THE POINCARÈ MAP, LIE GENERATOR, NONLINEAR INVARIANT, PARAMETER DEPENDANCE, AND DYNAMIC APERTURE FOR RINGS*

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Abstract

In earlier work related to the NSLS-II project we have outlined a control theory approach for the dynamic aperture problem. In particular, an algorithm for the joint optimization of the Lie generator and the working point for the Poincarè map. This time we report on how the Lie generator provides guidelines on acceptable magnitudes for e.g. the intrinsic nonlinear effects from insertion devices, and the nonlinear pseudo-invariant from the map normal form can be used to optimize the dynamic aperture. We also show how a polymorphic beam line class can be used to study the parameter dependence and rank conditions for control of optics and dynamic aperture.

INTRODUCTION

In earlier work [1] we have shown that joint optimization of the Lie generator of the Poincarè map and the working point is an effective approach to control the dynamic aperture (DA). The method is non-perturbative[1] in the sense that there is no need to bring the map into normal form, i.e., to integrate the equations of motion by perturbation theory. In particular, the corresponding Taylor map can be transformed into the factored form [2]

$$M = \ldots e^{:h_4:} e^{:h_3:} M_{\text{linear}} = A^{-1} \ldots e^{:h_4:} e^{:h_3:} RA$$

If the Lie generators are evaluated for a suitable amplitude, there is essentially only one free parameter, i.e., the weight on the generators driving resonances vs. tune shifts. In fact, it turns out that reasonable solutions for different lattices can be obtained without adjusting individual weights.

NONLINEAR DYNAMICS GUIDELINES: THE LIE GENERATOR

Having established a baseline lattice with adequate DA, the residual Lie generator from the sextupole scheme can then be used to provide insight into and guidelines for acceptable magnitudes of nonlinear terms from other sources. For example, to optimize the parameter choice for Insertion Devices (IDs), see Table 1 [3]. In particular, to view the IDs as an integral part of the system; rather than a (nonlinear) perturbation of the (linear) optics.

[1] The Taylor map has a finite radius of convergence.

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Table 1: Lie Generators from the Sextupole Scheme and a Single Insertion Device for Various Types

Lie Generator	Effect	Sextupole Scheme	DW	CPMU	SCU
h_{00220}	$\partial\nu_y/\partial J_y$	606.9	1,089.2	1,102.6	1,259.6
h_{00220}	$2\nu_y$	76.2	52.3	6.9	39.5
h_{00400}	$4\nu_y$	46.6	58.7	13.7	11.3

CELL TUNE VS. RESONANCES: THE NONLINEAR INVARIANT

Our earlier approach for the DA aperture optimization did not explicitly take into account the cell tune's closeness to the betatron resonances driven by the Lie generator. Therefore, we have evaluated how minimizing the coefficients for the nonlinear pseudo-invariant

$$K(\bar{J},\bar{\phi}) = e^{:g(\bar{J},\bar{\phi}):} k(\bar{J}) = \text{cst.}$$

where $[\bar{J},\bar{\phi}]$ are the action-angle variables for the linear optics, $:g(\bar{J},\bar{\phi}):$ a canonical transformation to Floquet space, and $k(\bar{J})$ a nonlinear rotation, compares in terms of the resulting DA. It is obtained from the map normal form [4]

$$M = A^{-1} \ldots e^{:h_4:} e^{:h_3:} RA \rightarrow A^{-1} e^{:g(\bar{J},\bar{\phi}):} e^{:k(\bar{J}):} e^{:-g(\bar{J},\bar{\phi}):} A$$

In other words, a perturbative approach; justified by the fact that the objective is to maximize the phase space region around the fixed point with regular motion.

In particular, the terms are of the form

$$K_i(\bar{J},\bar{\phi}) \propto \frac{J_x^{a_x} J_y^{b_y}}{\sin(\pi(n_x\nu_x + n_y\nu_y))}$$

where $[n_x, n_y]$ are integers, $[a,b]$ rational numbers, and $[\nu_x,\nu_y]$ the cell tune. Note the resonance denominator. Intuitively, we expect the resulting dynamics to be more regular since reducing the nonlinear part of the pseudo-invariant brings it closer to the linear invariant (action). To summarize, instead of controlling the dynamics by reducing the nonlinear terms in the equations of motion (Lie generator), we are now reducing the nonlinear terms in the corresponding (perturbative) solutions.

