INFLUENCE OF CHAOS ON RESONANCE CROSSINGS#

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Abstract

We undertake a study of particle dynamics in a model fixed-field alternating-gradient (FFAG) synchrotron in which space-charge plays a central role. The space-charge force corresponds to a Gaussian charge distribution in both transverse dimensions. The betatron-tune is linearly ramped through resonance. This ramping alone can cause particles to enter orbits that have chaotic motion. We found that space-charge can lead to spreading of the available tunes which can either increase or decrease the effects of resonance. By applying recently developed techniques to measure complexity in the orbital dynamics, we also determine whether chaoticity can arise in particle trajectories and subsequently influence resonance crossings. Furthermore, we can see that the chaoticity changes drastically in the area around a resonance crossing.

INTRODUCTION

This study originally intended to study the effect of noise on the resonance crossings using a model derived from Huang et. al[1]. However, before extra complications such as sextupole errors and dynamic noise could be introduced, important dynamical phenomena were observed, aided by the new complexity algorithmin the presence of only space-charge and focusing.

The model used is a symplectic map using the focusing parameters from the Fermilab booster. These are then perturbed by a space-charge kick at every half-cell. The distribution is assumed to be Gaussian at all times. The rms moments are then calculated after each turn and applied to the next turn to bring a small degree of self-consistency.

PATTERNS METHOD

A chaotic system is one where the orbits of a particle are very sensitive to initial conditions a small difference in starting points will cause the difference between the final orbits to grow exponentially with time.

The most common measurement of chaos is the largest Lyanpunov exponent, which detects the exponential change in initial conditions. Other methods exist, many of which take advantage of the difference in spectrum between a regular and a chaotic orbit.

These measures, unfortunately, can only tell if an orbit is at any time chaotic. In time-independent systems this is sufficient, but in time-dependent systems this can be problematic.

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The Patterns Method was developed by a member of Northern Illinois University's Beam Physics and Astrophysics Group Ioannis Sideris, to provide a real-time measure of the chaos of a particle[2].

This method looks for smooth curves within the signal of the orbit. This is done by comparing a point to the other points in an orbit and figuring out if it belongs to a smooth curve, as shown in the Figure 1.

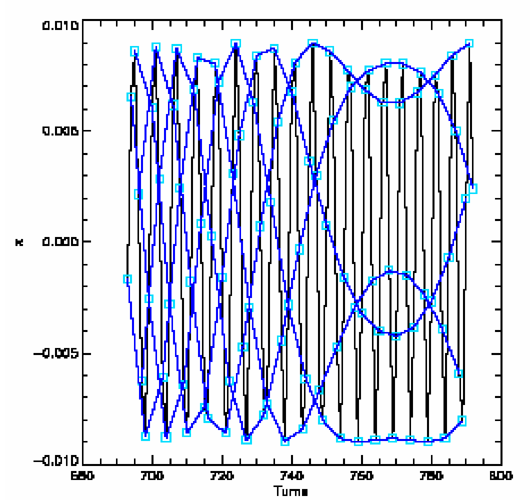

Figure 1: Smooth curves in a signal.

EMITTANCE

In order to get an informative view of the manner in which the beam evolves with respect to its relative regularity or chaos, the rms beam emittance values are plotted against the relative amount of chaos. Blue denotes regular orbits, green denotes weakly chaotic + regular orbits, and red denotes regular + weakly chaotic + chaotic. The graph has a solid line at 5000 since this is the number of test particles used. Important values of space-charge are detailed in Figure 2.

The plots in Figure 2 show a linearly tune-ramped system starting with initial x tune of 6.25 and initial z tune of 6.20. In the $5x10^{11}$ particles per meter space charge regime there is a strong regular epoch surrounding the resonance crossing. This does not exist in the $5x10^{12}$ regime, and in the $1x10^{13}$ regime is greatly reduced. A close inspection also shows the emittance in the 10^{12} regime is smaller than the $5x10^{11}$ regime. This seems unlikely, which is why a detailed knowledge of the mechanisms for emittance growth is needed.

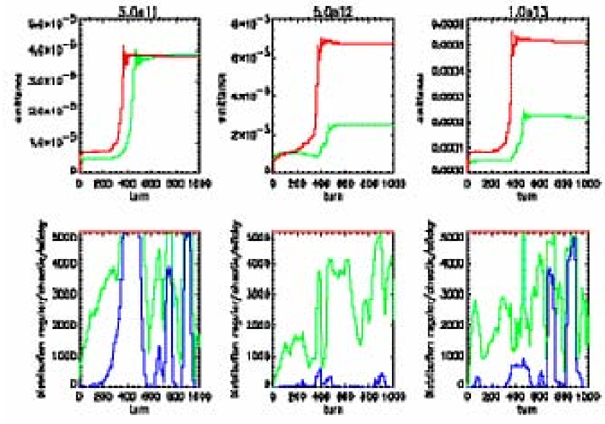

Figure 2: Plots of emittance and relative chaos. In the top row red denotes emittance in the *z* dimension, green is emittance in the *x* dimension. In the bottom row blue denotes regular orbits, green denotes weakly chaotic orbits + regular orbits, and red is the sum of all three.

