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DETERMINATION OF THE FIELD DEPENDENCE OF THE SURFACE RESISTANCE OF SUPERCONDUCTORS FROM CAVITY TESTS*

J. R. Delayen†[1], H. Park[1], S. U. De Silva,
Center for Accelerator Science, Old Dominion University, Norfolk, VA 23529
[1]also at Jefferson Lab, Newport News, VA 23606, USA

Abstract

Cryogenic tests of superconducting cavities yield an average surface resistance as a function of the peak surface magnetic field. An analytical formalism has been developed to extract the actual field dependence of the surface resistance from cavity tests and is applied to coaxial cavities and cavities of more complex geometries.

INTRODUCTION

The well-known expressions for the surface resistance of superconductors in electromagnetic fields, and its dependence on frequency, temperature, and a few materials parameters, were obtained as a perturbation theory under the assumption that the magnitude of the electromagnetic field is much smaller than the critical field. This resulted in a surface resistance independent of the magnitude of the electromagnetic field.

There have been several attempts at developing theories of the surface resistance at high rf field, up to the critical field, but, at present, there is no universally accepted consensus on the correct fully self-consistent theory.

Experimentally, cryogenic tests of superconducting cavities developed for particle accelerators have shown that superconductors can display a strong dependence of their surface resistance on the rf field. Furthermore, the field dependence can vary greatly depending on the history of the cavities: chemical treatment, high temperature and low heat treatment, impurity, ambient magnetic field during transition, cooling rate during, and so on.

Traditionally superconductors have shown an increase of their surface resistance with rf field which puts a limit to the accelerating gradients achievable for high-energy accelerators. More recently, processes have been developed that yield a decrease of the surface resistance with medium fields, a great benefit for accelerators operating in cw mode.

A strong dependence of the surface resistance on magnetic field is also often observed in cavities made by sputtering of Nb on Cu or in cavities made of Nb_3Sn on Nb

Developing a full understanding and theory of the rf field dependence of the surface resistance of superconductors will require accurate knowledge of that surface resistance as a function of the rf field, and its dependence on preparation and processing parameters.

Experimentally it has proven difficult to develop techniques where a superconductor was exposed to a uniform electromagnetic field. Until now, in all measurements, either the superconducting sample was exposed to a non-uniform field in a test cavity or, more often, an entire cavity was made of superconducting material that was exposed to a field that ranged from 0 to a maximum value.

In this paper we present a method and formulae that allow determination of the actual dependence of the surface resistance from experiments where an "average" surface resistance is derived from tests of superconducting resonators. An underlying assumption is still that, while the surface resistance has a magnetic field dependence, it does not have a dependence on location. This is not always true as it is known that superconducting cavities can have "hot spots" where the surface resistance is higher and with a stronger field dependence than in the rest of the cavity

ANALYTICAL METHOD

The geometrical factor G of a cavity is defined as

$$G = \omega\mu_0 \frac{\int_V |\mathbf{H}|^2 \,\mathrm{d}V}{\int_S |\mathbf{H}|^2 \,\mathrm{d}S}. \quad (1)$$

If the surface resistance is constant, then $G=QR_s$. If it is not, then $G/Q(H) = \bar{R}_s(H)$ where $\bar{R}_s(H)$ is an "average" surface resistance. It is related to the actual one by

$$\bar{R}_s(H_p)\int_S |\mathbf{H}(\vec{r})|^2 \,\mathrm{d}S = \int_S R_s\left(H(\vec{r})\right)|\mathbf{H}(\vec{r})|^2 \,\mathrm{d}S. \quad (2)$$

We define the function $a(h)$ as the fraction of the total cavity surface where $|H| \le hH_p$. The function $a(h)$ is continuous, monotonically increasing, with $a(0)=0$ and $a(1)=1$. It could also be interpreted as the probability distribution for the surface magnetic field, and its derivative $\mathrm{d}a/\mathrm{d}h$ as the probability density. For normal cavity geometries we also have $\mathrm{d}a/\mathrm{d}h|_{a=1} = \infty$.

