating cells for the electron accelerators [1, 2]. Depending on the generator and load positions, standing and travelling wave structures may be simulated.

Usually accelerating cells are connected in serial manner but it's possible to simulate more complex connection topologies by introducing additional  $C_{coup}$  and  $M$  components between non-adjacent cells to the circuit 1.

\* MatsievskiySV@gmail.com

Content from this work may be used under the terms of the CC BY 3.0 licence (© 2018). Any distribution of this work must maintain attribution to the author(s), title of the work, publisher, and DOI.

Amplitudes of the electrical field in cells may be obtained from the normalised amplitude vector using equation:

$$U = \sqrt{\frac{\pi r_{sh} f}{2Q}} X \quad (5)$$

### Implementation

Transitional solver, capable of solving systems of differential Eqs. (4) has been developed in R programming language; interface for it was implemented in form of the web application. Base equations set is defined for the electrically and magnetically coupled cells, however, new cell types can be added.

Numerical calculation of the DAE system is done using the deSolve library [4]. Its functions are implemented in Fortran, C and C++ which provides great performance even when used in context of the R language. Default method LSODA automatically switches between stiff and non-stiff systems, making solver very robust.

## MODELLING

In order to test transitional solver, two sections were modelled: 3-cell standing and travelling wave structures.

### Standing Wave Section

In Fig. 2 a) reflection of the 3-cell standing wave section is presented.

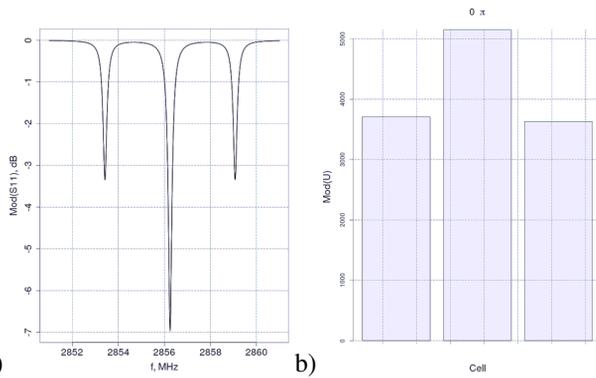


Figure 2: Standing wave section reflection (a), mode 0 field distribution (b).

First peak in Fig. 2 a) corresponds to mode 0, static field distribution for which is shown in Fig. 2 b).

In Fig. 3 transitional process in the structure operating in mode 0 is shown. In Fig. 3 a) oscillations for the first 200 oscillation periods are shown; Fig. 3 b) only shows oscillation envelope. After the 20000 periods of oscillations it appears, that relative amplitudes in the cells are close to those, obtained by the static solver.

For the  $\pi/2$  (Fig. 4 a) and  $\pi$  (Fig. 5 a) and 5 b) respectively. Again, relative field amplitudes after the end of the transitional process appear to match those, obtained by the static solver.

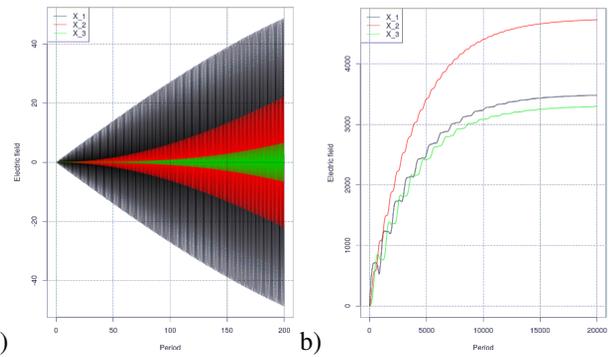


Figure 3: Transitional process in standing wave section for 0 mode of operation.

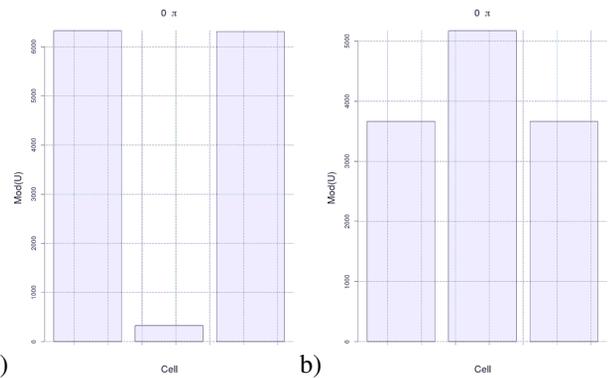


Figure 4: Standing wave mode  $\pi/2$  (a) and  $\pi$  (b) field distributions.

