NUMERICAL STUDY OF RF-FOCUSING USING FOKKER-PLANK EQUATION*

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Abstract

Based on the numerical solution of the Fokker-Plank equation we study the effect of longitudinal damping on the modulation of the bunch length in a storage ring with high RF voltage and momentum compaction.

INTRODUCTION

Reduction of the bunch length at the Interaction Point (IP) is very important for achieving very high luminosity in the electron-positron colliders with head-on collisions. A dynamic bunch length in the ring can help to avoid wake field effects [1]. We study the possibility of the rf focusing for the bunch shortening at the Interaction Region (IR) of a Super B-factory with low impedance cavities [2].

COMPUTER MODEL

In computer simulation we solve the Fokker-Plank equation of longitudinal motion of particles in a storage ring

$$\frac{\partial}{\partial t}\psi + \dot{x}\frac{\partial}{\partial x}\psi + \dot{p}\frac{\partial}{\partial p}\psi = \frac{\partial}{\partial p}\left\{ D_1 p\psi + D_2\frac{\partial}{\partial p}\psi \right\}$$

Phase distribution function $\psi = \psi(t,x,p)$ is a function of canonical coordinates: relative longitudinal position *x* and relative longitudinal momentum *p*. We assume that magnets are uniformly distributed in the ring

$$\dot{x} = \frac{1}{M_\rho}p \quad M_\rho = -\frac{E}{\alpha_\rho c} = const$$

E is a beam energy, α_ρ is a momentum compaction.

The radiation damping D_1 and the diffusion parameter D_2 have also uniform distribution in the ring. We may define them as a function of a radiation damping time τ_d and steady state energy spread σ_e

$$D_1 = \frac{2}{\tau_d} = const \quad D_2 = \frac{2}{\tau_d}\sigma_e^2 = const\ .$$

Time derivation of the longitudinal momentum is the force of the rf and the wake fields $F_{RF}(t,x)$ and $F_W(t,x)$

$$\dot{p} = F_{RF}(t,x) + F_W(t,x)$$

We assume that all cavities are concentrated at the rf region (RF). A bunch passes this place at the time $t = nT;\ n = 0,1,2...$, where T is the revolution period. RF region is opposite to IR, so the time when a bunch passes IR is $t = T/2 + nT;\ n = 0,1,2...$ .The rf force is

$$F_{RF}(t,x) = V_{rf}(x) \times \sum_n \delta(t - nT)$$

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The wake field force consists of the main three parts: the cavity wake fields V_{cav}^{wake}, acting naturally at RF region; the IR wake fields V_{IR}^{wake}, acting at the IR and the resistive-wall wake fields uniformly acting along the ring

$$F_W(t,x) = V_{cav}^{wake}(x) \times \sum_n \delta(t-nT) + V_{IR}^{wake}(x) \times \sum_n \delta(t - nT - \frac{T}{2}) + $$
$$+ V_{R-W}^{wake}(x)\frac{1}{T}$$

We include the IP wake fields because they are almost unavoidable due to the conjunction of the electron and the positron ring chambers.

For solving the Fokker-Plank equation, we use same computer algorithm, which was used for the stability study of the longitudinal motion in a ring [3], [4] and in simulations of electron cloud multipacting in a solenoidal magnetic field [5].

WAKE POTENTIALS

A low impedance cavity allows achieving higher threshold for the multi-bunch longitudinal instability. We studied different kinds of superconducting cavities, which can be used in a Super-B project [6]. In these simulations we use the wake potential of a cavity, which has shunt impedance *R/Q* of 12 Ohms at the main resonance frequency of 956 MHz. We used code “NOVO” [7] for calculating the wake potentials of a very short, 0.25 mm bunches in order to do dynamic simulations of a 1.8 mm bunch. The shape of a low impedance cavity and the wake potential of a 0.25 mm bunch are shown in Fig.1.