Because of the continuity and monotonicity of $a(h)$ we can make a change of variable in Eq. (2) and integrate over the magnetic field instead of the area.

$$\bar{R}_s(H) = \int_0^1 (hH)^2 \frac{\mathrm{d}a}{\mathrm{d}h}\mathrm{d}h = \int_0^1 R_s(hH)(hH)^2 \frac{\mathrm{d}a}{\mathrm{d}h}\mathrm{d}h. \quad (3)$$

We assume that the experimentally measured average surface resistance can be modelled by a sum of powers of the magnetic field:

$$\bar{R}_s\left(\frac{H}{H_0}\right) = \bar{R}_0 \sum_{\alpha_i} r_{\alpha_i}\left(\frac{H}{H_0}\right)^{\alpha_i}. \quad (4)$$

* Work supported by NSF Award No. PHY-1416051
† jdelayen@odu.edu

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The sum can be of any size and the coefficients α_i are not restricted to integers, they can be any non-negative real numbers. $\bar{R}_0$ is the zero-field surface resistance and H_0 is arbitrary; it is introduced to make the coefficients r_{α_i} dimensionless. We assume the same power expansion for the real surface resistance but with modified coefficients:

$$R_s\left(\frac{H}{H_0}\right)=R_0\sum_{\alpha_i}\beta(\alpha_i)r_{\alpha_i}\left(\frac{H}{H_0}\right)^{\alpha_i}. \quad (5)$$

Replacing $\bar{R}_s(H/H_0)$ and $R_s(H/H_0)$ by their power expansions in Eq. (3) and equating identical powers we obtain the coefficients $\beta(\alpha_i)$ relating the average and actual surface resistance.

$$\beta(\alpha)=\frac{\int_0^1 h^2\frac{\mathrm{d}a}{\mathrm{d}h}\mathrm{d}h}{\int_0^1 h^{2+\alpha}\frac{\mathrm{d}a}{\mathrm{d}h}\mathrm{d}h} \quad . \quad (6)$$

The function $\beta(\alpha)$ is continuous and, since $\mathrm{d}a/\mathrm{d}h>0$ and $0<h<1$, it is monotonically increasing with $\beta(0)=1$.

Equation (6) is more appropriate when the function $a(h)$ can be obtained analytically. For most cavities, it can only be obtained numerically; in that case it is more convenient to use an equivalent relationship derived from Eq.(6) by integration by parts.

$$\beta(\alpha)=\frac{2\int_0^1 h\left[1-a(h)\right]\mathrm{d}h}{(2+\alpha)\int_0^1 h^{1+\alpha}\left[1-a(h)\right]\mathrm{d}h} \quad . \quad (7)$$

In Eq. (7), $\left[1-a(h)\right]$ now represents the fraction of the cavity surface where $|H|>H_p$.

APPLICATION TO HALF-WAVE COAXIAL CAVITY

The results of the previous section will now be applied to a coaxial half-wave cavity specifically designed for the measurement of the field dependence of the surface resistance on niobium over a wide range of frequencies [1, 2]. Its length is L (459 mm), the radius of the center conductor is a (20 mm), and the radius of the outer conductor is b (101 mm). We also define the dimensionless parameters $\rho=a/b$ and $\delta=b/L$.

For such a cavity $a(h)$ and its derivative can be obtained analytically. The un-normalized contributions from the cavity components are:

Center conductor:

$$A_1(h)=4\rho\delta L^2\arcsin(h), \qquad \frac{dA_1}{dh}=\frac{4\rho\delta L^2}{\sqrt{1-h^2}}. \quad (8)$$

Outer conductor:

$$A_2(h)=\begin{cases}4\delta L^2\arcsin(h/\rho) & 0\le h<\rho\\ 2\pi\delta L^2 & \rho\le h\le 1\end{cases}$$

$$\frac{dA_2}{dh}=\begin{cases}\dfrac{4\delta L^2}{\rho\sqrt{1-(h/\rho)^2}} & 0\le h<\rho\\ 0 & \rho\le h\le 1\end{cases} . \quad (9)$$

