THE REGULAR AND RANDOM MULTI-POLE ERRORS INFLUENCE ON THE HESR DYNAMIC APERTURE

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Abstract

The High Energy Storage Ring (HESR) of FAIR project has the racetrack lattice, where each arc has the even number of super-periods S_{arc} and the tune with one unit smaller $\nu=S_{arc}-1$ in both planes. Due to this fundamental feature the total *n*-order multi-pole is entirely cancelled and the regular errors can be fully compensated inside of one arc. In case of the random multi-pole errors the dynamic aperture is determined by the structure resonances excitation. We consider both regular and random multi-pole influence on the dynamic aperture and the possible correction scheme.

INTRODUCTION

The HESR lattice consists of two arcs and two straight sections for target and cooling facilities with circumference ~500 m [1]. Figure 1 shows the common view of HESR lattice and the layout of half superperiod.

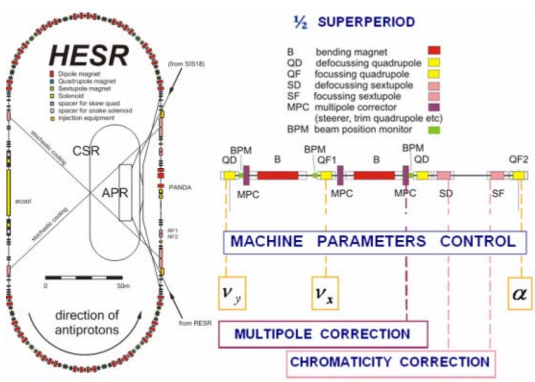

Figure 1: Schematic layout of the HESR lattice with the half superperiod of arc.

In order to minimize the preparation procedure for each experiment the HESR lattice has to have the decoupled functions responsible for global parameters of the machine like transition energy, zero chromaticity, dispersion suppressing and local parameters like beam luminosity on target, optimum parameters for cooling, injection system.

The arcs play the most important role for global parameters. We considered two options of lattice, and both have a racetrack shape with two arcs and two straight sections. In the first option the arc has the four-fold symmetry with four superperiods. In the second option the arc has the six-fold symmetry with six super periods. The phase advance $2\pi\nu_{arc}$ per arc is chosen $2\pi\cdot 3.0$ and $2\pi\cdot 5.0$ in first and second options correspondingly. To suppress dispersion function on the straight sections for the beam with non-zero momentum spread the arc has to satisfy the following conditions: the phase advance is integer and the total chromaticity is corrected to zero in the arc. Each super period consists of three FODO cells with 4 super conducting bending magnets (B=3.6T) and super conducting quadrupoles (G<75T/m).

The arc has the separated functions to control such machine parameters like [2]:

- the momentum compaction factor α is controlled by central focusing quadrupole QF2;
- the horizontal tune is controlled by focusing quadrupole QF1;
- the vertical tune is controlled by defocusing quadrupoles QD.

Figure 2 shows the $\beta_{x,y}$ and dispersion functions in the arc with six-fold symmetry and the elements placement.

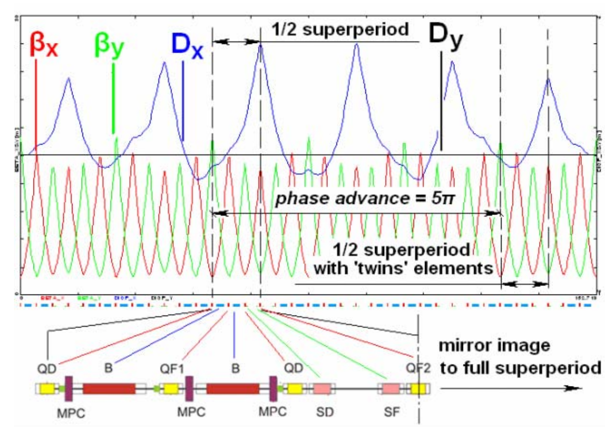

Figure 2: β-functions and dispersion on the arc (6-fold symmetry option).

THE HESR OPTICS AND BUILT-IN CORRECTION SCHEMES

Let us consider a common ideology of the lattice construction taking into account necessary correction schemes for the total chromaticity, non-linear chromatic sextupoles influence up to second order of a perturbation theory and the multipole regular errors in bend magnets or quadrupoles. As you see the HESR optics consists of the bend magnets, the quadrupoles, the sextupoles and the multipole correctors. Besides, due to the imperfections the multipoles errors are added in the lattice.

Linear optics. Chromaticity correction

In optics with quadrupoles and bend magnets we need to correct the total chromaticity effect. It is defined by all quadrupoles placed in the ring as the variation of the

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betatron tune $\nu_{x,y}$ with the relative momentum deviation $\delta = \Delta p / p$. For this purpose the sextupoles are installed and their integral contribution over whole ring circumference C into the chromaticity is equal:

$$\frac{\partial \nu_{x,y}}{\partial \delta} = \pm \frac{1}{4\pi} \int_0^C \beta_{x,y}(s) \cdot D(s) \cdot S(s) ds \,, \qquad (1)$$

where $S(s)$ is sextupoles strengths. It is clear to minimize the sextupole strengths and to split the chromaticity correction in the horizontal and vertical planes they have to be allocated in the maximum dispersion and different β_x and β_y values correspondingly.

