

STUDY ON GENERATION OF VARIABLE POLARIZED COHERENT THz RADIATION USING A CROSSED UNDULATOR

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Abstract

A variable polarized THz radiation source using a crossed undulator system has been developed at Tohoku University. In this scheme, two coherent undulator radiations from an extremely short electron bunch are used to control the polarization. They are linearly polarized radiations orthogonal to each other. Polarization of superimposed radiation is controlled by adjusting a relative phase between them. A compact planar undulator with seven periods has been designed for an experiment at our facility. The radiation frequency is 2.06 THz for electron beam energy of 22 MeV. The opening angle of the crossed undulator radiation was estimated to be 34 mrad (FWHM). Since the polarization state of the crossed undulator depends on observation angle, its angular dependence was evaluated. It was found that ideal polarization control is realized only in the angle range of 2.5 mrad, which is quite smaller than that of the radiation itself.

INTRODUCTION

Coherent THz light sources based on an accelerator have been developed at Research Center for Electron Photon Science, Tohoku University. A test accelerator called t-ACTS (test Accelerator as Coherent THz Source) has been constructed for generating coherent THz radiation from an extremely short electron bunch. It consists of a thermionic rf gun, an alpha magnet with an energy filter and 3 m long accelerating structure. An electron bunch is compressed down to shorter than 100 fs by means of a velocity bunching scheme in the traveling wave accelerating structure. Coherent transition radiation of a frequency range up to about 4 THz and coherent undulator radiation from 2.6 to 3.6 THz have been successfully observed [1].

At present, we are considering a variable polarized THz light source using a crossed undulator system. In order to carry out a demonstration experiment of its polarization control at t-ACTS, design consideration of the crossed undulator system and an investigation of its polarization properties are underway.

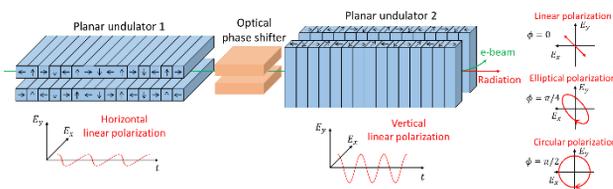


Figure 1: Concept of a crossed undulator.

CONCEPT OF A CROSSED UNDULATOR

Concept of the crossed undulator system is shown in Fig. 1. It consists of two planar undulators and an optical phase shifter between them. The undulators are arranged so that polarization direction of each radiation is orthogonal. Two linearly polarized coherent radiations are generated from an extremely short electron bunch passing through the undulators. They are superimposed both spatially and temporally by delaying the first radiation in the optical phase shifter using mirrors. Polarization is controlled by changing the optical path length in the phase shifter and adjusting phase difference between the two radiations.

PLANAR UNDULATOR

In order to carry out a demonstration experiment, the crossed undulator system must be designed considering some constraints and beam conditions at t-ACTS. Total length of each planar undulator must be shorter than 1 m because of a constraint of the experimental space. Radiation frequency must be lower than 3 THz because of beam conditions of the bunch compression mode of t-ACTS, namely, the electron energy of 22 MeV and the bunch length of approximately 80 fs. A planar undulator was designed taking these requirements into account. The undulator parameters are shown in Table 1. Block dimensions 60 mm, 30 mm and 20 mm are width, height and thickness, respectively.

Electric field radiated from an electric charge q is calculated from the Lienard-Wiechert potential. Its radiation field term is expressed as [2]

$$\vec{E}(\vec{r}, t) = \frac{q}{4\pi\epsilon_0} \left[\frac{\vec{n} \times \{(\vec{n} - \vec{\beta}) \times \dot{\vec{\beta}}\}}{Rc(1 - \vec{n} \cdot \vec{\beta})^3} \right]_{\text{ret}}, \quad (1)$$

where ϵ_0 is the dielectric constant of vacuum, R the observation distance, c the speed of light, $\vec{\beta}$ the velocity vector normalized by c and \vec{n} the unit vector in the observation direction. The bracket $[]_{\text{ret}}$ denotes that values inside it are evaluated at the retarded time. A frequency spectrum of the radiation field can be calculated by the Fourier transformation.

Angular distribution of the undulator radiation can be derived by changing observation point and calculating radiation field Eq. (1) emitted from an electron passing through the magnetic field in the undulator. The angular spread is 34 mrad in the x direction (the electron oscillation direction) and 96 mrad in the y direction in FWHM.

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Table 1: Undulator Parameters

Magnet material	Sm ₂ Co ₁₇
Residual magnetic field	1.09 T
Magnet array type	Halbach array
Block dimension	60 mm, 30 mm, 20 mm
Period length	80 mm
Number of periods	7
Gap	33 mm
Peak magnetic field	0.454 T
K value	3.39
Fundamental frequency	2.06 THz (E=22 MeV)

It should be considered that the angular spread of the crossed undulator radiation is the narrower one, 34 mrad because two undulators are arranged orthogonally.