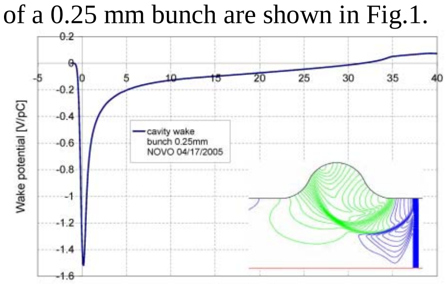

Figure 1: Wake potential of a 0.25 mm bunch and the shape of a 12 Ohm cavity.

It can be seen that the wake potential of this cavity has a capacitive character that may lead to a bunch shortening on contrary to resistive-wall wake potential (Fig. 2) that has an inductive character and produces an opposite effect - bunch lengthening.

For the IR we chose the PEP-II IR wake potential, assuming that the Super-B IR will be very similar to the PEP-II IR. This wake potential is shown in Fig. 3. IR wake fields are very strong fields. Beam energy loss of a bunch of 1.8 mm length in IR is comparable with the HOM energy loss in the twenty low impedance cavities.

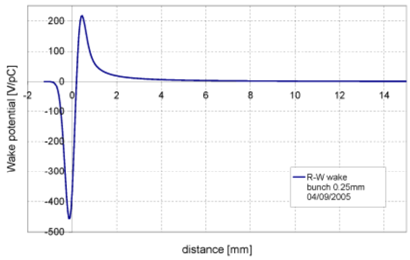

Figure 2: Resistive-wall wake potential of a 0.25mm bunch.

However, the main energy loss comes from the resistive-wall wake fields. The energy loss in a copper vacuum pipe (45 mm) is equivalent to the energy loss in the 45 cavities.

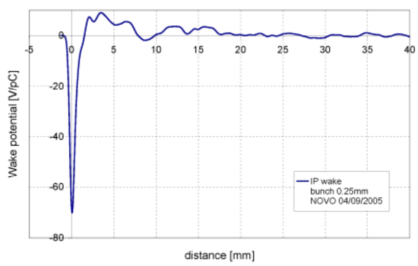

Figure 3: IR wake potential of a 0.25 mm bunch.

The cavity and the IR wake potentials of a 0.25 mm Gaussian bunch are used as primary wake functions for the calculating of the quasi Green functions. Wake potential of a bunch of an "arbitrary" shape is calculated as the convolution of the quasi Green function and the bunch density distribution. Details of using quasi Green functions can be found in the reference [3]. To calculate the resistive-wall wake potential we use analytical formulas [8].

PARAMETERS OF THE RING

The luminosity gain of the Super *B* Factory comes from the increase of the beam currents and shortening the bunch length. The beam energies are 8 GeV for the high-energy ring (HER) and 3.5 GeV for the low-energy ring (LER). In this paper we will study only LER longitudinal single bunch dynamics. The proposed total beam current for LER is 23 A, that corresponds to a bunch current of 3.3 mA or a bunch charge of 24.3 nC. The momentum compaction and the total rf voltage may verify to get the bunch length of 1.8 mm. We assume that bending radius ρ in all magnets is 13.5 m, so the energy loss per turn due to synchrotron radiation according to [9] is

$$U_{s.r.} = \frac{4\pi}{3} m_0 c^2 \left(\frac{E}{m_0 c^2}\right)^4 \frac{r_0}{\rho} = 0.97 \text{ MeV}$$

($m_0 c^2$ - electron rest mass, r_0 - a classical electron radius).

The damping time measured in number of turns is

$$\tau_d = \frac{E}{U_{s.r.}} = 3600$$

and the energy spread is approximately

$$\sigma_e = 0.28 \frac{E^2}{m_e c^2} \sqrt{\frac{\lambda_e}{\rho}} = 2.8 \text{ MeV}$$

(λ_e - electron Compton wavelength).

We also assume that one cavity can supply 1.5 MV to the beam, and then the number of cavities will be proportional to the total rf voltage.

RF FOCUSING

Keeping momentum compaction $\alpha \approx 0.001$ we need voltage of 66 MV (44 cavities) to make bunch length of 1.8 mm. This combination of parameters moves a synchrotron tune closer to the half-integer

$$Q_s = \sqrt{\frac{\alpha V_{rf} h}{2\pi E}} = 0.145$$

(harmonic number h=6984) and the dynamics effects become more noticeable. Simulation results are displayed in Fig. 4, which shows a bunch phase portrait (left plots) at the different ring locations. A bunch shape is shown in the right plots with wake potentials.