Non-linear optics. Chromatic sextupoles in the first order of perturbation theory

Together with sextupoles the high orders non-linear effects are added in the lattice optics. Considering sextupoles in the first approach of the perturbation theory within the scope of Hamiltonian formalism [3], we can get condition for cancelling of the total multipole of third order:

$$M_3^{total} = \sum_{n=1}^{S_{arc}} S_{x,y} \beta_x^{l/2} \beta_y^{m/2} \exp in(l\mu_x + m\mu_y) = 0 \,, \qquad (2)$$

where $S_{x,y}$ are the sextupole gradients, μ_x, μ_y - the phase advances per one superperiod. Thus, in the first approach we have an exact condition for compensating each sextupole's non-linear action by another one, located in the different half arcs and separated by $S_{arc}/2$ number of superperiods: the phase advance between them is equal $2\pi(\nu_{arc}/S_{arc}) \cdot (S_{arc}/2) = 2\pi(\nu_{arc}/2)$ and their non-linear actions are identical as far as possible (see Fig.2).

This principle can be applied by analogy to compensate the regular multipole errors in the bend magnets or quadrupoles as well, since for any element its twin exists on another half arc where non-linear kick is compensated.

Non-linear optics. Chromatic sextupoles in second order of perturbation theory

In the second order of perturbation theory sextupoles give contribution in the non-linear tune shifts $\zeta_{x,y,xy}^{sext}$ also. The reason of this effect is an excitation of the third order resonance with the sextupoles. On the other hand the non-linear tune shifts $\zeta_{x,y,xy}^{oct}$ arise already in the first approach due to the octupoles. It is given by expressions

$$\zeta_{x,y,xy}^{oct} \sim \int_0^{2\pi} \beta_{x,y,xy}^2 O_{x,y,xy}^2 R d\theta \,, \qquad (3)$$

accurate to a constant factor. In (3) $O_{x,y,xy}$ are the octupole gradients. Thus, rising the octupole components in the multipole correctors we can compensate non-linear sextupole influence in second approach.

MULTIPOLE CORRECTION SCHEME FOR REGULAR AND RANDOM ERRORS

A non-linearity origin follows obviously from a view of Hamiltonian function for charged particle motion [3]. Even in ideal optics and for the monochromatic beam each n-th multipole M_n in composition with the curvature h^m creates all higher multipoles M_{n+m}. In case of non-monochromatic beam $\delta \equiv \Delta p/p \neq 0$ each multipole of n-th order M_n gives all multipoles $M_{1 \div (n-1)}$ of 1÷(n-1)-th order in the place where D≠0. In case of the closed orbit distortion each n-th multipole M_n gives additionally all multipoles $M_{1 \div (n-1)}$.

Besides the built-in correction scheme specified above, let us consider the multipole correction scheme with multipole correctors by the example of imperfections in the bend magnets. It is clear that all high order multipole components appear in the bend magnets. The main effect of the high order errors influence is the non-linear tune shifts.

The regular errors in magnets

Let's consider the case of the regular errors in the bend magnets. Figure 3 shows the non-linear tune shift versus the sextupole and octupole component errors in the bend magnet measured in the units $10^4 \times \Delta b_{sext\_mag} / B_0$ and $10^4 \times \Delta b_{oct\_mag} / B_0$ correspondingly.

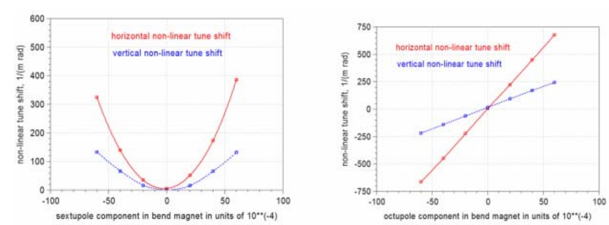

Figure 3: The non-linear tune shift vs. the sextupole $\Delta b_{sext\_mag} / B_0$ and octupole $\Delta b_{oct\_mag} / B_0$ components in bend magnet.

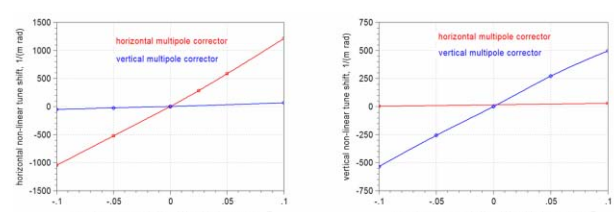

Figure 4: The non-linear tune shift versus the octupole components in multipole horizontal b_{oct}^{hcor} and vertical b_{oct}^{vcor} correctors.

From the numerical simulation we have found out that the horizontal tune shift is more sensitive to the errors in the bend magnets. In order to compensate the non-linear tune shifts we use the multipole correctors located near each quadrupoles (see Fig. 1, 2 and 4).