POLARIZATION

Consideration on Angular Dependence

Phase difference of the two undulator radiations is determined by difference between path lengths from each undulator to an observation point. However, the path length difference depends on observation angle. In addition, wavelength of undulator radiation also depends on observation angle. Therefore polarization of crossed undulator radiation has dependence on observation angle.

Consider a simple layout of the crossed undulator (Fig. 2). L_1 is a distance between the center of each undulator and L_2 is a distance from the center of the second undulator to an observation plane. θ_1 and θ_2 are observation angles from undulator U_1 and U_2 , respectively. R_1 and R_2 are observation distances from each undulator. Path length difference at an observation angle θ is expressed as

$$R_1 - R_2 = \frac{L_1 + L_2}{\cos \theta_1} - \frac{L_2}{\cos \theta_2}. \quad (2)$$

Subtracting the path length difference at $\theta = 0$, namely, L_1 , the difference $L_\delta(\theta)$ can be given by

$$L_\delta(\theta) = \frac{L_1(1 - \cos \theta_1)}{\cos \theta_1} + L_2 \left(\frac{1}{\cos \theta_1} - \frac{1}{\cos \theta_2} \right). \quad (3)$$

Note that θ_1 and θ_2 are functions of θ . Path length difference becomes larger by this L_δ at an observation angle θ than that on the axis ($\theta = 0$). On the other hand, angular dependence of wavelength of undulator radiation is

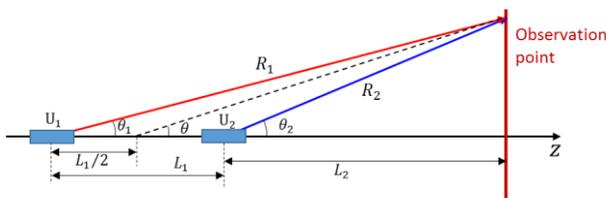


Figure 2: Schematic layout of two undulators and observation plane.

known to be expressed as

$$\lambda(\theta) = \frac{\lambda_u}{2\gamma^2} \left(1 + \frac{K^2}{2} + \gamma^2 \theta^2 \right), \quad (4)$$

where λ_u is period length of an undulator, γ the Lorentz factor of an electron, θ observation angle and K a deflecting parameter of the undulator.

Figure 3 shows calculation results of Eqs. (3) and (4) when $L_1 = 2.59$ m and $L_2 = 4.0$ m. The range of the horizontal axis corresponds to the angular spread of the crossed undulator radiation.

Since the wavelength shift is about 8 % ($\lambda(0) = 146$ μ m and $\lambda(17$ mrad) = 157 μ m), the phase difference in the front of light doesn't change largely even if θ changes. However, the phase difference differs by about $0.6\lambda_r$ (λ_r denotes wavelength on the axis) between the front and rear of the seven cycle light because $|\lambda(\theta_1) - \lambda(\theta_2)| = 13$ μ m. Therefore polarization change caused by this wavelength shift cannot be ignored. On the other hand, L_δ becomes nearly λ_r at 11 mrad and $2\lambda_r$ at 15 mrad, so that the influence on polarization is about three to four times larger than that of wavelength shift. Therefore the polarization largely changes in the range of the radiation angle mainly because of the shift L_δ .

Evaluation of Angular Dependence

Employing Stokes parameters (see APPENDIX), the polarization of the crossed undulator radiation was evaluated. Undulator radiations were assumed to be monochromatic and linearly polarized seven cycle lights, whose electric fields were sinusoidal as follows,

$$E_x(t) = E_{0x}(t)e^{i(k_x(\theta)z - \omega_x(\theta)t)}, \quad (5)$$

$$E_y(t) = E_{0y}(t)e^{i(k_y(\theta)z - \omega_y(\theta)t + \delta(\theta))},$$

where E_x (horizontal polarization) denotes the first undulator radiation and E_y (vertical polarization) the second one. k and ω are wave number and angular frequency, respectively, which were calculated taking the angular dependence into account. E_{0x} and E_{0y} were set to zero in the external time range of seven cycles. δ is phase difference between two radiations. It was evaluated as follows assuming the angular dependence $L_\delta(\theta)$ (Eq. (3)) and $\lambda(\theta)$ (Eq. (4)):

$$\begin{aligned} \delta(\theta) &= \delta_0 + \delta_1(\theta), \\ \delta_1(\theta) &\equiv -\frac{2\pi L_\delta(\theta)}{\lambda_y(\theta)}, \end{aligned} \quad (6)$$

where δ_0 denotes a phase difference on the axis. Radiation intensity I_0 , I_1 , I_2 and I_3 can be calculated by integrating the square of each polarization component of the electric field Eq. (5). Stokes parameters can be derived from the values of intensity.

