IMAGE CHARGE EFFECTS IN DYNAMICS OF INTENSE OFF-AXIS BEAMS*

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Abstract

This paper analyzes the combined envelope-centroid dynamics of magnetically focused high intensity charged beams surrounded by conducting walls. Similarly to the case were conducting walls are absent, we show that the envelope and centroid dynamics decouples from each other. Mismatched envelopes still decay into equilibrium with simultaneous emittance growth, but the centroid keeps oscillating with no appreciable energy loss. Some estimates are performed to analytically obtain some characteristics of halo formation seen in the full simulations.

INTRODUCTION

It has been established that in several situations magnetically focused beams of charged particles relax from nonstationary into stationary flows with concomitant emittance growth [1]. Such is the case of beams with an initially mismatched envelope, flowing along the magnetic symmetry axis of focusing systems. The initial oscillating envelope relaxes into the equilibrium solution with simultaneous emittance growth. Gluckstern [2] shows that mismatched beams induce formation of large scale resonant islands beyond beam border [3]. Beam particles could be captured by the resonant islands, departing from beam vicinity. That would cause noticeable emittance growth, along with associated decay into equilibrium. It is also a matter of recent interest to understand the dynamics of beams displaying some misalignment with respect to the symmetry axis of the focusing system.

MODEL

Our system is formed by a beam of charged particles moving along the inner channel of a circular conducting pipe with radius r_w. The beam is focused by a constant solenoidal magnetic field. Beam centroid swings around symmetry axis, so even beams with circular cross section will induce surface charges on the walls. Given the fact that as the beam swings it generates asymmetrical distribution of surface charges, which expect a complicated beam-envelope coupling not be easy to deal with. The beam of radius r_b is centered at centroid coordinate $\mathbf{r}_0 \equiv x_0\hat{\mathbf{x}} + y_0\hat{\mathbf{y}}$. The motion of a charged particle in the Larmor frame is represented by the dimensionless equation for the rescaled transverse coordinate $\mathbf{r} = (x/r_w)\,\hat{\mathbf{x}} + (y/r_w)\,\hat{\mathbf{y}}$ (centroid coordinates are normalized accordingly):

$$\mathbf{r}'' = -\mathbf{r} - \nabla_\perp \psi + \mathbf{F}_{image}. \tag{1}$$

Note that dimensionless pipe radius satisfies $r_w = 1$. The longitudinal velocity $v_{z,0}$ is approximately constant for fast particles, and primes denote derivatives with respect to the longitudinal scaled coordinate $s = \sigma_0^{1/2}\, z = \sigma_0^{1/2}\, v_{z,0}\, t$. $\sigma_0 = qB_z/2\gamma\beta mc^2$ is vacuum phase advance per unit axial length which measures focusing strength. $\beta = v_{z,0}/c$, $\gamma = (1-\beta^2)^{-1/2}$ is relativistic factor, q is individual charge of particles, m their mass and c speed of light. $\psi = \psi(\mathbf{r}, s)$ is dimensionless self-field electromagnetic potential acting on each particle and generated by the beam alone, which can be written in terms of the electrostatic potential ϕ as $\psi = q\phi/(\gamma m\beta^2 c^2 r_w^2 \sigma_0)$. The associated force reads $\mathbf{F}_{beam} \equiv -\nabla\psi$. $\mathbf{F}_{image} = \mathbf{F}_{image}(\mathbf{r}, s)$ is dimensionless force generated by surface charges whose structure may be determined by image charge considerations as one demands that the total electric field generated by the beam alone and surface charges. ψ is derived from Poisson equation

$$\nabla_\perp^2 \psi = -2\pi \frac{K}{N}\, n(\mathbf{r}, s), \tag{2}$$

with $K = 2Nq^2/\gamma^3 m\beta^2 c^2 r_w^2 \sigma_0$ as the beam perveance, N as the total number of particles per axial length, and $n(\mathbf{r}, s)$ as the dimensionless beam density.

When walls are absent centroid and envelopes become uncoupled [4, 5]. Even in the presence of conducting walls, surface charges induced by circular beams perfectly centered at symmetry axis do not act in the inner region $r < r_w$. These two facts suggest that centroid oscillations are moderate, an initially circular beam preserves its circular shape at least during initial stages of the dynamics. For a beam of known properties oscillating around the symmetry axis, we adopt a test particle approach and examine the occurrence of large resonant island encircling the beam. The presence of resonances allows beam particles to perform large excursions away from beam core, which would be highly suggestive of emittance growth and thermal relaxation.

