

# EMITTANCE MEASUREMENTS OF TRAPPED ELECTRONS FROM A PLASMA WAKEFIELD ACCELERATOR\*

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## Abstract

Recent electron beam driven plasma wakefield accelerator experiments carried out at SLAC showed trapping of plasma electrons. These trapped electrons appeared on an energy spectrometer with smaller transverse size than the beam driving the wake. A connection is made between transverse size and emittance; due to the spectrometer's resolution, this connection allows for placing an upper limit on the trapped electron emittance. The upper limit for the lowest normalized emittance measured in the experiment is 1 mm-mrad.

## INTRODUCTION

A 42 GeV electron drive beam with  $1.8 \cdot 10^{10}$  electrons with the normalized emittances of  $\epsilon_{N,x} = 60$  mm-mrad and  $\epsilon_{N,y} = 7$  mm-mrad was focused to a transverse spot size of  $10 \mu\text{m}$ , compressed longitudinally to an r.m.s bunch length of about  $12 \mu\text{m}$ , and was sent into a neutral lithium vapor with a full width at half maximum of 85 cm [1]. Through most of the vapor's length there was a uniform lithium density,  $n_p$ , of  $2.7 \cdot 10^{23} \text{ m}^{-3}$ , but at its edges the density had a roll-off that occurs over several centimeters. As the drive beam traversed the vapor, it ionized the lithium in its vicinity [2], drove a wakefield [3], and trapped electrons from the plasma [4]. As the electrons traveled through the roll-off they experienced a diminishing focusing force from the plasma. By quantifying the effect of this diminishing focusing force, the mean square transverse size on a downstream energy spectrometer is shown to be proportional to the transverse emittance. The trapped electrons appeared on this diagnostic with smaller transverse size than the drive beam.

## ENERGY SPECTROMETER

The electron energy was measured by imaging displacements after a magnetic dipole. Let the direction the beam travels be denoted as  $z$ , the vertical direction denoted as  $y$ , and the horizontal direction denoted as  $x$ . The center of the dipole was 2.18 meters downstream of the end of the heat-pipe oven and had an integrated magnetic field,  $\int B \cdot dl$ , set to either 1.2 Tm or 0.27 Tm. This dipole deflected electrons in the negative  $y$  direction

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with an angle inversely proportional to the electron's momentum. A 1.5 cm thick air gap was placed 86 cm downstream of the dipole center. Cherenkov radiation emitted from the electrons in the air gap was imaged by a camera located 2.5 m from the air gap. By imaging the position of the electrons in the air gap, their momentum was measured.

There were two distinct classes of  $x$  widths measured on the spectrometer. The thinner of the two appeared as long thin streaks in energy. Fig. 1 shows an example of the two different widths. The thin streaks were determined to be from plasma electrons by measuring how their maximum energy scaled with wakefield amplitude [5].

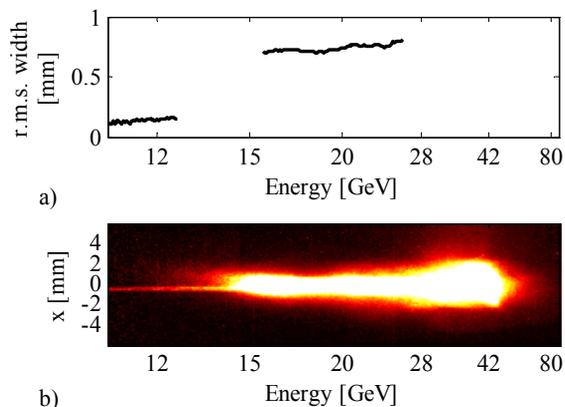


Figure 1: a) Root mean square width at energy spectrometer. b) Image from energy spectrometer viewed with a saturated color map. The thinnest part of the streak was determined to be trapped plasma electrons, while the wider part consists of beam electrons.

## FOCUSING

The relationship between the streak widths and their emittance can be found by examining the roll-off of the focusing forces in the plasma. The electric field from the drive beam was strong enough to completely expel electrons from its volume, which created an ion column in the plasma. Making substitutions into the equation of motion for an ultra relativistic electron oscillating through the beam axis of a cylindrically symmetric ion column [6] gives a differential equation that describes that particle evolution:

$$\ddot{x} + \frac{ne^2 x}{2\epsilon_0 \gamma mc^2} = 0, \quad (1)$$

where dots represent derivatives in  $z$ ,  $n$  is the ion density as a function of  $z$ ,  $m$  is the mass of an electron,  $e$  is the charge of a proton,  $\epsilon_0$  is the permittivity of free space, and  $\gamma$  is the electron's Lorentz factor.