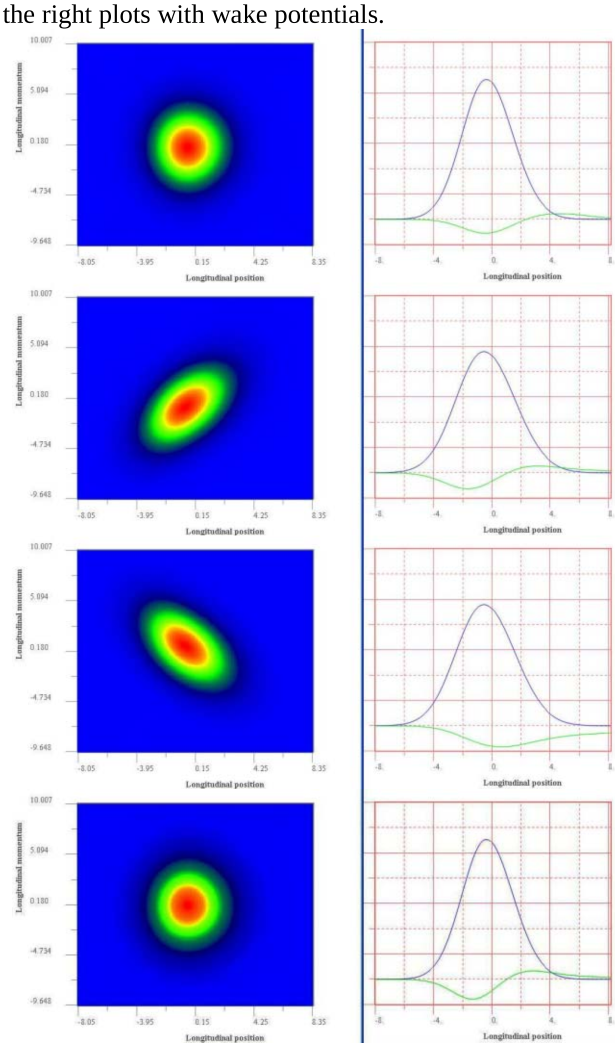

Figure 4: A bunch phase portrait (left plots) and a bunch shape (blue line) at different ring location: after the IR, before and after the RF, and before the IR. Green line shows the wake potential.

Fig. 5 shows relative position of a bunch at RF and IR through the several damping times.

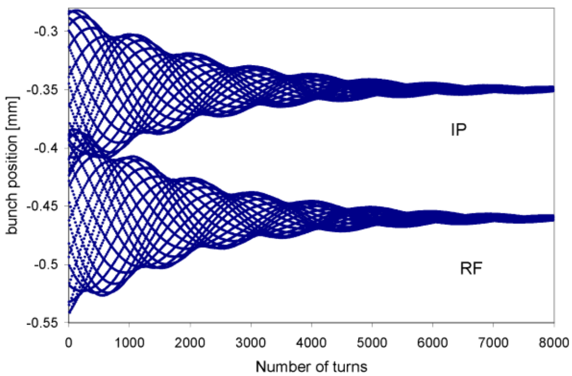

Figure 5: Relative bunch position at RF and IR.

A spectrum of synchrotron oscillations is shown in Fig. 6. The synchrotron tune is very close to the previously calculated value. The spectrum also contains the second and the third side band due to the bunch length variation.

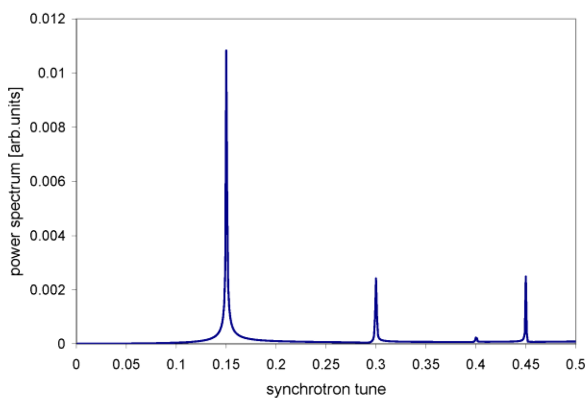

Figure 6: Synchrotron spectrum.