The envelope equation as governing equation for the beam radius $r_b = r_b(s)$ [7, 6] is:

$$r_b'' = -r_b + \frac{K}{r_b} + \frac{\epsilon^2}{r_b^3}, \tag{3}$$

where emittance $\epsilon = \epsilon(s)$ may not remain constant for mismatched beams. For space-charge dominated beams an approximate equilibrium of Eq. (3) is expressed like $r_{b,eq}^2 \approx K$

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at injection coordinate $s = 0$ when $\epsilon \to 0$. The image charge of the original beam is a point-like, or actually a line-like charge placed at

$$\mathbf{r}_{im} = \left(\frac{1}{r_0}\right)^2 \mathbf{r}_0. \tag{4}$$

and endowed with perveance $-K$, and the dynamical equation of centroid motion considering image charge effects is:

$$\mathbf{r}_0'' = -\mathbf{r}_0 + K\frac{\mathbf{r}_{im}(\mathbf{r}_0) - \mathbf{r}_0}{|\mathbf{r}_0 - \mathbf{r}_{im}(\mathbf{r}_0)|^2}, \tag{5}$$

which indicates that image force is independent of beam transverse size, as long as beam keeps its circular shape. Equation (5) has equilibria at $\mathbf{r}_0'' = 0$. If one writes $\mathbf{r}_0 = r_0\hat{\mathbf{r}}$, equilibria are located at:

$$r_{0,eq} = \begin{cases} \sqrt{-K+1} \text{ (unstable)}, \\ 0 \text{ (stable)}. \end{cases} \tag{6}$$

However, from envelope equation it follows $K = r_{b,eq}^2$ and one must satisfy the *filling* condition $r_{0,eq} + r_{b,eq} < 1$, therefore first equilibrium cannot be attained. This precludes the direct effects of unstable point on the centroid orbit with stationary envelope. However, one must still investigate how individual particles are affected as centroid moves according to Eq. (5), while envelope dynamics is governed by Eq. (3).

Envelope Mismatches

We look for large scale resonances in particle phase-spaces x, x' or y, y' as suggestive of a mechanism capable of extracting particles from beam core, which would increase emittance relaxing centroid or envelope motion. Fluctuations of beam envelope around proper equilibrium have been recognized by Gluckstern [2] as a cause for resonances which presence submit particles under evaporation-like process along resonances with simultaneous thermalization.

Particles moving away from beam surface along resonance manifolds. We used $\epsilon = 0$, $r_{b,eq} = 0.25$, and $K(\epsilon = 0) = r_{b,eq}^2$ along $r_b(s = 0) = 0.5$, recording phase coordinates whenever $r_b(s) = r_{b,eq}$ with $r_b' > 0$. As times goes on the beam core keeps executing damped oscillations until it settles down to a value approximately corresponding to the equilibrium envelope. Meanwhile the excess energy is continuously converted into low density halo until process exhausts. Figure 1 displays the rms radius $r_{rms} \equiv \sqrt{2 < x^2 + y^2 >}$ as a function of s and indicates asymptotic damping towards $r_{rms}(s \to +\infty) \equiv r_{rms}^* \sim 0.353$, and also shown the emittance growth obtained from full simulations. In the final state the core occupies a region delimited by $r \lesssim r_{core} \approx r_{b,eq}(\epsilon = 0)$ and at this point the halo radius is the same as resonance size $r_{halo} \sim 0.75$. We saw that the halo component of beam becomes a little distorted towards an elliptical shape as times advances. Is expected nonlinear anisotropic instabilities occurring for largely mismatched beams but does not seem to largely affect the agreement between simulations and estimates.

Mismatched Centroids with Matched Envelopes

The case where envelope is initially set to its equilibrium value $r_{b,eq}(\epsilon = 0)$ but centroid is set to move, with initial conditions $\mathbf{r}_0' = 0$ along $\mathbf{r}_0 = 0.2\hat{\mathbf{x}}$, give us the surprising result that despite the nonlinear forces on beam particles arising from image charges, no large scale resonance could be seen in the phase space plots, suggesting that centroid oscillations simply do not find any channel along which to decay with subsequent thermalization and emittance growth, as it occurred in previous case. As Fig. 2 reveals, no changes are seen in r_{rms} as a function of propagation coordinate s. Emittance, also seen in this figure, does not grow for entire run. No significant energy losses do occur for centroid. The agreement between full simulations and low dimensional model is remarkable. As long as the beam envelope is matched, centroid and envelope dynamics are uncoupled, just like in the case where walls are absent [4, 5].

Fully Mismatched Beams

We expect a negligible effect coming from image charge forces acting on beam particle. This follows from fact that centroid dynamics does not perturb the dynamics of particles far from the beam. Here the halo is again shaped according to envelope nonlinear resonances alone, similarly to case analyzed first. The resonances sizes arising from the envelope dynamics alone are still determining which extension can a particle move off the beam. Since these off-beam excursions are what essentially induces emittance growth, we are then inclined to conclude that even for fully mismatched beams, emittance growth is still essentially governed by the envelope mismatch.