The lithium vapor was created using a helium buffer gas in a heat-pipe oven. The density profile was found by measuring the temperature of the heat-pipe oven along  $z$  and then relating that to the density using the vapor pressure curve and the ideal gas law [7][8][9]. The density roll-off was fit to a Gaussian with an r.m.s width of  $\sigma = 3.97$  cm.

The relationship between emittance and beam size can be found by considering the evolution of the beam from inside the plasma to the spectrometer. The trapped electrons had energies up to 30 GeV. An electron of this energy completes a full radial oscillation from the ion column in 2.2 cm. Since the length of the roll-off is significant when compared to this oscillation length, the evolution of the electrons through the roll-off must be included in order to infer emittance from the beam size on downstream diagnostics.

The following equation represents propagation through the roll-off:

$$\ddot{x} + K \cdot \exp\left(-\frac{z^2}{2\sigma^2}\right) \cdot x = 0, \quad K = \frac{n_p e^2}{2\epsilon_0 \gamma m c^2}. \quad (2)$$

The propagation of electrons from the beginning of the roll-off,  $z = 0$ , to the position of the air gap,  $z = 3.04$  m, can be described by a transfer matrix,  $R$  [10]. The cosine-like term,  $R_{11} = C$ , and the sine-like term,  $R_{12} = S$ , determine the final position,  $x_f$ , from the initial position,

$x_0$ , and the initial angle,  $\dot{x}_0$ :

$$\langle x_f^2 \rangle = \langle x_0^2 \rangle C^2 + \langle x_0 \dot{x}_0 \rangle 2CS + \langle \dot{x}_0^2 \rangle S^2. \quad (3)$$

Eqn. 2 was solved numerically for  $C$  and  $S$  as a function of energy. Fig. 2 shows plots of  $C$  and  $S \cdot K^{1/2}$  as functions of electron energy. As is to be expected, the cosine-like and sine-like terms are  $\pi/2$  out of phase.

Given the oscillatory nature shown in Fig. 2 one might expect a modulation in streak widths as a function of energy; however, images taken from the experiment showed that the widths of the streaks didn't beat vs. energy. Because of the sinusoidal natures of  $C$  and  $S$ , the only way for the size not to beat vs. energy is for the  $C^2$  and  $S^2$  terms to have the same amplitude and for the amplitude of the  $C \cdot S$  term to be zero. Fig. 2 shows that  $C$  and  $S \cdot K^{1/2}$  have the same amplitude. In order for the beam size not to oscillate as a function of energy, the following criterion of the initial phase space must be satisfied:

$$\langle x_0 \dot{x}_0 \rangle = 0, \quad \langle \dot{x}_0^2 \rangle = K \langle x_0^2 \rangle. \quad (4)$$

This is the definition for the beam being matched to the plasma [8]. Therefore, the absence of beating vs. energy is evidence of the trapped electrons being matched to the plasma.

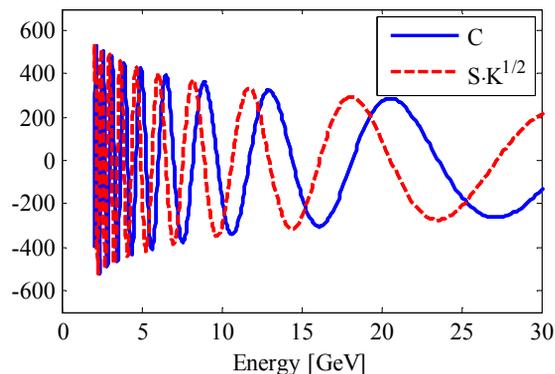


Figure 2: The cosine-like,  $C$ , and sine-like,  $S$ , terms of the transfer matrix from within the plasma to an energy spectrometer shown as functions of electron energy.

The conditions in Eqn. 4 can be substituted in Eqn. 3 to give the relationship between the size on the diagnostic and the size in the plasma:

$$\langle x_f^2 \rangle = \langle x_0^2 \rangle (C^2 + KS^2). \quad (5)$$

The conditions given in Eqn. 4 can be combined with the definition for emittance to give the relationship between the emittance and the beam size in the plasma. This can be inserted into Eqn. 5 to give the relationship between the normalized emittance,  $\epsilon_N$ , and the beam size on the energy spectrometer:

$$\epsilon_N = \frac{\gamma \sqrt{K} \langle x_f^2 \rangle}{(C^2 + KS^2)} \approx 0.0022 \text{ m}^{-1} \gamma^{1.06} \langle x_f^2 \rangle. \quad (6)$$

The factor in front of the beam size is fit well by a power law in  $\gamma$ . This fit was done for  $\gamma$  greater than 4,000 and less than 60,000, which was roughly the range of trapped electron energies measured with the spectrometer.

