TRANSITION FROM ISOTROPIC TO ANISOTROPIC BEAM PROFILES IN A UNIFORM LINEAR FOCUSING CHANNEL*

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Abstract

This paper examines the transition from isotropic to anisotropic beam profiles in a uniform linear focusing channel. Considering a high-intensity ion beam in space-charge dominated regime and large beam size-rms mismatched initially, observe a fast anisotropy situation of the beam characterized for a transition of the transversal section round to elliptical with a coupling of transversal emittance driven for collective instabilities of nonlinear space-charge forces.

INTRODUCTION

In space-charge dominated beams the nonlinear space-charge forces produce a filamentation pattern, which results in a 2-component beam consisting of a inner core and an outer halo [1]. When this core is mismatched in a uniform linear focusing channel, its envelope oscillates and the particles, represented by single test particle, oscillate about and through the core, this mechanism is called particle-core models.

Space-charge induced coupling between different degrees of freedom can be responsible for emittance growth or transfer of emittance from one phase plane to another [2]. In an analytical single-particle analyses Montague [2] pointed out that the space charge driven fourth order difference resonance may lead to emittance coupling. The coupling resonances driven by the beam space charge fields depending only on the relative emittances and average focusing strengths. In anisotropic beams, the emittance and/or external focusing force strength are different in the two transverse directions. Ikegami [2] and Hofmann [2] deal with effects of anisotropic of beam cores on halo dynamics. Most of the halo studies so far have considered round beams with axisymmetric focusing. Some news aspects caused by anisotropy-with the ratio of focusing strengths and/or emittances as additional free parameters- demonstrating an influence of the mismatch on halo size [2]. Simeoni *et al.* [3], recently, worked a nonlinear analyses of the transport of beams considering nonaxisymmetric perturbations. It is shown that large-amplitude breathing oscillating of an initially round beam couple nonlinearly to quadrupole-like oscillations in this case, the beam develops an elliptical shape.

This paper deals with the coupled motion between the two tranverse coordinates of a particles beam arising from the space-charge forces and, in particular, with the effects of the coupling in beam which have ratio $r_x/r_y \approx 1$, r_x and r_y are the envelope semi-axes-rms . A beam with non-uniform charge distribution always gives rise to coupled motion. However, it is only when the ratio $r_x/r_y \approx 1$ and large beam size-rms mismatched on the order of 100% [4] that the coupling can produce an observable effect in the beam as a whole. This effect arises from a beating in amplitude between the two coordinate directions for the single-particle motion and from the coupling between oscillations modes beam, -resulting in grow and transfer of emittance from one phase plane to another, in the beam develops an elliptical shape and therefore; in the transition from isotropic to anisotropic beam profiles.

ISOTROPIC TO ANISOTROPIC BEAM

We consider an axially long unbunched beam of ions of charge q and mass m propagating with average axial velocity $\beta_b c\hat{\mathbf{e}}_z$ along an uniform linear focusing channel, self-field interactions are elestrostatic. The beam is assumed to have an elliptical cross section centered at $x = 0 = y$ and vanishing canonical angular momentum $P_\theta \equiv \langle xy^{'} - yx^{'}\rangle = 0$, where x and y are the positions of the beam particles. We consider nonuiform density beam in space-charge dominated regime and $\epsilon_x = \epsilon_y = 1$ emittance, initially.

As demonstrated by Sacherer [4] and Lapostolle [4] envelope equations for a continuous beam are not restricted to uniformly charged beams, but are equally valid for any charge distribution with elliptical symmetry, provided the beam boundary and emittance are defined by *rms* (root-mean-square) values. Thus, we consider the parabolic density beam ($n_b = 2N_b/\pi r_x r_y \left[1 - x^2/r_x^2 - y^2/r_x^2\right]$) where N_b is the axial line density, $r_x = \sqrt{6\langle x^2\rangle}$ and $r_y = \sqrt{6\langle y^2\rangle}$ are ellipsis semi-axes *rms*. A main point is that for parabolic density distribution the fourth order space-charge potential driving the coupling is already present in the initial distribution, hence emittance exchange and beam develops an elliptical shape, immediately.

For a parabolic density $n(x, y)$, Poisson's equation $\nabla_\perp^2\phi = -qn/\varepsilon_0$, ε_0 is the permittivity of free space, provides the basis for obtaining the space-charge field component (assuming paraxial approximation). The density is assumed to be zero outside the ellipse. The solution has been given by Lapostolle [4]. The elestrostatic potential ϕ is given by :

$$\phi_{in} = \frac{2qN_b}{\pi\varepsilon_o}\left[\frac{x^2}{\alpha_x} + \frac{y^2}{\alpha_y} - \delta_x x^4 - \delta_y y^4 - \frac{1}{\alpha_x\alpha_y}x^2y^2\right] \quad (1)$$

inside the beam, where $\alpha_{x,y} = r_{x,y}(r_x + r_y)$,

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$\delta_{x,y} = (2r_{x,y} + r_y)/3r^3_{x,y}(r_x + r_y)^2$, and

$$\phi_{out} = \frac{q}{4\pi\varepsilon_0}\log\left[y^2 + x^2 + \Lambda + \sqrt{2}y\Delta^+ + \sqrt{2}x\Delta^-\right] + \frac{q}{2\pi\varepsilon_0\lambda^2}\left[y^2 - x^2 - \frac{y}{\sqrt{2}}\Delta^+ + \frac{x}{\sqrt{2}}\Delta^-\right] \quad (2)$$

outside the beam, where $\lambda = r_x^2 - r_y^2$, $\Lambda = \sqrt{(x^2 - y^2 - \lambda^2)^2 + 4x^2y^2}$ and $\Delta^\pm = \sqrt{\Lambda \pm (x^2 - y^2 \mp \lambda^2)}$.

