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DYNAMIC BEHAVIOUR OF ELECTRON BEAM UNDER RF FIELD AND STATIC MAGNETIC FIELD IN CYCLOTRON AUTO RESONANCE ACCELERATOR*

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Abstract

The cyclotron auto-resonance accelerator (CARA) is a novel concept of accelerating continuous gyrating charged-particle beams to moderately or highly relativistic energies, which can be used as the high power microwave source and applied in environment improvement area, particularly in the flue gas pollution remediation. In CARA, the continuous-wave (CW) electron beam follows a gyrating trajectory while undergoing the interaction with the rotating TE-mode rf field and tapered static magnetic field. Simulation models are constructed to study the effect of rf field and static magnetic field on electron beam in CARA, where the beam energy, trajectory and velocity component are analysed. The simulation results match reasonably well with theoretical predication, which sets up a solid foundation for future designs of CARA.

INTRODUCTION

The CARA may have application as a compact, low-energy injector for a multimegawatt gyroharmonic converter [1, 2] or for use in a source of radiation that requires low-energy electrons [3, 4]. An alternative technology to combat environmental pollution, electron beam dry scrubbing (EBDS), was introduced in 1970, and subsequently demonstrated in several manifestations [4]. CARA is an efficient process for converting rf energy into electron beam energy [4, 5]. The accelerated beam produced in CARA follows a gyrating trajectory, thus the beam is "self-scanning", requiring no other deflecting device or external field to sweep across a gas stream. So CARA is suitable for environmental applications include sterilization, flue gas and waste water treatment.

BASIC THEORY

When the static magnetic field satisfies a certain resonance condition in CARA, the gyrating electrons are maintained in phase synchronism with a rotating TE_{11} waveguide field, then electron beam can be continuously accelerated [2, 4].

The electron beam will gyrate under guiding magnetic field B_z. The rest electron gyration frequency is

$$\Omega_0 = eB_z / m_0 . \quad (1)$$

where e, m_o are the electron charge and rest mass respectively.

The synchronous axial guiding magnetic field is

$$B_z = m_0 \omega \gamma (1 - n\beta_z) / e . \quad (2)$$

where β_z, γ and ω are normalized axial velocity, relativistic factor and rf field frequency respectively. Refractive index n is expressed by

$$n = k_z c / \omega = c\sqrt{(\frac{2\pi f}{c})^2 - (\frac{1.841}{a})^2} / 2\pi f . \quad (3)$$

The relationship between the β_z and γ can be given

$$\beta_{z1} = \frac{1}{\gamma_1}[n_l(\gamma_1 - \gamma_0) + D^{1/2}] \quad (\gamma_1 > \gamma_0, \beta_{z1} > 0) . \quad (4)$$

where $D = (\gamma_0^2 - 1) - (1 - n_l^2)(\gamma_1 - \gamma_0)^2$, γ_0 is the beam's initial relativistic factor, β_{z1}, γ_1 and n_l denote the output parameters in CARA.

The maximum acceleration energy [2] in CARA is

$$\gamma_{1\max} = \gamma_0 + \left(\frac{\gamma_0^2 - 1}{1 - n_1^2}\right)^{1/2} . \quad (5)$$

Then the normalized transverse velocity β_T can be expressed by γ and β_z

$$\beta_T = \sqrt{1 - \frac{1}{\gamma^2} - \beta_z^2} . \quad (6)$$

There are many challenges in CARA:

(i) The maximum acceleration energy is related to n according to Eq. (5), so it is important to get a large n (close to 1) in order to achieve high energy electron beam. But n->1 implies large waveguide radius according to Eq. (3).

(ii) In order to achieve an effective acceleration, it is essential to find the appropriate rf field power and axial magnetic field to maintain the resonance condition.

(iii) The rf field strength and the slope of guiding magnetic field always have perturbations in practice, so the effect of the perturbation is of great significance.