## SYSTEM RESOLUTION

The transverse size not beating versus energy is evidence of the trapped electrons being matched into the plasma. However, the sizes of the streaks are small enough that it is important to worry about whether the resolution limit of the system is producing the absence of beating. In addition, since emittance is determined from transverse size, a resolution limited system could result in artificially large measured emittances. Two terms make up the resolution limit of the system: the resolution of the camera and the resolution limit from multiple Coulomb scattering in the various elements traversed by the electrons from the plasma to the energy diagnostic.

Camera resolution was measured by imaging a back illuminated 5  $\mu\text{m}$  pinhole located 2.5 m from the camera. A mask was placed over the camera lens to simulate a Cherenkov ring. The mask was translated along the lens to simulate the fact that different energy electrons hit the camera lens at different positions. The finite thickness of the air gap was simulated by translating the camera towards and away from the pinhole. These measurements gave the camera resolution limit as a function of the position on the camera. Using the dispersion of the dipole magnet this can be translated to a resolution limit vs. energy.

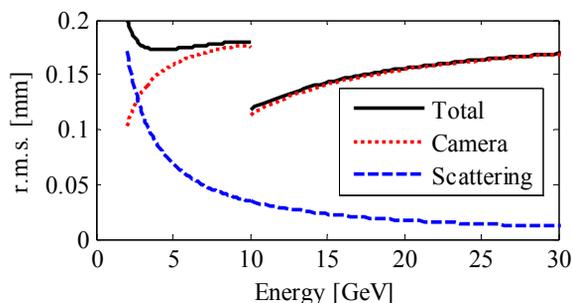


Figure 3: Resolution limited size on spectrometer due to multiple Coulomb scattering and the camera resolution. Above 10 GeV the resolution limit for  $|B \cdot dl| = 1.2 \text{ Tm}$ , and below 10 GeV the resolution limit for  $|B \cdot dl| = 0.27 \text{ Tm}$ .

There were several multiple Coulomb scatterers in between the plasma and the spectrometer, so even a beam that initially had zero emittance inside the plasma would acquire a finite size at our energy spectrometer. The resolution limit of the system from multiple Coulomb scattering comes from the calculated size at the spectrometer for a beam with initially zero divergence and size. The size was calculated from the angular scatter of each element and its distance to the spectrometer [11]. The following are the scatterers in the system along with their distances from the spectrometer and their thicknesses: At 193 cm, a 75  $\mu\text{m}$  Be foil; at 166 cm, another 75  $\mu\text{m}$  Be foil; at 4 cm, a 50  $\mu\text{m}$  Fe foil; and at 7.5 mm, a 420  $\mu\text{m}$  Si wafer. The total resolution of the system is the addition of the camera and multiple Coulomb scattering resolutions in quadrature. Fig. 3 shows the total resolution of the system for the two different integrated magnetic field settings.

The smallest streaks showed up with transverse sizes that were pushed right up against the resolution limit of the system: for these electrons only an upper limit can be quoted for the emittance. There were streaks that were not severely resolution limited, which didn't beat in size vs. energy. This shows that it is still appropriate to assume the trapped electrons were matched in the plasma.

## REMAINING ISSUES

There are a few remaining issues with this measurement. For the lowest energy trapped electrons on the spectrometer ( $\sim 2 \text{ GeV}$ ) the energy gain that occurs in the density roll-off can be significant, so using Eqn. 2 on

these electrons may not be appropriate. In addition, the system isn't exactly cylindrically symmetric, so the focusing forces will deviate slightly from that shown in Eqn. 1. It has also been shown that the fields from the trapped electrons may be strong enough to drive a wake in the helium buffer gas, which could change the propagation to the spectrometer [12]. Simulations will be performed to understand the errors introduced by these issues.

## CONCLUSION

A connection is made between a beam's transverse size on an energy spectrometer and its emittance, which allows for placing an upper limit on trapped electron emittance. The upper limit for the lowest normalized emittance measured in the experiment is 1mm-mrad.

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