End Plates:

$$A_3(h)=\begin{cases}0 & 0\le h<\rho\\ 2\pi\delta^2L^2\left(1-\dfrac{\rho^2}{h^2}\right) & \rho\le h\le 1\end{cases}$$

$$\frac{dA_3}{dh}=\begin{cases}0 & 0\le h<\rho\\ \dfrac{4\pi\rho^2\delta^2L^2}{h^3} & \rho\le h\le 1.\end{cases} \quad (10)$$

The function $a(h)$ for the half-wave cavity and its components from the center conductor, the outer conductor, and the two end plates is shown in Fig. 1. Its derivative is shown in Fig. 2.

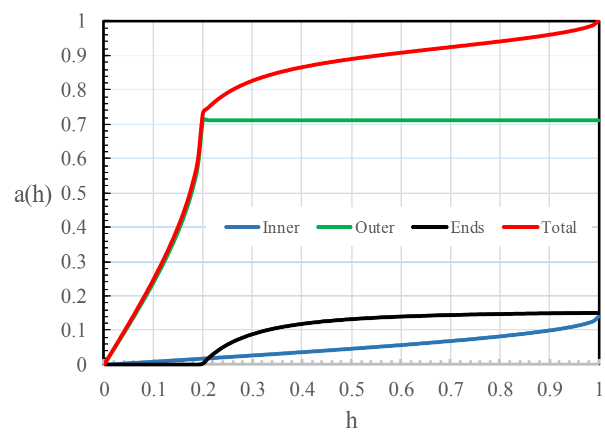

Figure 1: Fraction $a(h)$ of the cavity area where the surface magnetic field is $< hH_p$.

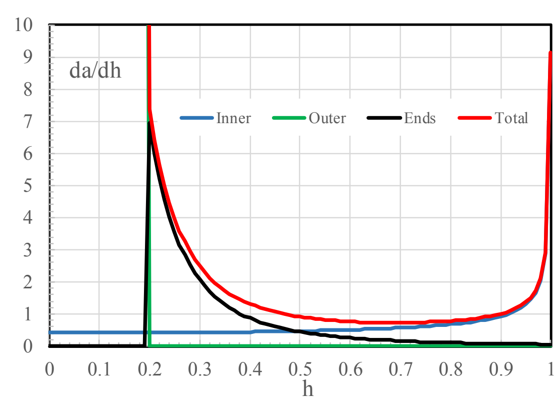

Figure 2: $\mathrm{d}a/\mathrm{d}h$ for the coaxial half-wave cavity.

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The correction function $\beta(\alpha)$ can be ontained analytically and is:

$$\beta(\alpha)=\frac{(1+\rho)/4+\rho\delta\ln(1/\rho)}{\frac{1+\rho^{1+\alpha}}{2\sqrt{\pi}}\frac{\Gamma(\alpha/2+3/2)}{\Gamma(\alpha/2+2)}+\frac{\rho\delta}{\alpha}(1-\rho^{\alpha})} \quad . (11)$$

The correction function $\beta(\alpha)$ is shown in Fig. 3 for various ratio of inner to outer conductor radii. The curve $\rho=0$ is equivalent to assuming that the observed field-dependence of the surface resistance is dominated by the center conductor.

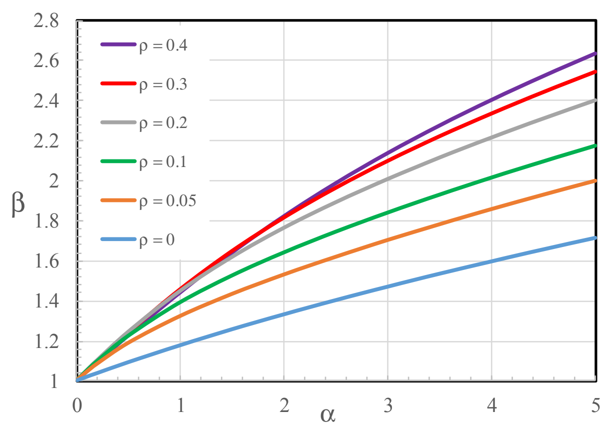

Figure 3: Correction function $\beta(\alpha)$ for different ratio of inner to outer conductor radii.