The conclusion from these results is that coupling of centroid and envelope dynamics is absent, similarly to what happens when walls are not present [4, 5]. It should be remarked that while the beam preserves axisymmetry with respect to its own center, it cannot exchange energy with centroid motion, since under this symmetry condition the centroid is unaware of beam size; Eq.(5). This feature would indeed suppress centroid damping or growth in agreement with the present results which indicate axisymmetric beam distortions. As the beam particles approach the wall, the beam tends to become a little distorted and one might perhaps expect to see some collective instabilities within the beam body. However, even under such extreme conditions, test particle models still agree with full simulations. Other regimes have been tested like that of higher centroid mismatches combined with matched envelopes, $\{x_0\}_{max} + r_{b,eq} = 0.7 + 0.25 = 0.95$ and agreement is still excellent. This indicates absence of important collective instabilities.

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CONCLUSIONS

We analyzed combined envelope and centroid dynamics of magnetically focused intense charged beams surrounded by conducting cylindrical walls. Even for the extreme cases no coupling involving these macroscopic quantities has been noticed.

In particular, when envelope is initially set at its matched equilibrium value, it stays there no matter how large might be the mismatched centroid excursions. This means that centroid dynamics cannot decay delivering its excess energy to internal energy of beam, which would cause thermalization and emittance growth. Once the centroid swings around symmetry axis, it keeps its oscillatory motion at least within computational time scales of our runs. Complementarily, if envelope and centroid are both initially mismatched, the envelope dynamics decays toward its matched equilibrium exhibiting emittance growth and other typical features like halo production, but centroid keeps oscillating again.

The present results extend previous investigation on the coupling of envelope and centroid dynamics in absence of surrounding walls [4, 5]. In these previous works it has been possible to show formally the uncoupled nature of combined dynamics. Here we made use of analytical estimates as well as Poicaré plots and full simulations to conclude likewise.

Although a good deal of research may be necessary along the lines presented here, it seems possible to look some implications of present findings. Once one is able to inject beams with matched envelopes, the centroid can be set to relatively large excursions without incurring into danger of beam losses via emittance growth. That might be useful in the design of oscillating devices in vacuum electronics.

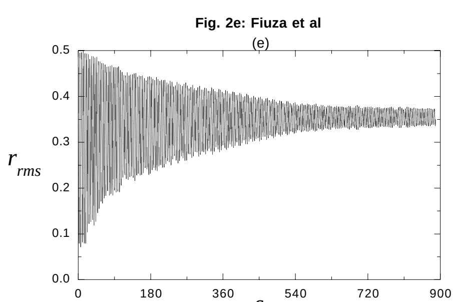

Figure 1: Decay of r_{rms}, where $\mathbf{r}_0 = 0$, $r_b(s = 0) = 0.5$, $K = r_b^2/4$ and $\epsilon(s = 0) = 0$.

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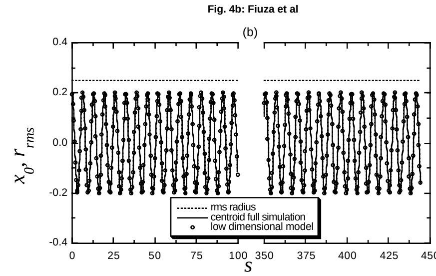

Figure 2: r_{rms} and x_0 as functions of s. Here $x_0(s = 0) = 0.2$, $r_b(s = 0) = 0.25$ and $K = r_b^2 = 0.25$.

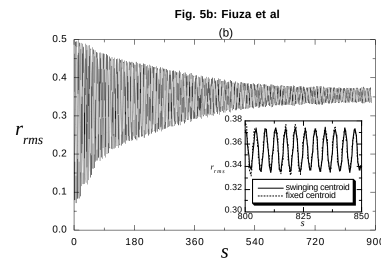

Figure 3: Decay of r_{rms} where $x_0(s = 0) = 0.2$, $r_b(s = 0) = 0.5$ and $K = r_b^2/4 = 0.25^2$.

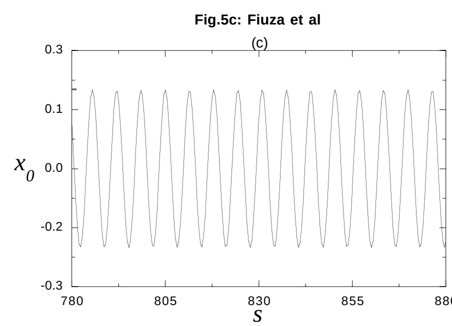

Figure 4: Emittance growth, with $x_0(s = 0) = 0.2$, $r_b(s = 0) = 0.5$ and $K = r_b^2/4 = 0.25^2$.

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