SIMULATION MODEL

For simplicity, a cylindrical waveguide is modelled. In order to achieve a high refractive index, the related parameters of this model are showed in Table 1. The simulation indicates that the rf field distribution in the cylindrical waveguide is the $TE_{11\text{-}35}$ mode of standing wave

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which only has the transverse electric component. Supposing the electron energy γ increases linearly on axial distance z, according to the previous research [2]. Then the related parameters vs axial distance z in CARA are obtained as the following Fig. 1 according to Eq. (2), (4) and (6).

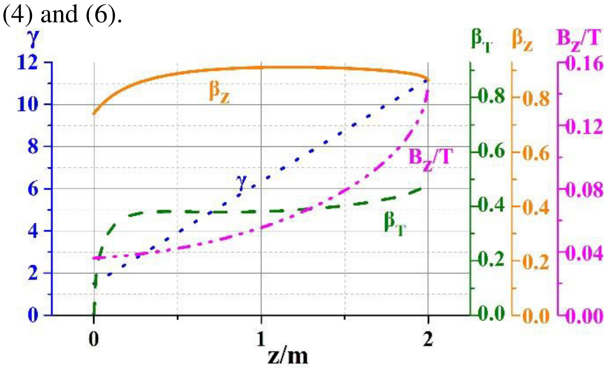

Figure 1: Dependence of initial guiding magnetic field B_z, electron energy γ, transverse velocity β_T and axial velocity β_z on axial distance z in CARA (from z=0 to 2 m).

Table 1: Related Parameter in CARA

Parameter	Value
Length of waveguide	2 m
Radius of waveguide	0.5 m
Refractive index n	0.9935
Initial γ of electron	1.4892
Rf field frequency	2.575 GHz

SIMULATION ANALYSIS

By changing the rf field scaling factor S_{rf} and the slope of the initial magnetic field B_z respectively, the effect of rf field and static magnetic field on electron beam behaviour in CARA.

Dynamic Behaviour under Different RF Field

The rf field strength can be adjusted by changing S_{rf}. Figure 2 shows the projection of the motion of electron on the y-z plane at different S_{rf} value for an initial axial magnetic field showed in Fig. 1. The number of revolutions decreases as the S_{rf} increases, which can be explained by $\Omega = eB_z / m_0\gamma$: the larger S_{rf} means larger acceleration gradient, then the electron can get larger energy γ, resulting in smaller cyclotron frequency. When the S_{rf} is too large to exceed a certain threshold, the electron will not be able to pass through the up-tapered magnetic field and reverse finally because of the Lorenz force caused by the transverse magnetic field component of rf field.

Figure 3 shows the dependence of related parameters on axial distance z at different S_{rf} value in CARA under initial axial magnetic field shown in Fig. 1. From Fig. 3, we can get that when S_{rf}=10, the γ, β_z and β_T curve are closest to the theoretical curve in Fig. 1, indicating the electron is maintained in phase synchronism with a rotating TE_{11-35} waveguide field. And the simulation curves agree well with the theoretical curve. The wiggling in these curves may be related to the non-resonance component in standing wave mode.

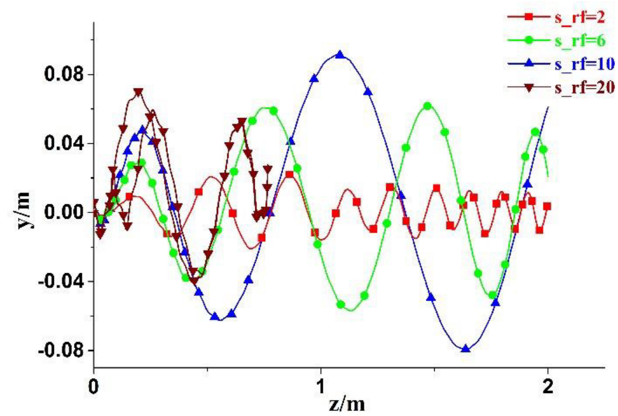

Figure 2: The projection of the motion of electron on the y-z plane under different rf field strength S_{rf}.

