PRECISE CALCULATION OF TRAVELING-WAVE PERIODIC STRUCTURE

L. Wang, Z. Li and A. Seryi, SLAC, CA 94025, USA

Abstract

The effects of the round edge beam hole on the frequency and wake field are studied using variational method, which allows for rounded iris disk hole without any approximation in shape treatment. The frequency and wake field of accelerating mode and dipole mode are studied for different edge radius cases, including the flat edge shape that is often used to approximately represent the actual structure geometry. The edge hole shape has weak effect on the frequency, but much effect on the wake field. Our study shows that the amounts of wake fields are not precise enough with the assumption of the flat edge beam hole instead of round edge.

INTRODUCTION

The wake field of the accelerating structure can cause emittance growth and an energy distribution or transverse instabilities. The wake field must be controlled to reduce the emittance growth and the instability. Therefore, a high precise calculation of the frequency, field and wake is desired. Various numerical methods were developed for the analysis of the higher order modes in a cell. The codes KN7C [1] and TRANSVRS [2] use field matching method and they can calculate the longitudinal and transverse modes with any phase advance per period, though imposing the approximation: flat beam hole surface parallel to the beam axis instead of round edge. Because of the assumption, the calculated wake field is not accurate enough [3]. Other numerical programs, such as URMEL [4], ABCI [5] and MAFIA[6], represent the structure by filling with a mesh so that these can calculate the modes in cells of any shape. However, they are difficult to estimate the field with exactly the same phase velocity to the beam.

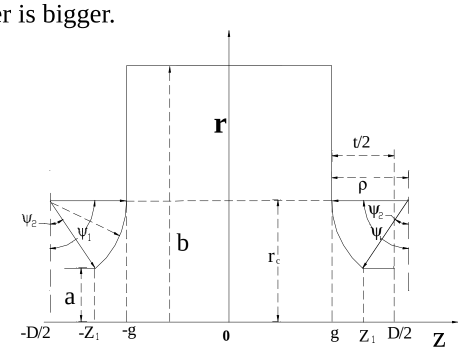
Here a variational method [7] is applied to study the x-band disc-loaded structure. The variational method can describe the structure geometry without any approximation. Therefore it is a better way in the geometry treatment than various mesh-based methods. This method can be used to calculate both monopole modes and all azimuthally varying modes with any phase shift per cell, which means that all kinds of field in the disk-loaded structure can be calculated. The fields in the variational method are given as a series expansion, which derives from the Maxwell equations. The calculated fields converge on the actual ones very fast when the number of terms in the series increases. Also, it uses less memory than the mesh-based method. The accuracy depends on the number of terms used. The frequency accuracy is around 10^{-7} for the acceleration mode with a number of terms more than 55.

ACCELERATING MODE

The accelerator structure with which we are concerned is a conventional disk-loaded cylindrical waveguide as shown in Figure 1. This paper studies the geometry with rounding curvature center just at the middle plane of the disk (Z_1=D/2 and $\psi_1=90^0$). Figure 2 shows the various possible dimensions of the disc-loaded structures with the same accelerating frequency 11.424*GHz* and a phase advance 120º per cell. The period of the cell is 8.8*mm*. The *b* can be fitted as a quadratic function of *a* as shown Figure 2. A bigger aperture *a* corresponds to a bigger radius *b*. A thinker disc (big *t*) also corresponds to a bigger radius *b*, but its effect on the radius *b* is smaller comparing with the effect of the aperture *a*.

Since we are able to calculate a disk-loaded structure with any rounding in its beam hole, it is interesting to see the difference of the accelerating mode for cells with hole-edge rounding and flat edge.

Figure 3 gives the frequencies of cells with flat edge for different disk thickness and disk-hole radius. There are the same *a*, *b* and *t* as those shown in Figure 2. Here, the aperture *a* is the distance from the center of the beam pipe to the bottom of the round disk. The frequencies are the modes with phase shift 120º per cell. The 2π/3 mode frequency approximately decreases linearly with disk-hole diameter for a given disk thickness. This is because the electric field around the disk-hole becomes weaker while the magnetic field becomes larger when disk-hole diameter is bigger.



D = disk spacing ρ *= edge radius* r_c *= radius of interface*
t = 2(D/2 − g) = disk thickness
2a = disk-hole diameter 2b = inside cylinder diameter
$2\delta = 2(D/2 - Z_1)$ *= flat land in disk hole*

Figure 1: Cross-section of disk-loaded waveguide.

In a cylindrically symmetric structure, the longitudinal wake-field at distance *s* behind the driving bunch is calculated in the frequency domain using [8, 9]

$$W_{\|m}(s) = \sum_n 2k_{mn}^{\|} e^{-\frac{\omega_{mn} s}{2Q_{mn} c}} \cos \frac{\omega_{mn}}{c} s \,, \quad (1)$$

with

email address: wanglf@slac.stanford.edu

$$k_{mn}^{\parallel} = \frac{1}{a^{2m}} \frac{|V_{mn}|^2}{4U_{mn}}, \quad V = \int E_z(z, r=a) e^{ikz} dz \,, \quad (2)$$

where V_{mn} is the stored energy, ω_{mn} the frequency of a resonant mode n, Q_{mn} the Q-value of the mode and $k_{mn}^{\parallel}$ the longitudinal loss factor per unit length of the mode calculated at r=a. All of the parameters above are those of the traveling wave.