The transverse orbit $x(s)$ of a beam particle satisfy the paraxial equation of motion

$$x^{''} + \kappa_0^2 x = \frac{-q}{m\gamma_b\beta_b^2c^2}\frac{\partial\phi}{\partial x} \quad (3)$$

with an analogous equation for orbit $y(s)$. Here, s is the axial coordinate of a beam , primes denote derivates with respect to s, and κ_0 is represented constant focusing force.

The envelope of the beam is an elliptical cross-section with *rms* radii r_j (henceforth, j ranges over both x and y) that obey the KV-*rms* envelope equations [4]

$$r_j^{''} + \kappa_0^2 r_j - \frac{2K}{r_x + r_y} - \frac{\epsilon_j^2}{r_j^3} = 0 \quad (4)$$

Here, $K = q^2N_b/\pi^2\varepsilon_0\gamma_b^3\beta_b^2mc^2$ is the dimensionless perveance of the beam. ϵ_j is *rms*-emittance of the beam along the j-plane.

The $\epsilon_j = \sqrt{\langle r_j^2\rangle\langle r_j^{'2}\rangle - \langle r_j r_j^{'}\rangle^2}$ can analytical been calculated following a model proposed to Lapostolle *et al.* [4] for nonlinear space-charge forces, and a sympletic mapping to Simeoni *et al.* [3] for large mismatch beam. According to Lapostolle *et al.* [4] nonlinear space-charge forces cause a change in the momentum components, which is equal to the product of the force and the time over which the force acts. In general, these changes in the momentum components change the phase-space distribution of the particles. Thus the particles experience a space-charge impulse, but do not propagate far enough for their positions to change appreciably. For a beam with large mismatch amplitudes, Simeoni *et al.* [3] has demonstrated that mismatch is like an impulse so fast that particle's positions not change, only momentum suffers a discontinuous variation. From eletrostatic potential and mismatch amplitudes on the order of 100%, the tranverse momentum impulse can be calculated. This results in a new phase-space distribution and new *rms* emittance. For example, in the x plane; the change in the momentum component is $\triangle p_x = qE_xh/v_b$, where $E_x = \partial\phi_x^{in}/\partial x$ is electric field, h is the effective amplitude of oscillations modes beams, and v_b is the beam velocity. Details of h can be found in paper's Simeoni *et al.* [3]. The impulse can also be expressed as change in the divergence angle, given non-relativistically in the paraxial aproximation by $\triangle x^{'} = qE_xh/m_bv_b^2$. If the final second moments of the particle distribution can be evaluted from the expression for x and $x^{'}$, the final *rms* emittance can be obtained.

Knowing that divergence is $x^{'} = x/r_x + qE_xh/m_bv_b^2$, the *rms* emittance for parabolic density yields

$$\epsilon_x = \left[K^2h^2\frac{r_x^2}{r_y^2}\left[\frac{1}{432}\frac{\Theta^2}{\Gamma^4} + \frac{7}{720}\frac{1}{\Gamma^4} - \frac{1}{360}\frac{\Theta}{\Gamma^4}\right]\right]^{\frac{1}{2}} \quad (5)$$

where $\Theta = \left(\frac{2r_x}{r_y} + 1\right)$ and $\Gamma = \left(1 + \frac{r_x}{r_y}\right)$. The result is easily transformed to the y plane interchanging r_x and r_y.

Numerical Results

It is easy to verify that there is a particular solution of the envelope equations (4) for which $r_j(s) = r_{b0} = \left[\left(K + (K^2 + 4\kappa_0^2\eta^2)^{1/2}\right)/2\kappa_0^2\right]^{1/2}$, where $\eta = \epsilon_x/\epsilon_y$ is ratio emittance. This corresponds to the so called matched solution for which a circular beam of radius r_{b0} preserves its shape throughout the transport along the focusing channel. Then, we transform the equations to a dimensionless form introducing the following dimensionless variables an parameters: $\tau = \kappa_0 s$ for the independent variable, $\tilde{r_x} = \sqrt{\kappa_0/\epsilon_y}r_x$ and $\tilde{r_y} = \sqrt{\kappa_0/\epsilon_y}r_y$ for envelope beam, $\tilde{x} = \sqrt{\kappa_0/\epsilon_y}x$ and $\tilde{y} = \sqrt{\kappa_0/\epsilon_y}y$ for test-particle, and $\tilde{K} = K/\epsilon_y\kappa_0$ for the scaled space-charge perveance. In addition, we introduce the following anisotropy parameters: the ratio emittance η, ratio of the envelope beam $\chi = r_x/r_y$ and the mismatch factor $\nu = r_x/r_{b0} = r_y/r_{b0}$.