To evaluate the relevance of the resulting pseudo-invariant, we first compute it as a function of $[\bar{J},\bar{\phi}]$ to 5th order[2] for a sample lattice and then evaluate it on a turn-by-turn basis for different betatron amplitudes by tracking, see Figs. 1-3. We conclude that it captures the

[2] By truncated power series algebra (TPSA) [5] and a polymorphic beam line class [6].

dynamics quite well up to an amplitude of ~10mm and starts to break down at ~15mm, i.e., at about half the DA.

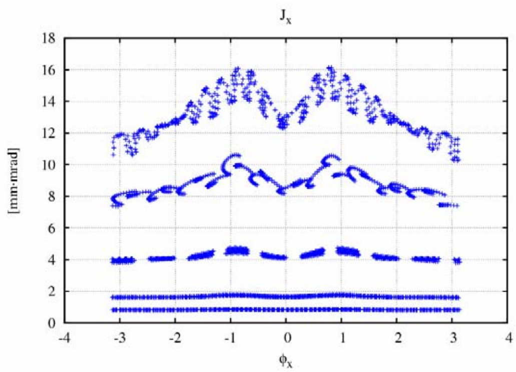

Figure 1: Variation of the Horizontal Linear Action with Betatron Phase (on a turn-by-turn basis).

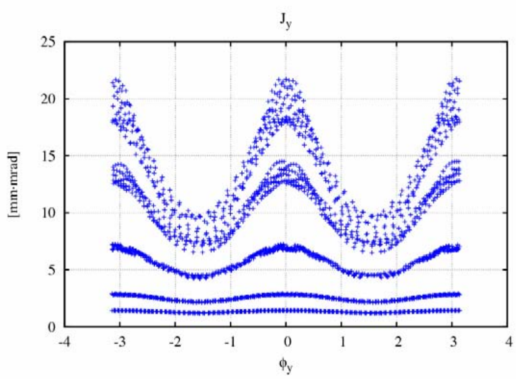

Figure 2: Variation of the Vertical Linear Action with Betatron Phase (on a turn-by-turn basis).

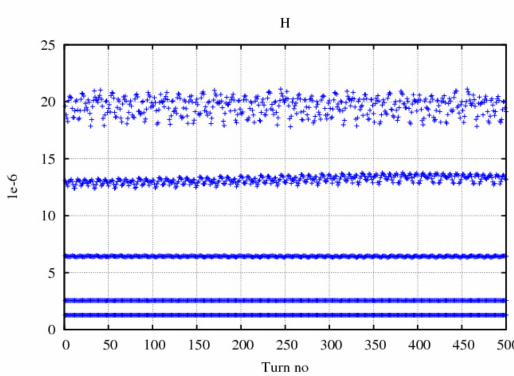

Figure 3: Variation of the 5th Order Pseudo-Invariant on a Turn-by-Turn Basis.

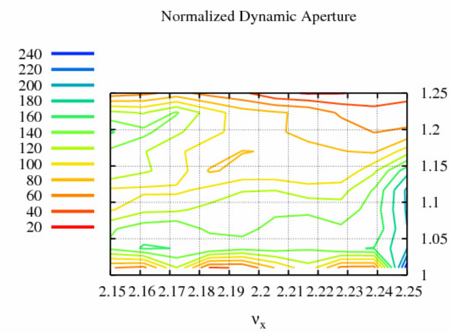

Figure 4: Tune Scan of Optimized DA (normalized with $\sqrt{\beta_x \beta_y}$, using Lie Generator).

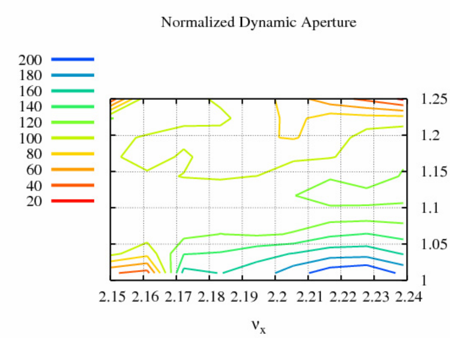

Figure 5: Tune Scan of Optimized DA (normalized with $\sqrt{\beta_x \beta_y}$, using pseudo-invariant).

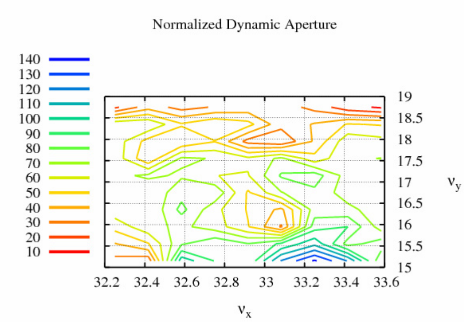

Figure 6: Tune Scan of Resulting DA for a Full Lattice with Magnet Misalignments (normalized with $\sqrt{\beta_x \beta_y}$).