The bunch length is shown in Fig. 7. It approaches 1.85 mm in IR and 2.1 mm in RF. Here the RF focusing effect is only 13 %.

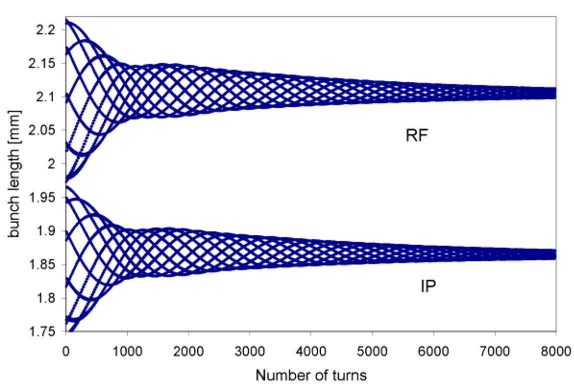

Figure 7: Bunch length in RF and IR.

Finally Fig. 8 shows the formation of the bunch energy spread. It approaches same value in RF and IR; however, it is larger than the initial number by 8 %.

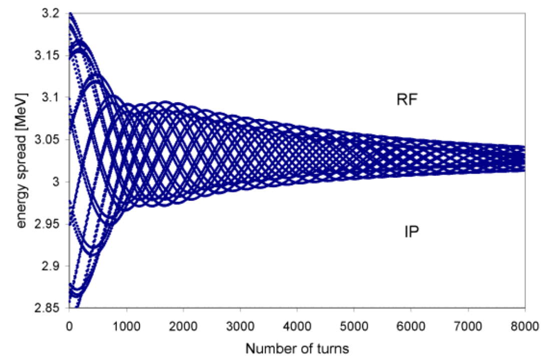

Figure 8: Bunch energy spread in RF and IR.

APPLICATION

As an application to other projects, we simulate the strong rf focusing experiment at DAPHNE [10] with some assumptions for the wake fields. We found very good agreement with a bunch compression number. Also we got very good agreement with multi-particle tracking simulation of rf focusing [1].

REFERENCES

[1] A.Gallo, P. Raimondi, M. Zobov "The Strong RF Focusing: a Possible Approach to Get Short Bunches at the IP", 30th Advanced ICFA Beam Dynamics Workshop on High Luminosity e+e- Collisions, October 13-16, 2003, Stanford, California

[2] J. Seeman, *et al.*, "Parameters of a SUPER-B-Factory design", PAC '05, Knoxville, Tennessee, May 2005, p. 2333.

[3] A.Novokhatski, "Computer Algorithm for Longitudinal Single Bunch Stability Study in a Storage Ring", SLAC-PUB-11251, May 2005.

[4] S.Heifets and A. Novokhatski "Coherent beam stability in low momentum compaction lattice", Phys. Rev. ST Accel. Beams, **9,** 044402 (2006)

[5] A.Novokhatski and J.Seeman, "Simulation of electron cloud multipacting in solenoidal magnetic field", Inst. Phys. Conf. Ser. No 175, Editor: M.Berz and K.Makino, Institute of Physics Publishing, Bristol, Philadelphia, 2002, p.249.

[6] A. Novokhatski, "Low R/Q Cavities for Super B-Factory," Super B-Factory Workshop, Hawaii, April, 2005.

[7] A. Novokhatski, "The Computer Code NOVO for the Calculation of Wake Potentials of the Very Short Ultra-relativistic Bunches", SLAC-PUB-11556, Dec 2005.

[8] M. Sands, K.Bane, The Short-Range Resistive Wall Wakefields, SLAC-PUB-95-7074, December 1995.

[9] M.Sands, "The Physics of Electron Storage Rings", SLAC Report No 121, November 1970.

[10] A.Gallo, *et al.*, "Proposal of a strong rf focusing experiment at DAFNE", EPAC 2004, Lucerne, Switzerland, June 2004, p.683.