We launch the beam with $\tilde{k} = 3$, $\kappa_0 = 1$, $\nu = 2.4$, $r_j = \nu r_{b0}$ and $\eta = 1$ initially, and integrate firstly the envelope equations up to $s = 50.0$. In Fig. (1) the beam

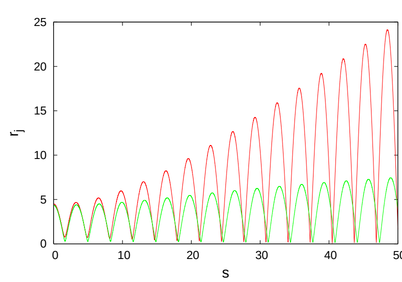

Figure 1: Evolution of the envelopes equations for r_x and r_y, represented by red and green lines, respectively

develops an elliptical shape with a increase in its size along of x-direction. This effect is caused for large beam size-rms mismatched its that coupled the oscillations modes beam, perturbing nonlinear space-charge force. Therefore perturbation induction coupling between different degrees of freedom with transference of emittance from one phase plane to another [2]. In Fig. (2) the initial transversal emittance is equal $\epsilon_{x0} = \epsilon_{y0} = 0.28$. It is observed emittance coupling caused for space charge driven Montague resonance [2]. Single particle resonances eventually yield *rms* emittance growth as more and more particles to be launched out of core. Assuming which beam is usually equipartitioned in x and y -($\xi = (r_y\epsilon_x)^2/(r_x\epsilon_y)^2$),

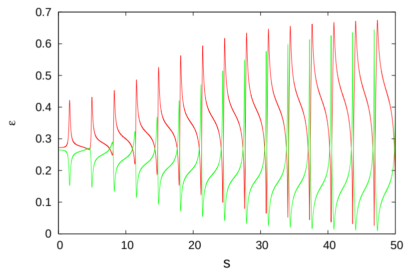

Figure 2: Emittance evolution for ϵ_x and ϵ_y, which are represented by red and green lines, respectively.

where ξ is the ratio of oscillations energies in the x and y directions,- but has different χ and large beam size-rms mismatched, the core resonances enable energy transfer from one plane to another [2]. We note that the exchange is accompanied by halo creation along one direction preferential, as illustrated in the Fig. (3). In the particle-core

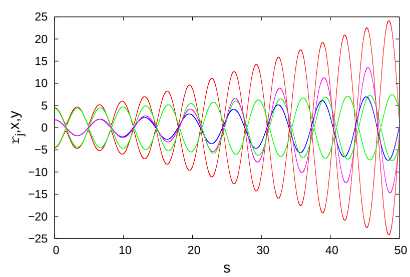

Figure 3: Test particle evolution for $x = 0.5r_x$ and $y = 0.5r_y$, initially, whose orbits x and y are represented by blue and pink lines, respectively.

model, the core is described by *rms* envelope equation (4), and the halo particles are modeled using test particles that subject to the external force and the time-dependent nonlinear space-charge force associated with the core. The test particles do not affect the motion of the core and are described by equation (3) taking ϕ_{in} for inside beam, and ϕ_{out} for outside beam.

For large beam size-rms mismatched and ratio envelopes beam small $\chi \ll 1$, the ratio of oscillations energies in the x and y directions remaines constant, $\xi = 1$. As illustrated in the Fig. (4) beam fast suffers anisotropization characterized for discontinuous variations in χ and η. The anisotropy leading to core-core resonance [2] in the presence of nonlinear space-charge forces was suggested as a possible approach to the equipartitioning question, since collisions cannot be made responsible for energy transfer in linacs.

CONCLUSIONS

The coupled motion between the two tranverse coordinates of a particles beam is arising from the space-charge forces. A beam with non-uniform charge distribution always gives rise to coupled motion. However, it is only when the ratio $r_x/r_y \approx 1$ and large beam size-rms mismatched that the coupling produces an observable effect in the beam as a whole. This effect arises from a beating in amplitude between the two coordinate directions for the single-particle motion and from the core-core resonances, resulting in growth and transfer of emittance from one phase plane to another, in the beam developing an elliptical shape along of the directions preferential and therefore in the fast transition from isotropic to anisotropic beam profiles. Future analyzes will demonstrate that the equilibrium state system is a anisotropic parabolic distribution.

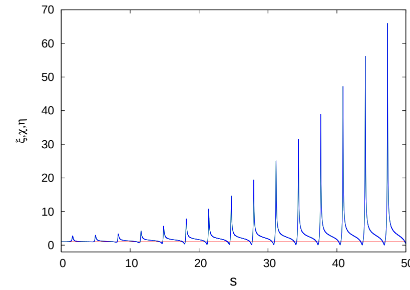

Figure 4: The ratio of oscillations energies ξ and anisotropic ratios, χ and η in beam propagation. ξ is represented by red line, and χ and η are overlapping represented by blue line.

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