TRACE-SPACE EVOLUTION

In order to understand the mechanisms of emittance growth the motions of the individual particles are plotted at regular intervals. Figures 3 through 5 show the different growth mechanisms between the $5x10^{11}$ and 10^{13} particles per meter regimes.

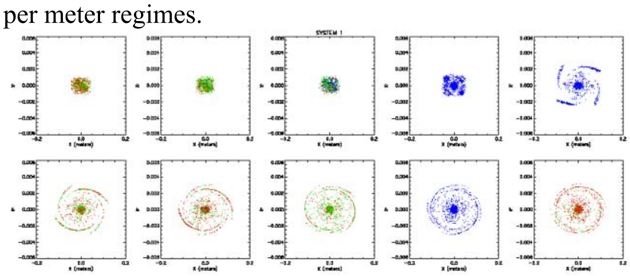

Figure 3: Slices at turn 100-1000 in 100 turn increments. Red points are chaotic, green points are weakly chaotic, and blue points are regular. These plots show the $5x10^{11}$ regime.

Figure 3 shows what appears to be the trapping of particles in islands which expand outwards before being smeared into a halo. In Figure 4 we do not see this phenomenon. However, when we increase the space-charge and see how the trace-space evolves in Figure 5, there is a definite return to the previous paradigm.

One fact that is immediately evident is that the $1x10^{13}$ regime spends its time with a much larger overall radius for its particles. Since the model's space-charge decreases exponentially with the distance from the center, it is reasonable to believe that the particles are so far away from the center that the space-charge force they feel is low enough that they act as though they are in a lower space charge regime.

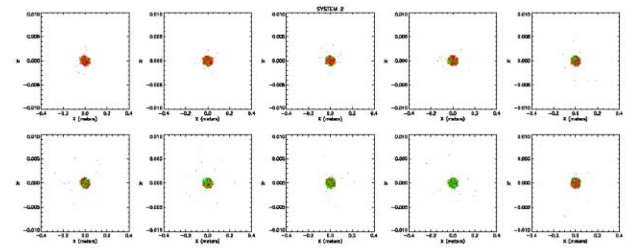

Figure 4: Slices at turn 100-1000 in 100 turn increments. Red points are chaotic, green points are weakly chaotic, and blue points are regular. These plots show the $5x10^{12}$ regime

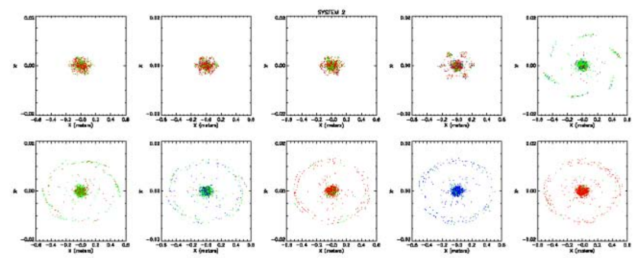

Figure 5: Slices at turn 100-1000 in 100 turn increments. Red points are chaotic, green points are sticky, and blue points are regular. These plots show the $1x10^{13}$ regime. This plot shows the crossing of the 4.0 resonance.

TUNES AND WHAT THEY REVEAL

Since the formation or lack thereof of islands in the beam seems to be the driving force behind emittance growth in this system, an understanding of these islands is necessary. As has been seen, both the space-charge and the betatron-tune of the system are instrumental in the formation and shape of the islands.

Since the space-charge changes the tune of the system we cannot simply use the tune that was input into the map. In this work the tune was determined by taking an initial and final condition, then assuming that space-charge was not involved, and finding the tune that would give the differences seen.

Average Tunes

An interesting experiment that will shed light on what occurs is to average the *x* and *z* tunes and plot them. These results are shown in Figure 7.

Due to the method used to find the betatron-tune, all that is plotted is the fraction, and not the total tune. The tunes in the low space-charge regimes follow the line determined by the ramping.