Figure 4 shows a calculated angular dependence of degree of circular polarization P_C (defined as S_3/S_0). Conditions were as follows: $L_1 = 2.59$ m, $L_2 = 4$ m, $\delta_0 = \pi/2$ and

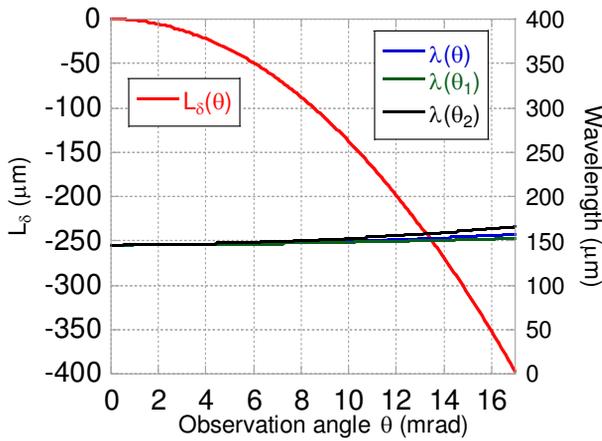


Figure 3: Angular dependence of L_δ and λ .

$E_{0x} = E_{0y}$. Wavelength of undulator radiation was calculated assuming parameters of Table 1. The red line and the green line denote P_C and intensity distribution of the single undulator radiation in the x direction, respectively. As the observation angle becomes larger, the phase difference changes in the positive direction and take the value of $+2\pi$ at approximately 11 mrad. Radiation with P_C larger than 0.9 is obtained in the angular range narrower than approximately 2.5 mrad, which is about 15% of the angular spread of the radiation. It was found from the result that ideal polarization control of the crossed undulator radiation can be realized only in the narrower angle range than its angular spread.

CONCLUSION

A variable polarized THz light source using a crossed undulator system has been developed at Tohoku University. A compact planar undulator was designed for the demonstration experiment: the period length is 80 mm, the number of periods seven, the K value 3.39 and the fundamental frequency 2.06 THz (for the electron energy of 22 MeV). The angular spread of the crossed

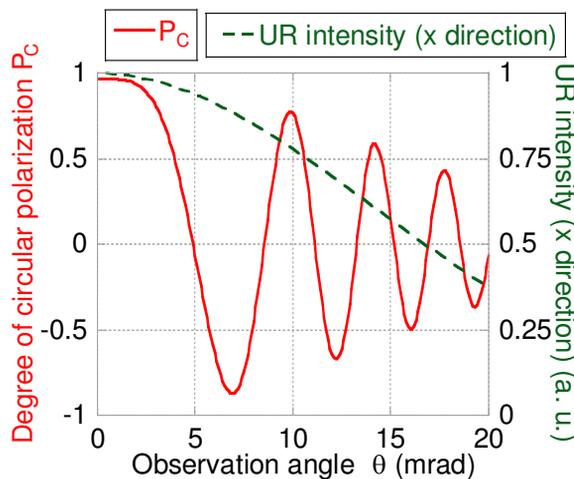


Figure 4: Angular dependence of degree of circular polarization.

undulator radiation is 34 mrad in FWHM. Polarization of the crossed undulator radiation depends on observation angle mainly because of angular dependence of path length of each undulator radiation. The angular dependence of degree of polarization was evaluated assuming sinusoidal radiation fields. It was found from the result that ideal polarization control can be realized only in the angle range of smaller than 2.5 mrad, which is narrower than that of the radiation itself.

ACKNOWLEDGEMENTS

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APPENDIX

The Stokes Parameters [3, 4]

The Stokes parameters are four observable quantities introduced by Stokes in 1852, which have dimensions of intensity of an electric wave. The polarization state of radiation can be practically described by these quantities. They are operationally defined by using a set of four filters F_1 , F_2 , F_3 and F_4 . Each has a transmittance of 0.5 for incident unpolarised light. Suppose that F_1 is isotropic irrespective of the polarization. F_2 and F_3 are linear polarizers whose transmission axes are horizontal and at $+45^\circ$, respectively. F_4 is a circular polarizer opaque to left polarization. Supposing that I_1 , I_2 , I_3 and I_4 are intensity of the transmitted light passing through only one filter F_1 , F_2 , F_3 and F_4 , respectively, Stokes parameters are defined as

$$\begin{aligned} S_0 &= 2I_0, \\ S_1 &= 2I_1 - 2I_0, \\ S_2 &= 2I_2 - 2I_0, \\ S_3 &= 2I_3 - 2I_0. \end{aligned} \quad (7)$$

S_0 is the total intensity of the incident light. S_1 reflects a tendency for the polarization to resemble either horizontal linear one ($S_1 > 0$), vertical linear one ($S_1 < 0$) or neither ($S_1 = 0$). Similarly, S_2 reflects a tendency for the light to resemble $+45^\circ$ linear polarization ($S_2 > 0$), -45° linear one ($S_2 < 0$) or neither ($S_2 = 0$). S_3 indicates a tendency for the polarization toward right-handedness ($S_3 > 0$), left-handedness ($S_3 < 0$) or neither ($S_3 = 0$).

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