As we can see the horizontal and vertical correctors influence on non-linear tune shifts are decoupled each from other and it allows to compensate the tune shifts in both planes independently.

The random errors in magnets

In case of the random errors we cannot define exact dependence between the errors in magnets and the corresponding tune shifts. Thus, we need to verify the correction scheme on particular example of random errors distribution.

The algorithm of sextupole errors simulation:

- The sequence of random variables under the normal law of distribution is created according to number of magnets;
- This sequence is used for definition sextupole errors in the bend magnets and the values of errors are adjusted for size with an multiplication by constant;
- Then for test distribution of particles we define as much as possible admissible size of errors at which there are no the particles losses. Taking this into account we fix one of the errors realizations which gives the maximal tune shift from a working point;
- For the test particles distribution and the chosen set of errors we build the dependence between tune shifts and particles amplitude. Then we check an opportunity of compensation by correctors;
- At the end we track the line of particles in the phase space and verify modification of this line after tracking in case of errors and their correction.

Figure 5 shows the dependence between tune shifts in horizontal (vertical) plane and the random errors size for sextupole b_2, octupole b_3 and decapole b_4 components in the bend magnets. Further we will consider the case with maximal tune shift from the working point ($\nu_x = 12.16, \nu_y = 12.18$) – the errors in sextupole component of the bend magnets.

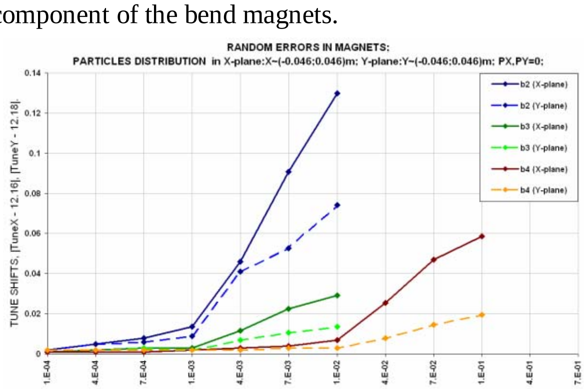

Figure 5: Betatron tune shifts vs. the random errors size (|tune $_{x,y}$ - (12.16; 12.18)|).

Figure 6 shows the dependence between tune shifts and particles amplitude in the worst case of the sextupole random errors set with / without octupole components correction by the multipole correctors. Thus, we have an opportunity to compensate non-linear tune shifts by the correctors.

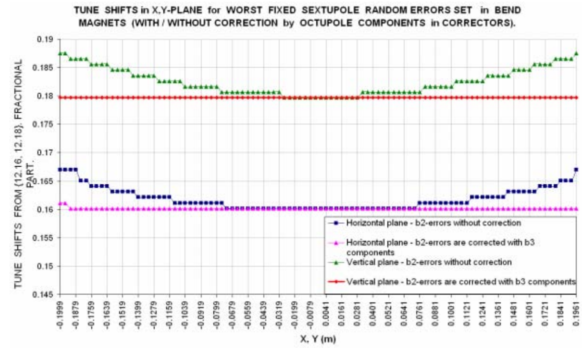

Figure 6: Tune shifts (from sextupole errors) vs. particles amplitude with / without octupole correction.

Figure 7 shows the qualitative attribute that a phase space filamentation due to non-linear effect increases effective phase space occupied the line in phase space, hence emittance increasing. Also we see at least there is a possibility of minimization of this effect by multipole correctors scheme.

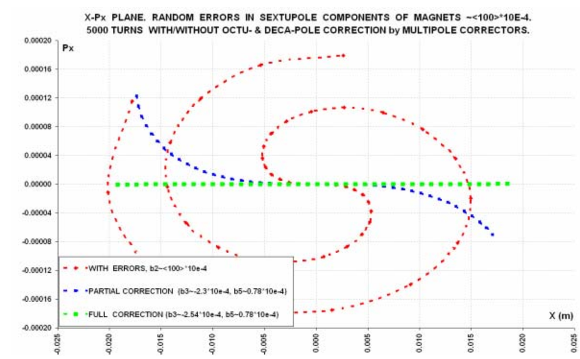

Figure 7: The phase space filamentation due to random sextupole errors in the bend magnets and the multipole correction scheme.

CONCLUSION

The lattice is developed with possibility to compensate high order non-linearity due to regular and random errors with mulipole correction scheme and compensation feature built-in in the lattice. We are thankful to R. Maier for attention to our work.

REFERENCES

[1]. Conceptual Design Report, GSI Darmstadt, 2006, http://www.gsi.de/GSI-Future/cdr/

[2]. Y. Senichev, et al., Lattice Design Study for HESR , Proceedings of EPAC, Lucerne 2004, p. 653 http://accelconf.web.cern.ch/accelconf/e04/PAPERS/MOPLT047.PDF

[3]. A. Chechenin, et al. The high order non-linear beam dynamics in High Energy Storage Ring of FAIR, Proceedings of EPAC, Edinburg 2006, pp.2083-2085