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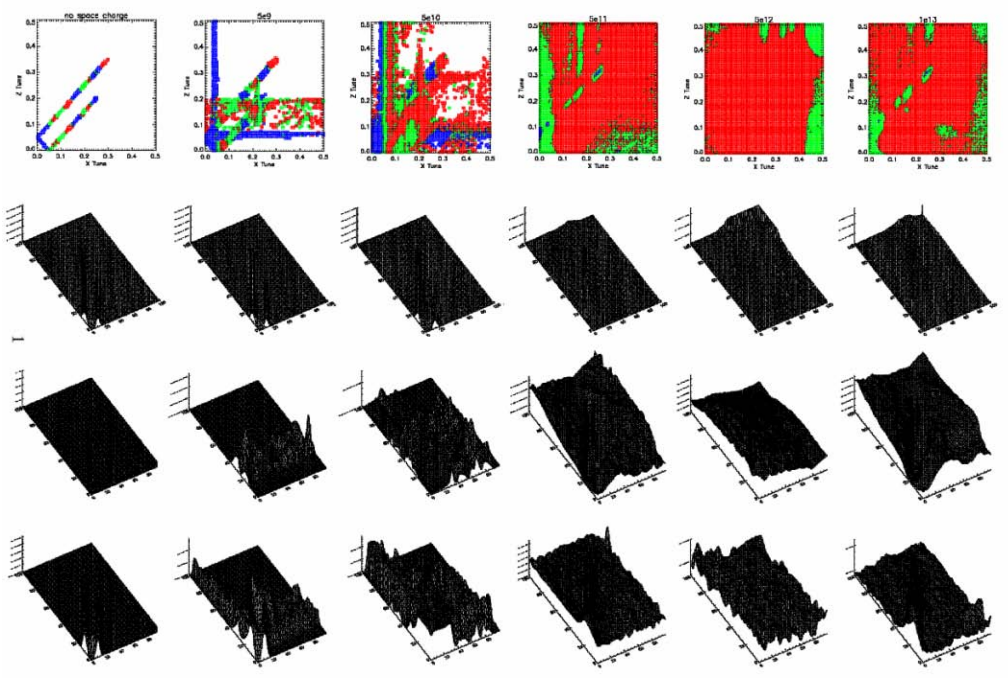

Figure 6: Top row: The average chaos of the bin red is chaotic, green is weakly chaotic, blue is regular. Second row: Shows the distribution in tune-space. Third row: The average space-charge kick. Fourth row: The average radius.

However, as the space charge is increased the early values of the tune do not follow the ramp line. In fact, in the $5x10^{12}$ regime the tunes do not hit the whole number resonance.

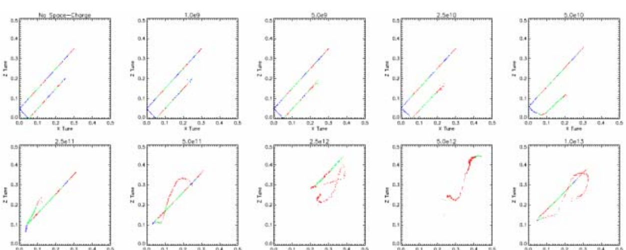

Figure 7: Average tunes for various space-charge values.

This lack of resonance could be the reason that the higher space charge regime does not exhibit the large emittance growth seen in the previous section.

Binned Tune Values

The average tune values shown give interesting results, though it must be remembered that they only show average values. The points they show could be where the majority of tunes are, or they could merely be showing an average of widespread values which is useless for comparison. Therefore, it is useful to see how these tunes are distributed throughout tune-space. Figure 6 shows the results of this process. The top row shows the average chaos in each bin in tune-space. The second row shows the distribution in tune-space, the third row shows the average space charge kick strength per tune bin. Finally, the last row shows the average radius per bin.

This series of plots shows that as space charge increases the number of available tunes increases, while the distribution does not leave the ramp line in significant amounts until the $5x10^{11}$ regime. The ramp lines which are prevalent in the later periods in most space charge regimes coincide with a decrease in space-charge as well as an increase in average radius.

CONCLUSIONS

- Space-charge increases the number of betatron-tunes that the particles comprising the beam can occupy.
- For many space-charge regimes the emittance growth mechanism involves the particles becoming trapped in expanding islands which pull them outwards before smearing them into a halo.
- For some values of space-charge, the tunes are not able to reach resonance. The emittance growth in these systems is far less severe.

REFERENCES

[1] Huang, x., et. Al., “Emittance Measurement and Modeling for the Fermilab Booster,” *Phys. Rev. S.T-A. And B.* **9**, 014202 (2006).

[2] Sideris, I. V., “A Measure of Orbital Stickiness and Chaos Strength,” *Physical Review E,* **73**, 066217 (2000).

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