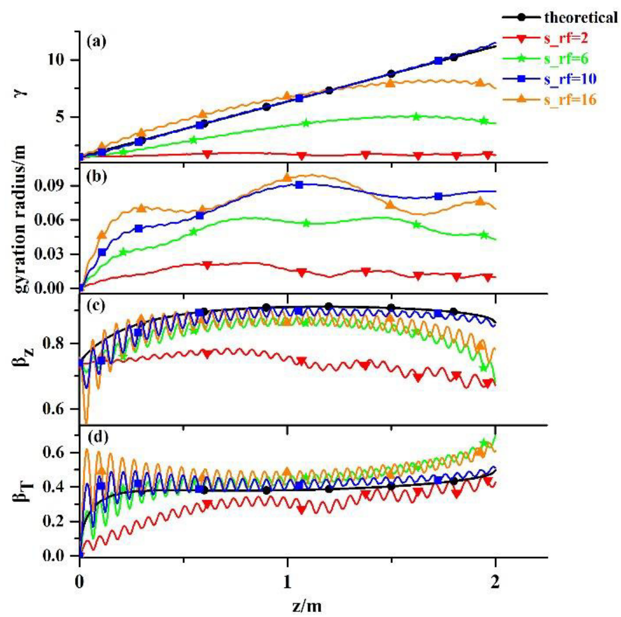

Figure 3: Dependence of (a) electron energy γ, (b) gyration radius ρ, (c) axial velocity β_z and (d) transverse velocity β_T on axial distance z at different S_{rf} value in CARA for a tapered axial magnetic field shown in Fig. 1.

During the movement of electrons, as z increases, β_T increases rapidly at the initial stage, then remains unchanged and finally increases slightly, which indicates that the electron can be accelerated transversely by the transverse electric field of rf field. The larger S_{rf} means the larger the acceleration gradient in CARA, and the slope of β_T curve is larger in the initial stage from z=0 to 0.5 [m]. The β_z does not change very much compared to the β_T. So in general, the γ is increasing almost linearly with z. The ρ also increases along z, which is related to the expression of $\rho = v_T m_0 \gamma / eB_z$: the radius is proportional to the β_T and γ.

When rf field deviates from the resonance, the γ and ρ curve increases first and then decreases, which means the electron oscillate during acceleration and deceleration due

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to none resonance. The oscillation amplitude increases with the S_{rf} for the β_T and β_z curves, and the curves are unstable compared to the resonant one when rf field is perturbed.

Dynamic Behaviour under Static Magnetic Field

The rf field scaling factor S_{rf} =10 remains a constant in this model. The perturbation magnetic field is set as $B_z' = B_z(z_0) + \alpha \cdot (B_z(z) - B_z(z_0))$, where B_z is the initial magnetic field shown in Fig. 1. By adjusting the parameter α, the perturbation magnetic field with different slope can be obtained.

Figure 4 shows the projection of the motion of electron on the y-z plane at different α value in CARA for a tapered axial magnetic field showed in Fig. 5 (a). The number of revolutions increases as the α increases, which can be explained by $\Omega = eB_z / m_0\gamma$: the larger α means larger magnetic field at each point along z axis, then the larger cyclotron frequency and gyration revolutions. The electron will reverse due to the mirror effect of the high magnetic field.

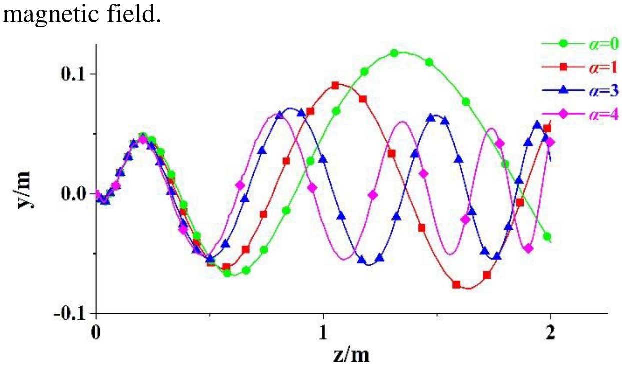

Figure 4: The projection of the motion of electron on the y-z plane under different magnetic field parameter α.