The basic algorithm for the DA-optimization is as before[3] and the result is shown in Figs. 4-5. Indeed, the $\mathrm{DA}\big/\sqrt{\beta_x \beta_y} = \mathrm{cst.}$ surfaces are broadened in the latter. The DA is shown in Figure 7. Moreover, the choice of working point can then refined by re-computing the DA (by tracking) for a full lattice with e.g. magnet misalignment errors for the previously determined quadrupole/sextupole tunes, see Figure 6.

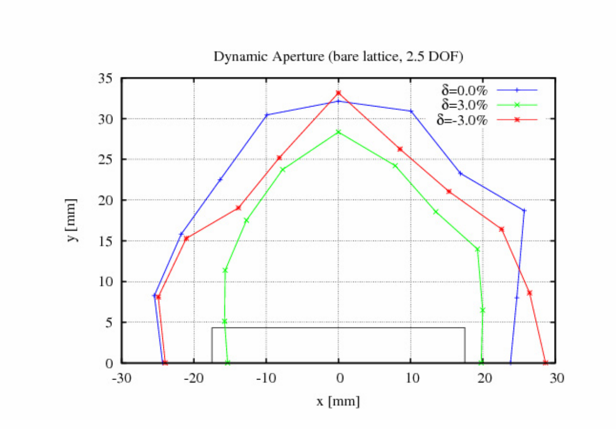

Figure 7: Dynamic Aperture for $\delta = 0$ and $\pm 3\%$.

PARAMETER DEPENDANCE: RANK CONDITIONS

The numerical/analytical model/framework, which is a prerequisite for this work, also enables one to compute the parameter dependence of any global property of the lattice, e.g. the multipole strength. In fact, the DA optimization algorithm uses this to compute the Jacobian for the nonlinear systems that must be solved to:

- adjust the cell tune in a controlled manner (quadrupoles),
- and to minimize the Lie generator[4] or pseudo-invariant (sextupoles)

=> straightforward to implement e.g. a gradient search.

Another use of the Jacobian is to check the rank conditions of the governing equations for the control of a (linearized) realistic system; by singular value decomposition.

For example, even though the CDR lattice [3] had quadruplets in the short and long straight sections, we experienced convergence issues with the (local) control of the optics perturbations from the IDs [9]. In theory, this is modeled by two decoupled 4×4 systems. Intuitively, we concluded that this originated from degeneracy. Indeed, removing one of the quadrupoles in each quadruplet from the algorithm led to convergence. Later, this was confirmed by rank analysis of the Jacobian for general optics tuning with the short and long matching sections (10 constraints) for a modified CDR lattice, i.e., with triplets in the short straights, see Table 2. The approach can be generalized to analyze the entire super cell.

[3] Note, the use of an artificial thin phase-space rotation is to simplistic since it ignores the change of optics for the sextupoles associated with a realistic cell tune adjustment. In fact, this is precisely what led us to implement the tune scan approach outlined in [7].

[4] This is also how we diagnosed that the rank of the 9×9 sextupole response matrix for the first order terms in the SLS sextupole scheme is only 8, due to optics [8]. In particular, control of h_{20001} requires adjusting the phase advance over lattice sections.

Table 2: Rank Conditions for Control of the Impact of IDs on the Linear Optics.

	Gradients	
	Quadruplets	Triplets in Short Straights
Singular Values	10×8	10×7
	3.5E+00	3.3E+00
	2.1E+00	1.9E+00
	1.3E+00	1.0E+00
	4.0E-02	3.5E-02
	1.1E-02	5.9E-03
	2.7E-04	3.6E-04
	3.1E-05	3.4E-05
	3.8E-06	

CONCLUSION

We have outlined how, after having established a baseline lattice with adequate DA, the Lie generator can be used to obtain insight into and provide guidelines for the acceptable magnitude of nonlinear terms from other sources e.g. insertion devices. We have also evaluated how well the nonlinear pseudo-invariant captures the dynamics and how to use it to control the dynamic aperture. Finally, we have shown how a polymorphic beam line class can be used to study the parameter dependence and rank conditions of the governing equations for control of optics and dynamic aperture.

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