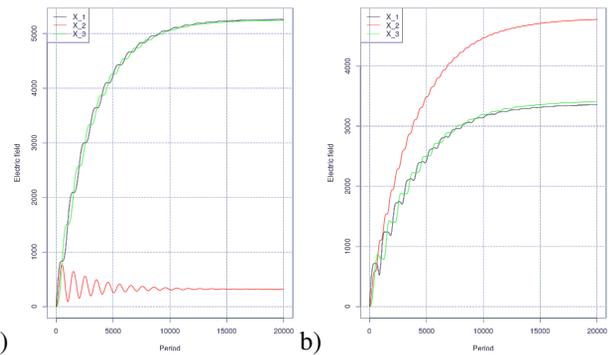


Figure 5: Standing wave mode  $\pi/2$  (a) and  $\pi$  (b) transitional processes.

Table 1: Standing Wave Relative Field Amplitudes in Cells

Mode	Static solver	Transitional solver
0	1.02/1.42/1	1.06/1.43/1
$\pi/2$	19.65/1/19.6	20.27/1/20.2
$\pi$	1/1.41/1	1/1.42/1.01

Relative field amplitude comparison for the standing wave section is shown in the Table 1. For the mode 0 maximum field difference is 4%, for the mode  $\pi/2$  – 3% and for the mode  $\pi$  – 2%.

## Travelling Wave

In Fig. 6 a) reflection of the 3-cell travelling wave section is presented.

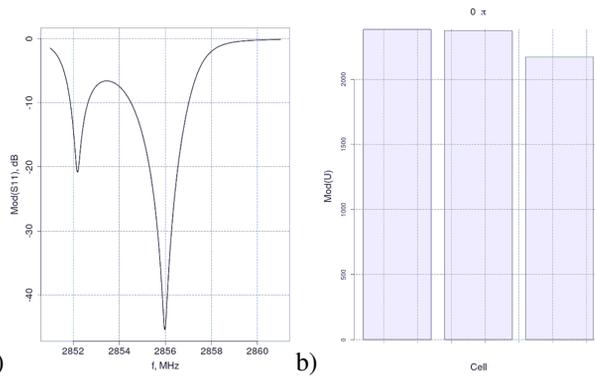


Figure 6: Travelling wave section reflection (a), mode  $2\pi/3$  field distribution (b).

Second peak in Fig. 6 a) corresponds to mode  $2\pi/3$ . Static field distribution for this mode is shown in Fig. 6 b). In Fig. 7 the transitional process for this mode is shown.

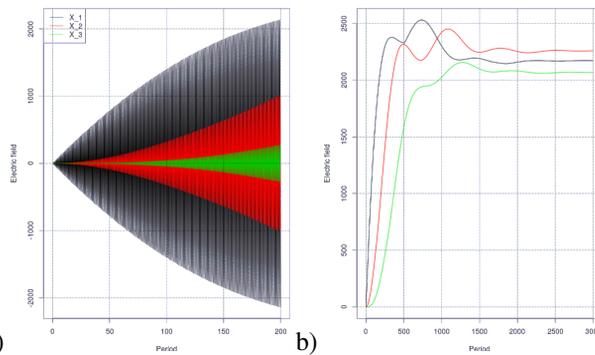


Figure 7: Travelling wave section reflection (a), mode  $2\pi/3$  field distribution (b).

Table 2: Travelling Wave Relative Field Amplitudes in Cells

Mode	Static solver	Transitional solver
$2\pi/3$	1.1/1.09/1	1.05/1.09/1

Relative field amplitude comparison for the travelling wave section is shown in the Table 2. For the mode  $2\pi/3$  maximum field difference is 5%.

## CONCLUSION

Calculation of the transitional processes using the equivalent circuit method was discussed in this paper. One of the possible approaches to this problem – direct solution of the system of DAE – was described. Calculations of the standing wave and travelling wave sections were presented. Maximum field difference between static and transitional solvers is 4% for the standing wave section and 5% for the travelling wave section.

## ACKNOWLEDGEMENTS

The collective of authors would like to thank N.P. Sobenin for the useful discussion of the work and its results.

## REFERENCES

- [1] V. Kaluzny, O. Kaluzny, A. Y. Smirnov, and O. A. Adonev, “Linear accelerator and deflector rf section analysis and optimization with the equivalent circuit scheme,” *Journal of Technical Physics*, vol. 87, no. 7, pp. 101–109, 2014, ISSN: 0044-4642.
- [2] V. Kaluzny, “Analysis of dlw accelerating structures with equivalent scheme,” *Engineering physics*, no. 3, pp. 27–37, 2006.
- [3] V. Kaluzny and O. Kaluzny, “Transitional process analysis in multicell cavities with electrical and magnetic coupling,” *Engineering physics*, no. 3, pp. 46–59, 2008.
- [4] K. Soetaert, T. Petzoldt, and R. W. Setzer, “Solving differential equations in R: Package deSolve,” *Journal of Statistical Software*, vol. 33, no. 9, pp. 1–25, 2010, ISSN: 1548-7660. doi: 10.18637/jss.v033.i09. <http://www.jstatsoft.org/v33/i09>