Figure 5 shows the related parameters on axial distance z for a tapered axial magnetic field shown in Fig. 5 (a). During the movement of electrons, the γ, ρ and β_T curves have a clear upward trend while β_z curve almost unchanged after a little growth at the beginning. The axial magnetic field B_z is not perturbed when $\alpha = 1$, and the electron is maintained in phase synchronism condition. When the magnetic field is perturbed, the larger the value of α, the greater the γ and β_T while the smaller the ρ and β_z from Fig. 5. When electron gyrates, the transverse electric field accelerates it, resulting β_T increases. The larger the value of α, the more the revolution numbers. So the electron will get more acceleration from rf field, then the γ and β_T are larger. The larger the axial magnetic field has a larger radial component of magnetic field which causes a reduction in β_z. According to $\rho = v_T m_0 \gamma / eB_z$, although the energy increases as α increases, the gyration radius ρ decreases as the increase in magnetic field B_z dominates.

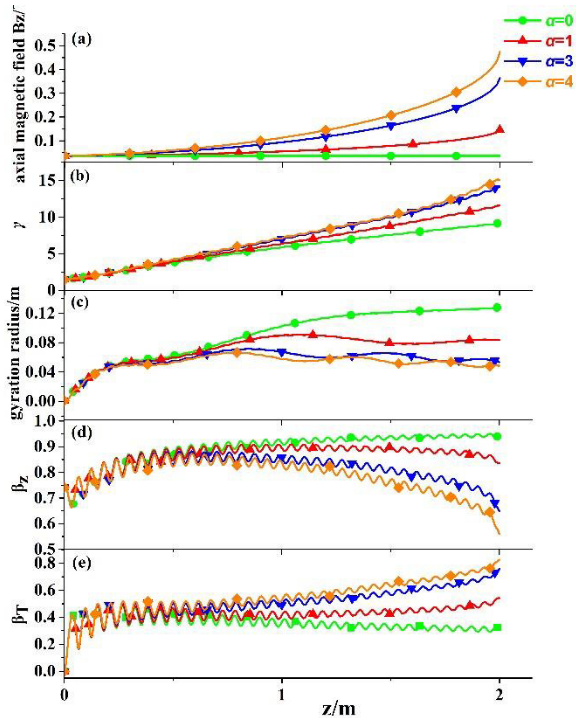

Figure 5: The (a) axial magnetic field B_z distribution along z axis under different α value, dependence of (b) electron energy γ, (c) gyration radius ρ, (d) axial velocity β_z and (e) transverse velocity β_T on axial distance z under a tapered axial magnetic field shown in (a).

SUMMARY

In this paper, we have made an analytical study and simulation of CARA about the effect of rf field and axial magnetic field on electron beam in computer simulation. In summary, we can draw the following conclusions:

(i) When the electron is maintained in phase synchronism with a rotating TE_{11-35} waveguide field using up-tapered axial magnetic field, with axial distance z, γ grows almost linearly, ρ increases and then remains almost unchanged, β_T has a rapidly growth at the beginning and then remains almost a constant, and β_z increases slightly then remains almost unchanged.

(ii) As S_{rf} increases, the revolution number decreases, and the growth rate of γ, ρ and β_T increase obviously while the β_z decreases. The electron will reverse when the S_{rf} is too big. The γ, ρ and β_z at the exist of the waveguide (z=2 m) have the maximum value while the β_T has the minimum value as rf field scaling factor S_{rf} increases.

(iii) As α increases, the revolution number, γ and β_T increase obviously while ρ and β_z decreases. The electron will reverse when the slope exceeds a threshold value.

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