Figure 4 shows the loss factors of the structure with flat and round edge. The loss factor approximately linearly decreases with the aperture. The structure with a flat edge has a bigger loss factor with a maximum difference 4%. The difference becomes smaller when the aperture a becomes larger.

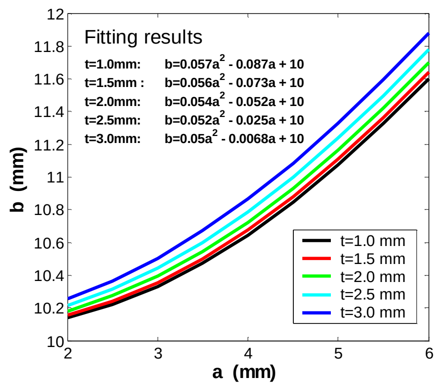


Figure 2: Relation between the aperture a and the radius b of a disc-load structure with round beam hole to maintain the accelerating mode frequency and phase constant.

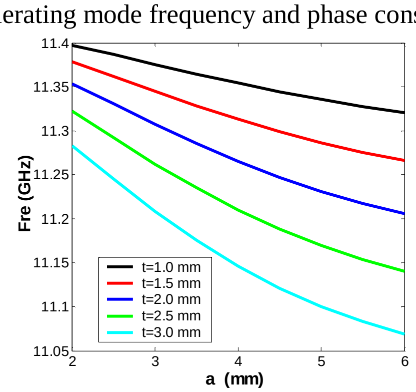


Figure 3: Frequency of 2π/3 mode with flat edge approximation.

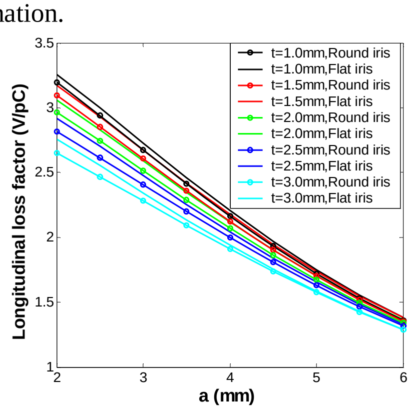


Figure 4: Comparison of the longitudinal loss factor for the structures with round edge and flat edge.

DIPOLE MODE

In a cylindrically symmetric structure, the transverse wake-field at distance s behind the driving bunch is [8, 9]

$$W_{\perp m}(s) = \sum_n 2k_{mn}^{\perp} e^{-\frac{\omega_{mn} s}{2Q_{mn} c}} \sin \frac{\omega_{mn}}{c} s \,, \quad (3)$$

with

$$k_{mn}^{\perp} = \frac{cm}{a^{2m}\omega_{mn}} \frac{|V_{mn}|^2}{4U_{mn}}, \quad V = \int E_z(z, r=a) e^{ikz} dz \,, \,(4)$$

Similar to monopole mode, the wake filed with exactly the same phase velocity to the beam is calculated. The HEM11 mode is usually assumed to be the most dangerous mode. Figure 5 and 6 shows the frequency and loss factor of the HEM11 mode. Both the frequency and loss factor show a stronger nonlinearity comparing with the accelerating mode.

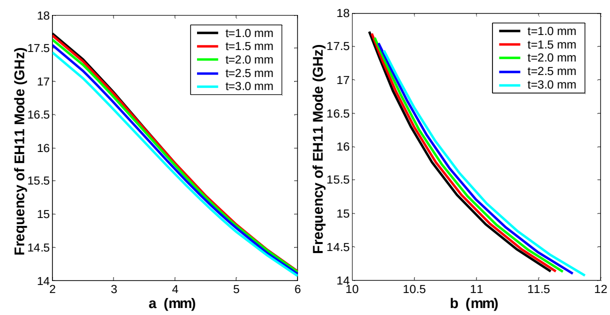


Figure 5: Frequency of the HEM11 mode.

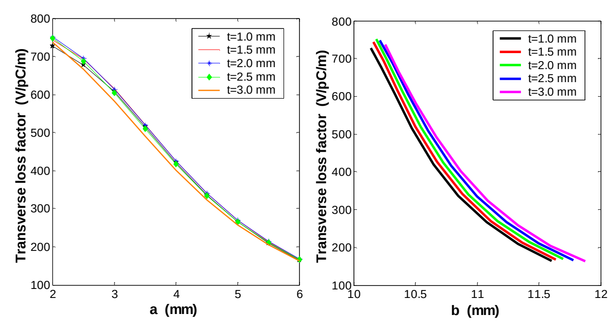


Figure 6: Loss factor of the HEM11 mode.

The synchronous phases, frequencies and transverse loss factors per cell for structures with flat beam-hole and round beam-hole are shown in Table 1. The structure has the following parameters: t=1mm, D=8.7474mm, b=10.1412mm, a=2mm, ρ=0.5mm,δ=0mm. We found in this case that the rounding of the beam-hole edge change the loss factor of the dangerous HEM16 mode by more than 15%, although it has little effect on the mode frequency. The wake of the 8 HEM modes in Table 1 is shown in Figure 7.