Figure 3 shows an example of a Q-curve for the half-wave cavity operating in the fundamental TEM mode at 325 MHz and at 4.35 K.

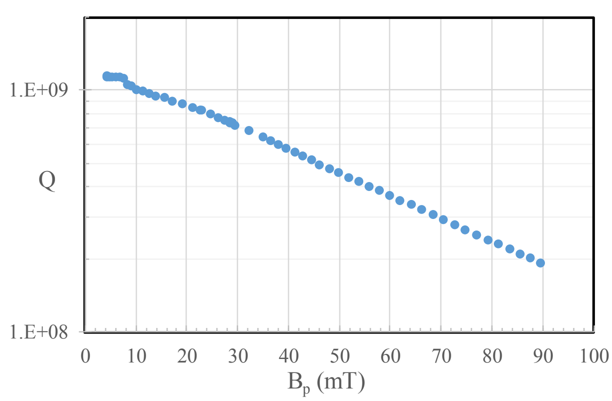

Figure 4: Q-curve for the half-wave cavity at 4.35 K in the fundamental 325 MHz mode.

From the Q-curve and the geometrical factor (59 Ω) the average resistance is calculated and shown as blue dots in Fig. 5. In this example the experimental data is fitted by a third-degree polynomial, shown as the purple line. The surface resistance is in nΩ and the normalizing field is $B_0=100$ mT.

$$\bar{R}_s\left(\tfrac{B}{B_0}\right)=48.2\left[1+1.54\left(\tfrac{B}{B_0}\right)+2.03\left(\tfrac{B}{B_0}\right)^2+3.20\left(\tfrac{B}{B_0}\right)^3\right]$$

From Eq. (11) the correction coefficients re calculated:

$$\beta_0=1,\ \beta_1=1.46,\ \beta_2=1.79,\ \beta_3=2.04$$

This leads directly to an expression to the actual surface resistance as a function of magnetic field shown as the red curve in Fig. 5:

$$R_s\left(\tfrac{B}{B_0}\right)=48.2\left[1+2.24\left(\tfrac{B}{B_0}\right)+3.63\left(\tfrac{B}{B_0}\right)^2+6.52\left(\tfrac{B}{B_0}\right)^3\right]$$

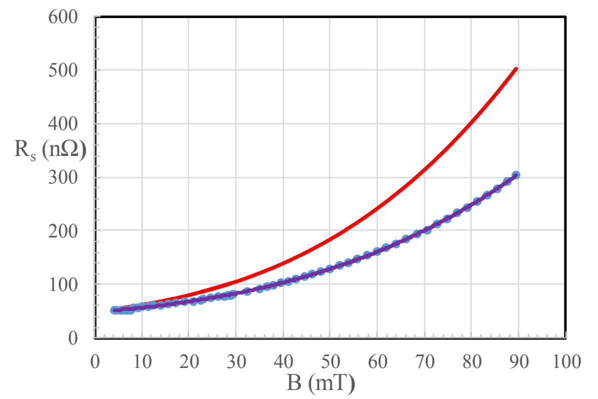

Figure 5: Average surface resistance (blue dots), polynomial fit (purple curve) and derived actual surface resistance (red curve) from the Q-curve of Fig. 4.

CAVITIES OF MORE COMPLICATED GEOMETRY

In most cases the function $a(h)$ and its derivative cannot be obtained analytically. For cavities that still display axial symmetry (e.g. quarter-wave or TM "elliptical" cavities), $a(h)$ can be obtained by a line integration of the magnetic field over the profile. For cavities with full 3-D geometry, $a(h)$ must be obtained by sampling the magnetic field over the whole surface and building the "probability" distribution and density. In both cases $\beta(\alpha)$ is more easily calculated from Eq. (7). As a check on the accuracy of the determination of $a(h)$, for any cavity it should satisfy $\mathrm{d}a/\mathrm{d}h|_{a=1}=\infty$.

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