Figure 8 and 9 show the electric fields of the HEM11 and HEM16 mode. Both the real part and image part of the field are not zero. When URMELT code is used to calculated the wake, it can model the structure very close to the exact dimension with an assumption that the phase shift per cell is a simple rational number times π. This assumption can cause large error. For instance, if the phase shift per cell is 360^0 for the HEM16 mode shown in Table 1 (round beam-hole case), then the frequency and loss factor become 36.294GHz and 0.86×10^{17}V/pC/m^2, respectively. Although this approximation doesn't change its frequency much, it causes 17.4% decrease of the loss factor.

Table 1: Frequency, phase and loss factors of the HEM modes of the structures with flat and rounded beam holes

Mode	Phase (degree)		Frequency (GHz)		k (10^{17}V/C/m^2)	
	flat edge	round hole	flat edge	round hole	flat edge	Round hole
HEM11	186.8011	186.1853	17.78347	17.72507	0.843451	0.834845
HEM12	220.8614	220.3722	21.02584	20.97909	0.009015	0.011052
HEM13	271.6385	270.2597	25.86010	25.72866	0.533224	0.531805
HEM14	326.6087	325.2278	31.10338	30.98147	0.045503	0.043849
HEM15	344.9131	344.9845	32.83308	32.83792	0.078118	0.064719
HEM16	383.1862	381.2430	36.48405	36.29512	1.166120	1.013410
HEM17	410.8394	409.3971	39.10258	38.98092	0.001775	0.003737
HEM18	436.3155	435.1684	41.52120	41.39898	0.102232	0.075065

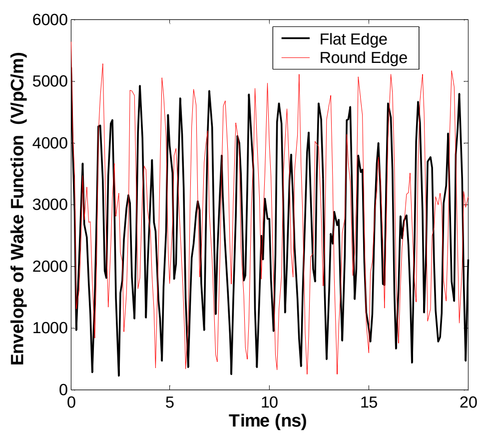


Figure 7: Transverse Wakefield of a structure with flat edge and round beam hole.

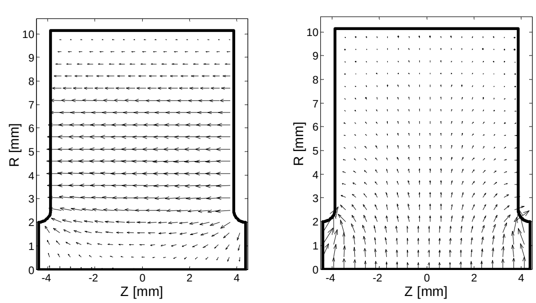


Figure 8: The electric field of HEM11 mode: real part (left) and image part (right).

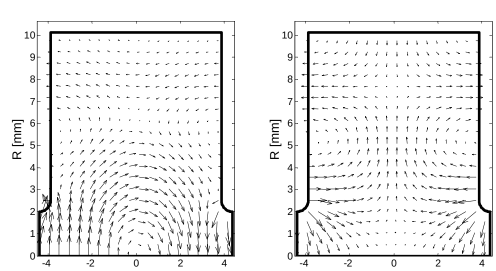


Figure 9: The electric field of HEM16 mode: real part (left) and image part (right).

SUMMARY

The exact synchrotron modes of the disc-loaded structure are precisely calculated without any geometry approximation. The approximation in the shape and phase shift causes a large error of the wake.

ACKNOWLEDGEMENTS

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REFERENCES

[1] E. Keil, Nucl. Instrm. Methods, 100, 419, 1972.
[2] K. L. F. Bane and B. Zotter, Proceedings of the XI-th International Conference on High Energy Accelerator, Geneva, 1980.
[3] M.TAKAO, T.HIGO and K. TAKATA, Evaluation of Wake Fields of Disk Loaded Structure for Japan Linear Collider, KEK Report 91-4, 1991.
[4] T. Weiland, Nucl. Instrm. Methods, 216, 329, 1983.
[5] Y. Chin, LBL-35258,CBP Note-069,CERN SL/94-02 (AP)
[6] The MAFIA Collaboration, MAFIA.
[7] L. Wang,Y. Lin and T. Higo, Nucl. Instr. and Methods A Vol.481, 95 (2002)
[8] A. Chao, Physics of Collective Beam Instabilities in High Energy Accelerators, Wiley-Interscience Publication, John Wiley & Sons, Inc., 1992.
[9] K. L. F. Bane, P. B. Wilson and T. Weiland, Wake fields and wake field acceleration, AIP Conf. Proc. 127, 875, 1985.

